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Direct Measurement of the Molecular Attraction of Solid Bodies. I. Statement of the Problem and Method of Measuring Forces By Using Negative Feedback

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The importance both in principle and practice of the study of the molecular attraction of macroscopic objects, as a function of the distance between their surfaces, is pointed out. The method of calculating the interaction, based on the assumption of additivity of the London forces, is criticized. The physical basis and the consequences of E. M. Lifshitz's macroscopic theory of the attraction of macroscopic bodies is briefly presented. We discuss the main difficulaties of the experimental measurement of the molecular attraction of solids, as a function of their distance of separation (gap width), and their elimination in the microbalance with negative feedback which we have constructed. Special attention was given to the complete elimination of surface charges by ionizing the air in the neighborhood of the bodies.

1. INTRODUCTION

IN addition to the usual valence forces, which have relatively short range and practically vanish for interatomic distances of several Angstrom units, there are forces of attraction between any atoms or molecules, whose magnitude decreases much more slowly with distance. Such (molecular) forces are the cause of fundamental phenomena in molecular physics and physical chemistry: surface tension, capillarity, capillary condensation, physical adsorption, and other phenomena. Molecular forces determine most of the properties of liquids--their viscosity, heat of vaporization and mutual solubility.

The existence of attractive forces between atoms and molecules naturally gives rise to similar forces of "molecular attraction" between two macroscopic bodies which are brought very close to one another. The existence of such forces between colloidal particles, and their role in coagulation was suggested in Ref. 1, and was considered in the first quantitative theory of the stability of colloids[2,3], together with the repulsive forces between the diffuse double layers of approaching particles. However, some authors[4] believe that the coagulation of colloidal systems can be explained, without assuming the existence of long range forces, in terms of the van der Waals attraction between the molecules which make up the colloidal particles. Thus the problem of calculating the force or the energy of molecular interaction of macroscopic particles as a function of their separation is one of the basic problems in the theory of the stability and coagulation of colloids.

Despite the great theoretical and practical importance of molecular forces, the study of their nature and the development of a theory began relatively recently. The first correct picture of the nature of molecular forces was proposed by P. N. Lebedev, and also by B. B. Golitsyn. In 1894, Lebedev[5], in discussing the ponderomotive action of waves on resonators, wrote: " In Hertz's researches, in the interpretation of light vibrations as electromagnetic processes, there is hidden another problem which has previously not been considered--the problem of the sources of radiation of those processes which occur in a molecular vibrator when it gives off light energy into the surrounding space; this problem leads us, on the one hand, into the field of spectrum analysis, while on the other hand it leads us quite unexpectedly to one of the most complicated problems of contemporary physics, the study of molecular forces. This latter point follows from the following considerations: From the standpoint of the electromagnetic theory of light we must admit that there are ponderomotive forces between two radiating molecules, just as there are between two vibrators in which electromagnetic oscillations are excited. They are the consequence of the electrodynamic interactions of variable electrical currents in the molecules (according to Ampere's law) or of varying charges in them (according to Coulomb's law). We must therefore admit that there are intermolecular forces which are closely related in origin to the processes of emission of light."

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* F. B. Leib participated in the development of the technique.

"Of greatest interest and difficulty, because of its complexity, is the situation which exists in a physical object in which many molecules interact simultaneously, and their vibrations, because of the close proximity of the molecules are not independent of one another. It it ever becomes possible to solve this problem, then we shall be able, by using the data of spectral analysis, to calculate in advance the magnitudes of the intermolecular forces which are due to the coupled light emission of the molecules, deduce their temperature dependence and, by comparing these calculated quantities with experimental observations, solve the fundamental problem of molecular physics--whether all the so-called 'molecular forces' reduce to the previously known mechanical action of light radiation mentioned above, i.e., to electromagnetic forces, or whether some other forces of hitherto unknown origin are involved."

The first quantitative theories of molecular forces could, however, be developed only after the structure of atoms and molecules was clarified. One of them is the completely general quantum-mechanical London theory of molecular forces. For the case where the intermolecular distance is large compared to molecular diameters, i.e., mainly in gases, the theory gives forces which decrease with the inverse seventh power of the distance between the molecules. However, in solids, where the molecules do not rotate freely, forces can exist which drop off much more slowly with distance. At the same time, for the small intermolecular distances which are characteristic of condensed bodies, there may be a strong influence of forces such as those associated with quadrupole moments, which decrease more rapidly with distance. Considering all this, the existing attempts at quantitative verification of the theory of molecular forces must be regraded as being incapable in principle of giving sufficiently accurate and convincing results. Actually, all these attempts are based on the comparison with theory of effects of an integral nature, in which those terms predominate which depend on short range interaction, i.e., on interactions of molecules at distances of the same order as their radii. Among these phenomena are the value of the constant a in van der Waal's equation, the heats of sublimation and vaporization and the energies of adsorption and wetting. In this case accurate comparison with theory is made difficult by the fact that for such small separations there is, strictly speaking, no single theory of molecular forces which is applicable. In addition, the result depends on a superposition of forces of different character (for example, quadupole forces) some of which depend on the unknown orientations of the molecules and the asymmetry of their fields of force.

The current theories of molecular forces could be tested much more severely if their action were measured at distances large compared with molecular diameters. Of special interest from this point of view would be measurements of the resultant molecular attraction of two solid bodies separated by a gap which is many molecular diameters in width, i.e., measurements analogous to the experiments of Cavendish with gravitational forces and the experiments of Coulomb with the forces acting between electrical charges. In contrast to measurements of the forces of adhesion[6], such experiments would enable us to test existing theories of molecular interaction (provided the theory is supplemented by a method for summing the interactions over molecules constituting the macroscopic body), at distances for which only one type of force remains, and the corresponding limitations of the applicability of the theory are removed. In the first place, it might be possible to check the quantum mechanical theory of dispersion forces and thus establish a picture of their true origin.

At the same time it is evident that such measurements have great intrinsic interest, expecially if we consider on the one hand, the broad development in past years of applications of molecular long range forces to fundamental problems of colloid chemistry and surface phenomena, and on the other hand the doubts as to the existence of such long range forces which have been expressed, for example, by Langmuir. Finally, the measurement of molecular attraction of macroscopic bodies is also important for testing the methods of summation of molecular interactions (cf. below).

As far as we know, until recently (1951(no such experiments, even of a qualitative nature, had been published. The reason for this is undoubtedly the very great experimental difficulties, which will be discussed in detail later.

The problem of the present work includes the development of a method for direct measurements of molecular attraction of solid bodies, and the application of the results to the testing of existing theories and to the problem of coagulation of aerosols and colloids. At the same time, the paper contains an analysis of the status of the question of the existence and magnitude of molecular forces for macroscopic objects.

II. DESCRIPTION AND CRITIQUE OF CURRENT THEORIES OF MOLECULAR FORCES BETWEEN MICRO- AND MACROSCOPIC OBJECTS

According to the calculations of London[7], the interaction energy of atoms or molecules, at distances large compared to their dimensions, is inversely proportional to the sixth power of their separation. London's law can be expressed as follows:

$$U_{\text{disp}} = C / r^6, \tag{1}$$

where U is the energy of molecular interaction of the two particles when separated by a distance r, and C is a positive number which has a fixed value for each type of atom. C is calculated using the matrix elements of the electric moments of the two atoms. Since the force is given by $F = -dU/dr$, the van der Waals forces of molecular attraction vary inversely as the seventh power of the distance between molecules.

London's theory has limited applicability. London's calculation loses its validity not only at small atomic separations, where the wave functions of the atoms overlap, but also at large distances where one must include the effects of electromagnetic retardation, which are associated with the finite velocity, c, of propagation of electromagnetic waves. The retardation was taken into account by Casimir and Polder[8], using quantum electrodynamics. They used the same perturbation method as London, but thier perturbation operator included, in addition to the electrostatic attraction, the action of the radiation field of one atom on the other, and vice versa. Casimir and Polder express the effect of the retardation by means of a monotonically decreasing correction factor, which is equal to unity for small distances and is proportional to r^{-1} for large distances. Thus their theory includes that of London as a special case.

According to the theory of Casimir and Polder, for $r \gg \lambda_i$, where λ_i are the wavelengths of absorption (emission) of the atom, the interaction energy of two atoms having static polarizability α is

$$U = -(23/4\pi)\,\hbar c\alpha^2 / r^7, \tag{2}$$

or

$$U = -C_1 / r^7, \quad \text{where} \quad C_1 = 251 e^2\alpha^2.$$

Here $\hbar$, c and e have their usual meanings. In this limiting case, the attractive force $(-dU/dr)$ between two atoms varies with distance as r^{-8}.

Let us now consider the resultant molecular interaction between macroscopic objects. Usually one assumes the additivity of the London forces, and calculates the attraction between objects composed of large numbers of molecules by taking their energy of attraction to be equal to the sum of the energies of attraction between all pairs of molecules constituting the bodies. Thus, de Boer[9] and Hamaker[10] find the interaction of two bodies, containing q molecules per unit volume, by intergrating the elementary interactions which satisfy London's law. Hamaker derives formulas for the energy and force of attraction between two spheres, a sphere and an infinite plane, and between two parallel infinite planes. If the shortest distance between the surfaces is much smaller than their radii of curvature, then the interaction energy is given, in the first case, by

$$U = -AR / 12H, \tag{3}$$

and the force by

$$F = AR / 12H^2; \tag{3'}$$

in the second case the energy is

$$U = -AR / 6H, \tag{4}$$

and the force

$$F = AR / 6H^2; \tag{4'}$$

for two infinite plates, the energy per unit surface is

$$u = -A / 12\pi H^2, \tag{5}$$

and the force acting on unit surface is

$$f = A / 6\pi H^3. \tag{5'}$$

Here R is the sphere's radius and H the minimum separation of the bodies; A is a constant, introduced by Hamaker, which depends on the nature of the bodies and is equal to $\pi^2 q^2 C$.

If we make similar calculations including retardation effects, in the limiting case of large separations, we get for the energy per unit area of parallel plates

$$u = -A_1 / 30\pi H^3 \tag{6}$$

and for the force acting on unit surface

$$f = A_1 / 10\pi H^4. \tag{6'}$$

where $A_1 = \pi^2 q^2 C_1$.

The justification for extending the additivity property of the London forces to the case of condensed bodies has not been established either theoretically or experimentally. This procedure would be justified only for the unrealizable case of two highly rarefied bodies, i.e., gases, separated by a gap. For condensed bodies, the atomic and molecular constants, in particular α and I, are changed from their values for isolated atoms and molecules because of the mutual influence of neighboring particles. Consequently, the contributions of individual molecules to the molecular interaction depend on their coordination and concentration, and for surface molecules, on the number of neighbors. Thus if we accept strict additivity we should, to be consistent, take for α and I their values for isolated molecules, which are known to be in error. If, however, we don't do this, then it is difficult to get the "true" values of α and I, since for condensed systmes they are hard to determine and are frequently altogether unknown.

Besides the lack of rigor in such an approach, it should be remarked that it is always very difficult in practice to compute the constants A and A_1 even for isolated atoms and molecules. In most cases the calculation does not lead to quantitative results, and it remains unclear how this can be done. The values of α and I are not known for many atoms. In such cases there is nothing one can do except to get the polarizability from refraction data on solids, and substitute quantities which are characteristics of condensed media into the London formula which is valid for the interaction of individual atoms.

A completely new and general theory of the molecular attraction of condensed bodies has been developed by Lifshitz[11] on an essentially different basis. Lifshitz's theory is a macroscopic theory, and is characterized by the absence of any special assumptions concerning the nature of the interactions of individual atoms. The fundamental idea of the theory is that the interaction of bodies is considered to be established through the medium of the fluctuating electromagnetic field, which is always present in the interior of any absorbing medium and which even extends beyond its boundaries. This field does not vanish even at absolute zero, where it is related to the zero-point oscillations of the radiation field. Such a method possesses complete generality, since it is applicable to any bodies at any temperature; retardation effects are automatically included in it.

Lifshitz derives a formula for the attractive force f acting per cm^2 of surface of each of two bodies separately by a slit of width H. This formula simplifies in two limiting cases:

1. If H is small compared to the fundamental wavelengths in the absorption spectra of the material, then

$$f \approx \frac{\hbar}{8\pi^2 H^3} \int_0^\infty \left(\frac{\varepsilon - 1}{\varepsilon + 1}\right)^2 d\xi, \tag{7}$$

i.e., the force is proportional to H^{-3}. Here $\hbar$ is Planck's constant, ϵ is the dielectric constant of the material and is a function of the frequency ω, $i\xi$ is the imaginary part of ϵ(ω is treated as a complex quantity in the theory);

2. If H is large compared to the characteristic wavelengths, then it can be shown that

$$f = \frac{\hbar c}{H^4} \frac{\pi^2}{240} \left(\frac{\varepsilon_0 - 1}{\varepsilon_0 + 1}\right)^2 \varphi(\varepsilon_0), \tag{8}$$

where c is the velocity of light, ϵ_0 is the static value of the dielectric constant, and $\varphi(\epsilon_0)$ is given by the following Table:

$1/\varepsilon_0$	0	0.025	0.1	0.25	0.50	1
$\varphi(\varepsilon_0)$	1	0.53	0.41	0.37	0.35	0.35

Consequently, in this case, the force is proportional to H^{-4} and depends only on ϵ_0.

However, as already noted, precise quantitative computations according to Lifshitz's theory, over the whole range of values of H, are impossible because of the lack of knowledge of the necessary optical constants.

III. METHOD OF MEASUREMENT

In any method of direct measurement of molecular attraction, the experiment always reduces to the measurement of two quantities, the force of interaction between two neighboring objects and the distance between them

This task is fraught with exceptional experimental difficulties, which can be overcome successfully by suitable choice of the form and material of the specimens to be investigated.

1. Samples Used in Measurements

For various reasons it is advantageous to have one of the objects flat and the other spherical in shape. We therefore measured the attractive force between a plate 4 × 7 mm and spherical lenses having radii of curvature $R = 10$ cm and $R = 25$ cm. This choice of sample shape simplifies the adjustment of the surfaces, which is more complicated if two plates are used, while the shortest distance between the bodies can be measured sufficiently accurately from the diameter of Newton's rings. In addition, such samples enable one to study the dependence of the forces on the radius of curvature of the spherical surface, and thus to separate the molecular forces, which are proportional to the radius of the spherical surface, from various masking effects which are apparently associated with surface electrification. The relation which expresses the proportionality of the molecular attraction to the sphere radius was obtained in the form[12]:

$$F(H) = 2\pi R u(H), \tag{9}$$

where $F(H)$ is the attractive force between the sphere and flat plate, R is the radius of the sphere and $u(H)$ is the interaction energy per cm^2 for a pair of infinite plates of the same material in the same medium, H the minimum distance between the surfaces. Using this formula we can, from measurements of the attractive force between a sphere and a flat plate, calculate the energy of interaction between two infinite flat plates, a quantity which is independent of the radius of curvature R.

As basic material for preparation of specimens we chose quartz glass and K-8 glass, since their transparency makes possible the use of the most accurate optical methods for determining the size of the gap between the surfaces, the surfaces of these materials can be easily given a high polish, and finally their surfaces are not damaged by the various cleaning methods which must be used. All these features of the selected surfaces make it possible to obtain and measure very small gaps between the surfaces.

The measurement of the attractive force between selected samples was carried out in air and in vacuum. The interaction of the two bodies should depend in no way on whether there is air or vacuum in the gap between them. But each case has its advantages and drawbacks from the point of view of experimental technique, and the comparison of results of the two types of measurements serves as an important method for checking the validity of the measurements. Carrying out experiments in vacuo proved to be more accurate and convenient. The reason for this was that viscous forces in the air in the gap, which arise when the gap width is changed, can become comparable to the molecular forces even for slow changes in gap width, so that measurements in air had to be done by waiting before taking a reading for some time while the gap was kept fixed. This slowed down the measurements, and it was often not possible to manage to take a reading at a moment when it was free of fluctuations. In addition, in the experiments, we did not succeed completely in avoiding vibration of the beam (of the balance) caused by convection currents.

Our experiments in vacuum were carried out at a residual pressure between 0.1 mm and several millimeters of Hg.

2. Method of Measurement of Force of Interaction. The Feed-back Balance.

The basic difficulty in the measurement of the force of molecular attraction between bodies is that these forces F are perceptible only for very small separations, and increase very rapidly with further increase in the distance H, so that (dF/dH) is negative and large in absolute value. Therefore, if we bring the two surfaces sufficiently close to one another, they will adhere to one another. Obviously, to solve this problem we need a balance which, on the one hand, has a high restoring torque, but on the other hand must have high sensitivity, i.e., we must satisfy requirements which are incompatible for ordinary balances.

We succeeded in finding a way out of these difficulties by applying to the balance a method analogous to negative feedback[13]. The idea of the method is that a displacement x of the balance beam (or a rotation through angle α) by some means produces an electrical current i which gives rise to an electromagnetic reaction M, which in turn acts on the beam balance and tends to restore it to its equilibrium positions. In the ordinary sense the balance is thus made less sensitive, since the same overloads produce smaller deviations of the beam from its equilibrium position, but since in a balance with negative feedback the overload is estimated not from the displacement or the angle of rotation of the beam but rather from the strength of the current as measured by a microammeter or galvanometer, the sensitivity can actually be greatly increased. At the same time, the feedback reduces the period of vibration of the balance, and makes feasible practically instantaneous establishment of equilibrium.

It is easy to realize the simplest and most convenient case where the torque M determined by the feedback is many times greater than the parallel torque which is caused by the shift of the center of gravity of the beam from its lowest position. It is this latter torque which ordinarily determines the sensitivity, the period and other meteorological characteristics of the balance. Thus the device which we simply call a feedback radically changes all the characteristics of the balance, so that instead of considering the torque due to the gravity force

$$m = l_0\alpha,$$

acting on the beam balance, we need to consider the torque M which is produced by forces of electromagnetic origin which act on the balance and depend on the angle of rotation. This torque is $M = l\alpha$, where the constant $l\,(l \gg l_0)$ depends on the beam tracking device used with the balance and on the coefficient of proportionality between the current i and the torque M, but not on the weight of the balance. For a given overload in one pan of the balance relative to the other, the deflection angle of the balance will be l/l_0 times smaller than for the same balance without feedback; at the same time the period will be reduced by a factor of $\sqrt{l/l_0}$.

One can easily control the feedback parameters and, consequently, the characteristics of the balance. This flexibility of the method is especially important for the solution of our problem, first because the force and its gradient change markedly with gap size, and secondly because of the possible presence of various extraneous effects whose magnitude and variation with distance are very difficult, and sometimes impossible, to estimate in advance.

3. Plan and Principle of Operation of the Apparatus[14,15].

a) *The Balance.* The force of interaction between the flat surface of the plate P and the convex surface of the spherical lens L was measured by means of a special beam balance (Fig. 1). The length of the beam K was 35 mm, its weight 0.1 gm. The plate P was placed on the end of the beam and the lens L on a support which was independent of the beam, so that the gap H between the convex underside of the lens and the upper surface of the plate was sufficiently small. A small mirror S was attached to the other end of the beam. Attached to the beam was an agate prism a riding on an agate fulcrum b. Rough balancing was done with a glass rod c, weighing 10-50 mg, which could be shifted along the beam as a rider. The beam was rigidly attached to a coil frame R (with 15-20 turns of wire), placed in the field of a permanent magnet (cf. Fig. 2). The magnetic field B was ≈ 850 gauss.

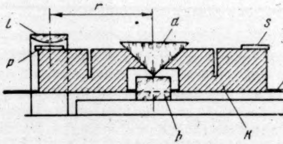

FIG. 1. Plan of beam balance with feedback.

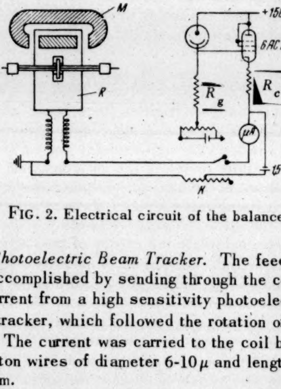

FIG. 2. Electrical circuit of the balance

b) *Photoelectric Beam Tracker.* The feedback was accomplished by sending through the coil R the current from a high sensitivity photoelectric beam tracker, which followed the rotation of the beam. The current was carried to the coil by Wollaston wires of diameter 6-10μ and length ~30 mm.

The servo system for the rotation of the beam balance consisted of a raster-type photocell relay and a single stage amplifier. The relay was placed at a distance of 35 mm above the beam so that its optical axis *OO* (Fig. 3) was parallel to the axis of rotation of the beam. The circuit of the photocell relay is shown in Figs. 2 and 3.

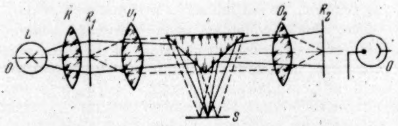

FIG. 3. Raster photorelay.

Ths light source *L* was a 50 watt incandescent lamp with a short thin filament. By means of the condenser *K* the light from the source was shone on the linear (typographic) raster R_1 (a glass plate with alternating transparent and opaque bands of equal width), and after passing through the objective O_1 was focussed on the mirror *S* (the corresponding light paths are shown by the solid lines). By means of the objective O_1, mirror *S*, and a second objective O_2 of the same focal length (7.5 cm), a real image of the raster R_1 was produced in the plane of the second raster R_2 having the same ruling of 60 lines-cm (the corresponding light paths are shown by the dotted lines). The size of the image of raster R_1 corresponded with the size of R_2, since they were situated in the focal planes of two identical objectives (the very slight difference in focal lengths of the objectives O_1 and O_2 was easily compensated by a small displacement of the rasters out of the focal planes of their objectives.

The planes of the rasters were perpendicular to the plane containing the beam and its axis of rotation, while the lines of the rasters were perpendicular to the beam and its axis of rotation. The slightest rotation of the mirror changed the position of the image of the first raster relative to the second raster, thus increasing or decreasing the size of the openings. Figure 4 shows schematically how the size of the light-transmitting surface changes as a function of the relative position of the image of raster R_1 (the grey bands) and raster R_2 (the black bands). After passing through the second raster, the light fell on an antimony-cesium vacuum photocell (ACV-3) which controlled the grid of an amplifier tube.

The photocell current was amplified using the simple circuit shown in Fig. 2. A battery connected through a potentiometer in the grid circuit

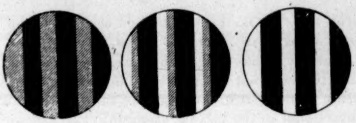

FIG. 4. Relative position of the rasters.

of the 6AC7 applied a negative potential to it. The load resistor R_g was 11 megohms. The negative feedback in the amplifier (by means of the resistor R_f in the cathode circuit) assured good stability. The anode current *i*, partially compensated by the current i_1 (from a 1.5 volt dry battery) flowed through the balance coil *R*. By changing the current i_1 by means of a resistance box *K*, the current in the coil could be regulated without leaving the steep part of the tube characteristic.

c) *Principle of Operation of the Apparatus.* At a definite null position of the balance beam the current i_0 in the coil was zero. Rotation of the beam through a small angle changed the light transmission through the second raster and the illumination of the photocell, producing a current $i_0 = k\alpha$, where the output current *k* is a constant of the tracking device. By sending the current *i* through the balance coil in the required direction, we produced a feedback; the coil was in the field of a magnet, so that the beam balance was subjected to a torque

$$M = ni = nk\alpha = l\alpha,$$

where *n* and *l* are constants, with *n* depending only on the number and shape of the turns of wire and on the magnetic field strength.

By bringing the convex surface of the lens, which was set on a platform having a very fine screw mechanism, closer to the surface of the plate, we could decrease the gap *H* between them until the attractive force *F* made its appearance. When this occurred, the beam deflected to an angle (which was extremely small for high output current from the servo, i.e., for "tight" feedback) at which the torque *Fr*, where *r* is the moment arm of force *F*, was balanced by the feedback torque *M*. The force was calculated from the formula

$$F = ni/r, \tag{10}$$

where *i* is the current recorded by a microammeter connected in the plate circuit (μA in Fig. 2), *r* is the distance from the knife edge of the prism *a* to that point on the surface of the plate *P* which corresponds to the shortest distance between the surfaces (cf. Fig. 1); the method of determining the constant n is described below.

Thus. by means of the feedback, the molecular attraction was automatically balanced by a torque, acting on the coil in the magnetic field, which was proportional to the current strength, The molecular attractive force could be determined by measuring the current.

Since we could achieve only a rough balancing of the beam by means of the glass rider c (Fig. 1), the current i_0 was actually not equal to zero, but corresponds to some zero value of the torque M_0 which maintained equilibrium of the balance for gaps H where there was still no attractive force between the surfaces. Upon the appearance of the attractive force, the current i increased by an amount Δi_0, and the force was computed from the formula

$$F = n\Delta i_0 / r. \tag{11}$$

If any distrubances produced repulsive forces, these resulted in a decrease of the current i_0 ($\Delta i_0 < 0$).

4. *Method for Adjusting the Gap.*

One of the most difficult points in our work was that of obtaining a stable gap size of a fraction of a micron between the surfaces, and of achieving smooth motion of one body relative to the other.

Of all the methods tried by us for adjusting the gap H, the best was one using the same negative feedback. A sufficiently small adjustment of the gap (down to $0.1\,\mu$) was produced by a micrometer displacement of the raster R_1 (Fig. 3) in a direction perpendicular to its lines. This produced an electromagnetic torque which was not balanced by gravity and which shifted the beam until it reached the angle corresponding to the new equilibrium position. By moving the raster R_1 to one side or the other, we raised or lowered the end of the beam, thus increasing or decreasing the gap between the surfaces, the current i_0 in the coil meantime remaining constant until the gap H become so small that the molecular interaction between the bodies could be detected.

The null current i_0 remains constant and the relations (10) and (11) are valid only if the intrinsic restoring torque of the balance in the absence of feedback is negligibly small compared to the torque M. It is clear that the gravity torque will be completely absent when the distance d from the centerof gravity to the point of support (the knife edge) is zero. By using the feedback, which reduces the oscillation period and can make it sufficiently small, it is possible (contrary to the case for an ordinary balance) to go arbitrarily far in the direction of decreasing d, down to $d = 0$.

In our balance, the center of gravity was located practically on the knife edge of the prism. We used as criterion for sufficiently small distance d, the requirement of constancy of the current i_0 in the coil for a wide range of positions of the beam when, of course, the gap H was so large that there was still no force between the plate P and lens L (Fig. 1). We found that the current I_0 remained constant to within an accuracy sufficient for our experiments ($\pm 0.1\,\mu A$ and less) for $d \leq 0.025$ cm $= d_0$. The distance d is related to the period T, in the absence of the feedback. We adjusted d by determining experimentally the period T_0 corresponding to d_0; T_0 was approximately 6-8 sec. T and d are very sensitive to the slightest changes in balance, so that before each experiment we had to set the required values of $T > T_0$ (or $d < d_0$). This was done by using a glass rider c (Fig. 1) of approximate weight.

Coincidence of the center of gravity with the knife edge greatly reduces the sensitivity of the apparatus to vibrations of the support, since the vibrations are mainly transmitted through the point of support. However, turning of the base can be transmitted to the balance beam through the medium of the viscous layer between the lens and plate.

5. *Adaptability of the Method*

Since the current strength and the torque of the interaction between the coil and the magnet are proportional (with coefficient n), the sensitivity of the feedback balance is independent of the sensitivity and other characteristics of the servo; the balance gives a linear relation between the force acting between the bodies (the "load") and the current in the coil, independently of whether the amplifier characteristic is linear or not. Such a balance allows sensitivity adjustments by changing the number of windings in the soil and the intensity of the magnetic field, or by simply shunting the coil. The possibility of changing the sensitivity of the balance by simple "electrical" means is especially valuable.

At constant sensitivity the value of the coefficient of feedback l depends on the current output k, which we should vary over a wide range by using various parameters on the amplifier circuit: by changing the cathode resistor R_c (Fig. 2), by operating on different parts of the plate characteristic, or finally by using the amplifier with a tube having a flatter characteristic and less noise.

Evaluation of the current output in amps/radian was done in the following way. The frame coil *R* (Fig. 2) was short-circuited. A special locking device *5* (Fig. 5) in the beam was held fixed so that there was a small gap between lens and plate. The change in plate current, corresponding to a small deflection of the beam which was set by using the locking device, was measured with a microammeter. The deflection angle was taken to be equal to the ratio of the change in the gap to the length of the beam arm. (The method for determining the gap is described in a later paper.) When investigating interactions which did not require very high values of the coefficient l, we reduced the output current from the servo, since this decreased the effect of vibrations on the balance beam and consequently reduced the readings on the microammeter.

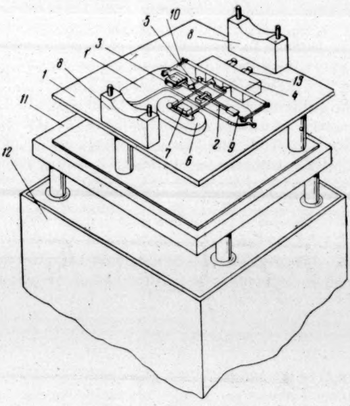

FIG. 5. Overall view of the equipment for measuring molecular forces.

In applying the negative feedback method to an ordinary balance, slow changes of the output current coefficient, in contrast to short period fluctuations, are not harmful since the equilibrium of the balance is maintained by a gradual deflection of the beam, while the plate current by means of which the load is determined remains constant. In our apparatus, both rapid and slow changes in the servo are dangerous. We require a stable position of equilibrium of the beam for a time long enough for measurement of the gap between the surfaces and the corresponding current. This is especially true for measurements in air, since in that case motion of the beam gives rise to resistive forces, caused by the viscosity of the air, which distort the forces measured.

To assure stable conditions, we used only storage and dry batteries as voltage sources; the amplifier circuit was shielded by placing it inside a wire screen, and the platform on which the balance was mounted was grounded.

Using the method described, one can measure forces of interaction between bodies within the limits of $1\text{-}2 \times 10^{-4}$ dyne to ~ 20 dynes, even when the force decreases rapidly with separation. Thus, one can measure a force, whose gradient is 10^6 dynes/cm to an accuracy of 0.02 dyne. The vibration period of the beam was approximately 5×10^{-3} sec when such tight feedback (servo output current about 500 amps/radian) was used.

These features of the balance enabled us to overcome the considerable and special difficulties of our problem.

6. *Auto-oscillations.*

It should be noted that auto-oscillations of large amplitudes can develop in a feedback circuit. This phenomenon is associated with inertia of the servo, i.e., with the fact that the current output of the servo lags the deflection of the beam.

This phenomenon can be eliminated by simply including a phase-shifting network in the amplifier circuit. In measuring molecular forces, oscillations are generated only at large separations of the surfaces, when the damping effect of a thin air layer is no longer present. Even for measurements in vacuum (1×10^{-2} mm Hg) this damping effect was sufficient, and we did not have to introduce any phase-shifting network.

7. *Layout of the Apparatus.*

The arrangement of our apparatus is shown in Fig. 5. A massive brass plate *1*, for bracing all the parts of the apparatus, rests on three supports which are tall enough to give access to the control screws under the platform. On the platform are: the plate *1′* supporting the beam *2*, the lens support *3*, a mechanism *4* for moving the glass fiber, the lock stop *5*, the magnet *6* with core *7*, and the supports *8* for the raster relay. The plate *1′* rests on three supports. The heights of two of them can be changed by means of differential micrometer screws located under the platform *1*. The purpose of this arrangement will be explained below. The agate fulcrum *b* (Fig. 1) is cemented to the plate, and serves as the support for the agate prism *a* of the balance beam.

The beam was *U*-shaped and made of 0.16 mm thickness of aluminum. Three notches were made in the beam: the agate prism was cemented in the central notch with shellac, the feedback coil frame was cemented into the other two notches. The rectangular base of the coil frame is made of 0.06 mm aluminum and has ribs for stiffening it. The ends of the enamelled copper wire, having a diameter 40 μ, of which 15-20 turns are wound on the frame, are soldered to thin Wollaston wires (6 μ diameter) and led out below the mechanism for moving the glass rider, to the terminals *13*. The frame is 20 × 40 × 2 mm. The mirror *S* and a quartz, or glass, plate *P* are attached to the ends of the beam by means of aluminum couplings. Figure 6 shows the appearance of the plate and the lens.

There is a hole in plate 1′ (Fig. 5) through which the lens and its support pass freely. The screws through this plate, which were mentioned earlier, serve to tilt the beam and the plate to various angles relative to the lens, which is necessary for shifting the point of contact of the surfaces under investigation.

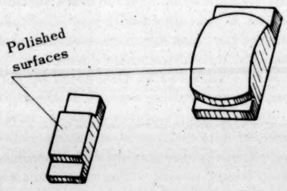

FIG. 6. Test specimens.

The glass rider *9* (Fig. 5), which is used for rough balancing and for calibration of the balance, rests in the *U*-groove of the beam and is moved along it by the slider *10*. The slider is shifted by means of a screw under the platform, which sets the horizontal carriage *4* in motion. To prevent contact between the lens and plate, and to separate them in case of an intentional or accidental contact, the special lock-stop *5* was provided. Contact of the stop with the beam is made through the crossed edges of two corundum crystals (in order to reduce the adhesive forces), one of which is cemented to the coupling at the left end, the other to the plate of the stop. Vertical displacement of the stop is done by means of a differential screw brought out under the platform. The magnet *6* and the coil *7* rest freely on the platform. All the parts of the apparatus which are located between the supports of the photorelay are enclosed by a squat brass housing. Small glass windows are cut in the housing at the positions of the lens and the mirror cemented to the beam.

In the apparatus for vacuum work, the housing was sealed to the platform by means of a rubber gasket, and all the micrometer screws transmitted their motion through sylphons soldered to the bottom side of the platform.

The supports *8* hold the raster photorelay (cf. the diagram in Fig. 5) which is mounted inside a brass tube 40 mm in diameter and 300 mm long.

8. Calibration of the Balance

The force of interaction between the lens and plate is found from the relation

$$F = ni/r = \gamma i.$$

The methods for measuring the current *i* and the distance *r* were described above; we must now explain how the coefficient *n* was determined.

The calibration of the balance (i.e., the determination of *n*) was done by using the glass rider *c* (Fig. 1), which could be moved along the beam. With a large gap between the surfaces (when no molecular attractive forces are present), the current i_0 in the coil was measured with a microammeter for various positions of the glass rider, which were read on the ocular scale of a microscope. The coefficient *n* was calculated by using a graph of the coordinate *y* of the end of the glass rider rod versus current i_0, using the formula

$$n = P\Delta y / \Delta i_0,$$

where *P* is the weight of the glass rod.

Figure 7 shows one of the calibration curves; in this case *n* was 2.51 mg/mA. In our experiments the lever arm *r* was 1.9 cm on the average, which corresponded to a balance sensitivity of

$$\gamma = n/r = 1.32 \text{ mg/mA}.$$

The measured coefficient *n* was in good agreement with the value 2.55 mg/mA calculated using Ampere's alw. Moreover, the accuracy of this computation is undoubtedly lower than the accuracy of determination of *n* from the calibration curve, so that the computation can serve only as a check on the absence of gross errors in the calibration.

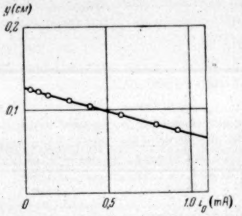

FIG. 7. Calibration curve. Weight $P = 46.60$ mg, Number of Turns $N = 15$.

The calibration of the balance was done in the circuit prepared for the experiment, immediately before and after the measurement, and the current was always measured with the same instrument. Thus all the factors which might effect the graduation of the instrument and its sensitivity during measurements, were also present during calibration and were automatically taken into account.

The small spread of the points on the calibration curve (Fig. 7) guaranteed determination of n to an accuracy of ± 0.4%. The value of r was 19 ± 0.5 mm. The resulting inaccuracy in determining γ was ± 3% ($\gamma = 1.32 \pm 0.04$). The precision of measurement of the force F was almost entirely dependent on the error in measuring the current whose value, because of vibrations of the support, fluctuated by $\pm 0.1\,\mu$A in the most favorable case, while the absolute value of the current was in the range 0.2-$2\,\mu$A. The lower limit of measurability of forces was set by these parasitic vibrations, which gave rise to an error of $\pm 0.13\gamma = \pm 0.13$ microgram $\approx 0.13 \times 10^{-3}$ dyne.

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A Contribution to the General Theory of Statistical Equilibrium of a System of Interacting Particles

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Equations for the equilibrium distribution functions are obtained from the general dynamic equations, without making the assumption of the canonical character of the initial equilibrium distribution. In place of this a series of physical conditions are imposed of the type of the condition of diminishing correlations with increase of the distances between the particles.

IF, for systems consisting of N similar interacting particles occupying a volume V, we postulate the existence of a class of distribution functions[1,2]

$$n_s(t, \mathbf{x}^1, \ldots, \mathbf{x}^s), \tag{1}$$

$$f_s(t, \mathbf{x}^1 \ldots, \mathbf{x}^s, \mathbf{y}^1, \ldots, \mathbf{y}^s),$$

$$g_s(t, \mathbf{x}^1, \ldots, \mathbf{x}^s, \mathbf{y}^1, \ldots, \mathbf{y}^s, \mathbf{z}^1, \ldots, \mathbf{z}^s), \ldots$$

($\mathbf{x}^i$ is a vector determining the position of the particles, $\mathbf{y}^i$ a velocity vector of the particles, $\mathbf{z}^i$ an acceleration vector, etc.), and if we subject these distribution functions to the conditions

$$\int \ldots \int f_s(t, \mathbf{x}, \mathbf{y}) \prod_{i=1}^{s} d\mathbf{y}^i = n_s(t, \mathbf{x}), \tag{2}$$

$$\int \ldots \int g_s(t, \mathbf{x}, \mathbf{y}, \mathbf{z},) \prod_{i=1}^{s} d\mathbf{z}^i = f_s(t, \mathbf{x}, \mathbf{y});$$

$$\int \ldots \int f_{s+1}\, d\mathbf{x}^{s+1}\, d\mathbf{y}^{s+1} = Af_s, \tag{3}$$

($\mathbf{x}$ and $\mathbf{y}$ denote, respectively, the sets of vectors $\mathbf{x}^1, \ldots, \mathbf{x}^s$ and $\mathbf{y}^1, \ldots, \mathbf{y}^s$; A is a normalizing factor), and if, besides, we require fulfillment of the equations

$$\lim_{\delta\tau \to 0} \frac{1}{\delta\tau} \tag{4}$$

$$\times \left\{ \int \ldots \int g_s(t - \delta\tau, \mathbf{x} - \mathbf{y}\delta\tau, \mathbf{y} - \mathbf{z}\delta\tau, \mathbf{z}) \prod_{i=1}^{s} d\mathbf{z}^i - \int \ldots \int g_s(t, \mathbf{x}, \mathbf{y}, \mathbf{z}) \prod_{i=1}^{s} d\mathbf{z}^i \right\} = 0$$

and of anologous equations for the other distribution functions*, then it is possible to obtain a system of basic equations of the kinetic theory

$$\frac{\partial f_s}{\partial t} + \sum_{i=1}^{s} \frac{\partial f_s}{\partial \mathbf{x}^i} \mathbf{y}^i = \frac{1}{m} \sum_{i,j}^{s} \frac{\partial f_s}{\partial \mathbf{y}^i} \frac{\partial \Phi^{ij}}{\partial \mathbf{x}^i} + \frac{1}{m} \sum_{i=1}^{s} \iint \frac{\partial \Phi^{i,s+1}}{\partial \mathbf{x}^i} \frac{\partial f_{s+1}}{\partial \mathbf{y}^i}\, d\mathbf{x}^{s+1}\, d\mathbf{y}^{s+1} \tag{5}$$

in the absence of external forces, for an interaction law given by a mutual potential Φ.

As we demonstrated[2], N. N. Bogoliubov's equations for the system under consideration, the Maxwell distribution, and (for $s=N$) the Gibbs distribution all follow from Eq. (5), in the determination of the eqilibrium state as well as of the stationary state of a system in which the distribution function f_s factors into the product of two independent functions giving, respectively, the distributions of the velocities and of the coordinates [$f_s = \psi_s(\mathbf{x})\,\varphi_s(\mathbf{y})$]. A generalization of the results[2] in the case of complex systems, consisting of particles of different kinds, has also been derived[3].

In constructing a kinetic theory independent, in a certain sense, of the results of statistical mechanics, the theorem proved in Refs. 2 and 3 should be considered as a sufficient condition for obtaining an equilibrium solution. It is necessary to prove the necessity of the above-indicated properties of the equilibrium distribution functions for the dynamic states of complex particles. It is desirable to derive this proof with the least possible use of special assumptions about the character of the law of interaction between the particles.

Let us write Eq. (5) in the form[3,4]

$$\frac{\partial F_s}{\partial t} = [H_s; F_s] + \frac{1}{v} \int \left[\sum_{i=1}^{s} \Phi_{i,s+1}; F_{s+1} \right] d\mathbf{q}_{s+1}\, d\mathbf{p}_{s+1}, \tag{6}$$

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* In the work of Born and Green[1] and in our previous article[2] this condition is inaccurately written. The correct expression is the one presented above.

where F_s represents the function f_s, normalized in the manner of Bogoliubov, and where the momenta appear in place of the velocities.

Let us assume that the density of our system is small enough that the expansion

$$F_s = F_s^0 + v^{-1}F_s^1 + v^{-2}F_s^2 + \dots \tag{7}$$

is meaningful. Considering henceforth a system in the absence of external forces, for the sake of simplicity, we shall look for solutions of the equations

$$[H_s;\, F_s] + \frac{1}{v}\int\Big[\sum_{i=1}^{s}\Phi_{i,\,s+1},\ F_{s+1}\Big]d\mathbf{q}_{s+1}\,d\mathbf{p}_{s+1} = 0, \tag{8}$$

for the stationary state, subject to the conditions (7), assuming central forces of interaction. Besides the conditions enumerated in Eqs. (2)-(4), the equilibrium distribution functions must satisfy boundary conditions for relaxation of the correlations, having the form

$$F_s(\mathbf{q}_1, \dots, \mathbf{p}_s) \to \prod_{1\leqslant i\leqslant s} F_1(\mathbf{q}_i, \mathbf{p}_i) \tag{9}$$

for all $|\mathbf{q}_i - \mathbf{q}_j| \to \infty$.

Let us substitute the expansion (7) into Eq. (8) and consider the corresponding systems of equations. For the zero-order approximation of the unary function (single-particle distribution) we obtain

$$m^{-1}(\mathbf{p}_1 \operatorname{grad} F_1^0) = 0, \tag{10}$$

from which it follows that F_1^0 can depend only on $\mathbf{p}_1$.

For the zero-order approximation to the s-dimensional distribution function, we have the following equation

$$\sum_{\alpha=1}^{3}\left\{\frac{\partial U_s}{\partial q_1^\alpha}\frac{\partial F_s^0}{\partial p_1^\alpha} + \dots + \frac{\partial U_s}{\partial q_s^\alpha}\frac{\partial F_s^0}{\partial p_s^\alpha} - \frac{1}{m}\left(p_1^\alpha\frac{\partial F_s^0}{\partial q_1^\alpha} + \dots + p_s^\alpha\frac{\partial F_s^0}{\partial q_s^\alpha}\right)\right\} = 0, \tag{11}$$

and conditions on the relaxation of the correlations of the form

$$F_s^0 - \prod_{1\leqslant i\leqslant s} F_1^0 \to 0$$

for all $|\mathbf{q}_i - \mathbf{q}_j| \to \infty$. The equations of the characteristics

$$\frac{dq_i^\alpha}{\partial H_s/\partial p_i^\alpha} = \frac{dp_i^\alpha}{-\partial H_s/\partial q_i^\alpha},$$

$$\alpha = 1, 2, 3;\ i = 1, \dots, s$$

possess $6s - 1$ integrals of the motion: $J_k(\mathbf{q}_1, \dots, \mathbf{p}_s) = c_k$.

Let us assume that the transition to the limit $|\mathbf{q}_i - \mathbf{q}_j| \to \infty$ is "adiabatic"; i.e., let us assume that in the passage to the limit to be considered the limiting functions F_s^0 depends on the same integrals of the motion as the initial function F_s^0. Since F_s^0 is an implicit function of q and p,

$$\lim F_s^0(J_1, \dots, J_{6s-1}) = F_s^0(\lim J_1, \dots, \lim J_{6s-1}).$$

If we set $\lim J_k = J_k^*$ then, from the condition on the relaxation of the correlations, we obtain

$$F_s^0(J_1^*, \dots) = F_1^0(\mathbf{p}_1)\dots F_1^0(\mathbf{p}_s) \tag{12}$$

or

$$\ln F_s^0(J_1^*, \dots) = \sum_{i=1}^{s}\ln F_1^0(\mathbf{p}_i). \tag{13}$$

Because of the fact that the F_s^0 are functions of the "s-body" problem, while the F_1^0 are functions of the "single-body" problem, it follows from (13) that in the limit $\ln F_s^0$ depends only on additive integrals of the motion and is made up of a linear combination of them. Not considering the uniform translational and rotational motion of the system as a whole, we obtain

$$\ln F_s^0(\mathbf{J}_1^*, \dots) = \alpha_s - \beta W_s^*, \tag{14}$$

where $W_s^* = \lim H_s$. In this way the solution sought for will have the form

$$F_s^0 = c_s \exp(-\beta H_s) \tag{15}$$

in connection with the "adiabatic" character of the limiting transition being considered; in particular

$$F_1^0 = c_1 \exp(-\beta p_1^2/2m).$$

In agreement with the conditions on the relaxation of the correlations, $c_s = (c_1)^s$, and from the normalization conditions it follows that

$$c_1 = \left(\int \exp(-\beta p^2/2m)\,dp\right)^{-1}.$$

For the first approximation to the single-variable function

$$-m^{-1}(\mathbf{p}_1 \operatorname{grad} F_1^1) + \int [\Phi_{12};\ F_2^0]\,d\mathbf{q}_2\,d\mathbf{p}_2 = 0 \quad (16)$$

and since

$$\int\left[\Phi_{12};\ F_2^0\right]d\mathbf{q}_2 d\mathbf{p}_2 = \sum_{\alpha=1}^{3} c_2 \left[\int \exp\left(-\frac{\beta p_2^2}{2m}\right)d\mathbf{p}_2\, \frac{p_1^\alpha}{m} \exp\left(-\frac{\beta p_1^2}{2m}\right) \times \frac{\partial}{\partial q_1^\alpha}\int \exp(-\beta\Phi_{12})\,d\mathbf{q}_2 + \exp\left(-\frac{\beta p_1^2}{2m}\right)\int \frac{\partial}{\partial q_2^\alpha}\exp(-\beta\Phi_{12})\,d\mathbf{q}_2 \times \int \frac{p_2^\alpha}{m}\exp\left(-\frac{\beta p_2^2}{2m}\right)d\mathbf{p}_2\right] = 0,$$

we deduce that F_1^1 can depend only on $\mathbf{p}_1$. From the normalization condition it further follows that $F_1^1 = 0$. Analogously, we find that all the higher approximations for the single-variable functions vanish.

Passing to the binary functions (joint distributions), let us examine the equation

$$[H_2;\ F_2^1] + \int\left[\sum_{i=1}^{2}\Phi_{i3};\ F_3^0\right]d\mathbf{q}_3 d\mathbf{p}_3 = 0; \quad (17)$$

Using (15) we can write

$$\int\left[\sum_{i=1}^{2}\Phi_{i3};\ F_3^0\right]d\mathbf{q}_3\,d\mathbf{p}_3 \quad (18)$$
$$= \sum_\alpha c_3 \exp(-\beta\Phi_{12}) \exp\left[-\frac{\beta}{2m}(p_1^2 + p_2^2)\right] \times \int \exp\left(-\frac{\beta p_3^2}{2m}\right)d\mathbf{p}_3 \left[\frac{p_1^\alpha}{m} \times \int \frac{\partial}{\partial q_1^\alpha}\exp[-\beta(\Phi_{13} + \Phi_{23})]\,d\mathbf{q}_3 + \frac{p_2^\alpha}{m}\int\frac{\partial}{\partial q_2^\alpha}\exp[-\beta(\Phi_{13} + \Phi_{23})]\,d\mathbf{q}_3\right].$$

On the other hand, setting

$$F_2^1 = F_2^0 \overset{*}{\varphi}_2, \quad (19)$$

for the particular solution of the inhomogeneous Eq. (17), we get

$$[H_2;\ F_2^1] = [H_2;\ F_2^0\overset{*}{\varphi}_2] = [H_2;\ F_2^0]\,\overset{*}{\varphi}_2 + [H_2;\overset{*}{\varphi}_2]\,F_2^0 = [H_2;\ \overset{*}{\varphi}_2]\,F_2^0; \quad (20)$$
$$[H_2;\ \overset{*}{\varphi}_2] + \sum_\alpha\left\{\frac{\partial\Phi_{12}}{\partial q_1^\alpha}\,\frac{\partial\overset{*}{\varphi}_2}{\partial p_1^\alpha} + \frac{\partial\Phi_{12}}{\partial q_2^\alpha}\,\frac{\partial\overset{*}{\varphi}_2}{\partial p_2^\alpha} - \frac{p_1^\alpha}{m}\,\frac{\partial\overset{*}{\varphi}_2}{\partial q_1^\alpha} - \frac{p_2^\alpha}{m}\,\frac{\partial\overset{*}{\varphi}_2}{\partial q_2^\alpha}\right\}.$$

Noting that

$$c_3\int\exp(-\beta p^2/2m)\,d\mathbf{p} = c_2,$$

we find a particular solution in the form

$$F_2^0\overset{*}{\varphi}_2 = F_2^0\int\exp[-\beta(\Phi_{13} + \Phi_{23})]\,d\mathbf{q}_3. \quad (21)$$

The general solution of Eq. (17) is represented by the expression

$$F_2^1 = F_2^0\overset{*}{\varphi}_2 + b_2' F_2^0, \quad (22)$$

where the constant b_2^1 must be chosen so that F_2^1 satisfies the condition on the relaxation of the correlations: $F_2^1 \to 0$ for $|\mathbf{q}_i - \mathbf{q}_j| \to \infty$. Correspondingly b_2^1 is given by

$$b_2^1 = -\int(1 + f_{13} + f_{23})\,d\mathbf{q}_3,$$

where $f_{ij} = \exp(-\beta\Phi_{ij}) - 1$, and F_2^1 can be represented in the form

$$F_2^1 = F_2^0\int f_{13}f_{23}\,d\mathbf{q}_3. \quad (23)$$

For F_3^1 we obtain in an analogous manner

$$F_3^1 = F_3^0\int\exp[-\beta(\Phi_{14} + \Phi_{24} + \Phi_{34})]\,d\mathbf{q}_4 + b_3' F_3^1, \quad (24)$$

where

$$b_3^1 = -\int(1 + f_{14} + f_{24} + f_{34})\,d\mathbf{q}_4.$$

Let us further consider the equations

$$[H_2;\ F_2^2] + \int\sum_{1\leqslant i\leqslant 2}[\Phi_{i3};\ F_3^1]\,d\mathbf{q}_3\,d\mathbf{p}_3 = 0. \quad (25)$$

Substituting F_3^1 from (24) we obtain

$$[H_2; F_2^2] + \sum_\alpha F_2^0 \Big[\frac{p_1^\alpha}{m} \int \frac{\partial}{\partial q_1^\alpha}$$

$$\times \exp[-\beta(\Phi_{13} + \Phi_{23})] \overset{*}{\varphi}_3 d\mathbf{q}_3$$

$$+ \frac{p_2^\alpha}{m} \int \frac{\partial}{\partial q_2^\alpha} \exp[-\beta(\Phi_{13} + \Phi_{23})] \overset{*}{\varphi}_3 d\mathbf{q}_3 \Big]$$

$$+ b_3^1 \sum_\alpha F_2^0 \Big[\frac{p_1^\alpha}{m} \int \frac{\partial}{\partial q_1^\alpha} \exp[-\beta(\Phi_{13} + \Phi_{23})] d\mathbf{q}_3$$

$$+ \frac{p_2^\alpha}{m} \int \frac{\partial}{d q_2^\alpha} \exp[-\beta(\Phi_{13} + \Phi_{23})] d\mathbf{q}_3 \Big] = 0.$$

Seeking a particular solution of the inhomogeneous equation in the form $F_2^2 = F_2^0 \varphi_2^{**}$, where φ_2^{**} does not depend on the momenta, we obtain

$$\frac{\partial \varphi_2^{**}}{\partial q_1^\alpha} = \int \frac{\partial}{\partial q_1^\alpha} \exp[-\beta(\Phi_{13} + \Phi_{23})]$$

$$\times \Big\{ \int \exp[-\beta(\Phi_{14} + \Phi_{24} + \Phi_{34})] dq_4 \Big\} dq_3$$

$$+ b_3^1 \frac{\partial}{\partial q_1^\alpha} \int \exp[-\beta(\Phi_{13} + \Phi_{23})] d\mathbf{q}_3,$$

from which we find for the desired solution

$$F_2^0 \varphi_2^{**} = \frac{1}{2} F_2^0 \int \exp[-\beta(\Phi_{13} + \Phi_{23} \tag{26}$$

$$+ \Phi_{14} + \Phi_{24} + \Phi_{34})] d\mathbf{q}_3 d\mathbf{q}_4$$

$$+ b_3^1 F_2^0 \int \exp[-\beta(\Phi_{13} + \Phi_{23})] d\mathbf{q}_3.$$

We can then write the general solution in the form

$$F_2^2 = F_2^0 \varphi_2^{**} + b_2^2 F_2^0 \tag{27}$$

and from our boundary conditions we obtain for the constant b_2^2 the expression

$$b_2^2 = \frac{1}{2} \int (1 + f_{14} + f_{24} + f_{13} + f_{23} \tag{28}$$

$$+ f_{34} + f_{13} f_{14} + f_{13} f_{24} + f_{14} f_{23}$$

$$+ f_{23} f_{24} - f_{13} f_{34} f_{14} - f_{23} f_{34} f_{24}) d\mathbf{q}_3 d\mathbf{q}_4.$$

Finally, F_2^2 is represented by the product

$$F_2^2 = F_2^0 L_{12}^{34}, \tag{29}$$

where the symbol L_{12}^{34} designates a collection of integral terms determining all possible correlational relations between a fixed group of particles (1, 2) and the "running" group of particles (3, 4). In this fashion we have for the binary functions

$$F_2^0,\ F_2^1 = F_2^0 L_{12}^3, \quad F_2^2 = F_2^0 L_{12}^{34}, \ \ldots$$

and for the ternary functions

$$F_3^0,\ F_3^1 = F_3^0 L_{123}^4, \quad F_3^2 = F_3^0 L_{123}^{45}, \ \ldots$$

In connection with the fact that we are at the moment interested only in the question of the general structure of the solution (in the sense of its multiplicity with respect to the dependence on coordinates and momenta) and not in the problem of the exact construction of all the higher approximations, we shall examine only the "principal" parts of the particular solutions of the corresponding inhomogeneous equations, i.e,, expressions of the form

$$\bar{\mathrm{F}}_s^i = \frac{F_s^0}{i!} \int \mu_s \mu_{s+1} \ldots \mu_{s+i-1} d\mathbf{q}_{s+1} \ldots d\mathbf{q}_{s+i}, \tag{30}$$

$$\mu_s = \exp\Big(-\beta \sum_{j=1}^{s} \Phi_{j,\, s+1}\Big);$$

where the superscript i indicates the order of the approximation. Postulating for $\overline{F}_{s+1}^{i-1}$ the form (30), let us consider the equation

$$[H_s; \bar{F}_s^i] + \int \Big[\sum_{j=1}^{s} \Phi_{j,\, s+1}; \bar{F}_{s+1}^{i-1} \Big] d\mathbf{q}_{s+1} d\mathbf{p}_{s+1}. \tag{31}$$

Substituting $\overline{F}_s^i = (F_s^0/i\,!)\gamma_s^i$, we obtain

$$\frac{1}{i} \sum_\alpha F_s^0 \Big\{ -\frac{p_1^\alpha}{m} \frac{\partial \gamma_s^i}{\partial q_1^\alpha} - \ldots - \frac{p_s^\alpha}{m} \frac{\partial \gamma_s^i}{\partial q_s^\alpha} \Big\}$$

$$+ \sum_\alpha \int \Big\{ \frac{\partial \Phi_{1,\, s+1}}{\partial q_1^\alpha} \frac{\partial F_{s+1}^0}{\partial p_1^\alpha}$$

$$+ \ldots + \frac{\partial \Phi_{s,\, s+1}}{\partial q_s^\alpha} \frac{\partial F_{s+1}^0}{\partial p_s^\alpha} \Big\} \gamma_{s+1}^{i-1} d\mathbf{q}_{s+1} d\mathbf{p}_{s+1}$$

Comparing corresponding terms, we can write:

$$\int \frac{\partial}{\partial q_1^\alpha} (\mu_s) \int \mu_{s+1} \ldots \mu_{s+i-1} d\mathbf{q}_{s+1} d\mathbf{q}_{s+2} \ldots d\mathbf{q}_{s+i}$$

$$= \frac{1}{i} \frac{\partial \gamma_s^i}{\partial q_1^\alpha}$$

and since

$$\frac{\partial \mu_s}{\partial q_1^\alpha} \mu_{s+1} \ldots \mu_{s+i-1} d\mathbf{q}_{s+1} \ldots d\mathbf{q}_{s+i}$$

$$= \frac{1}{i} \frac{\partial}{\partial q_1^\alpha} \int \mu_s \mu_{s+1} \ldots \mu_{s+i-1} d\mathbf{q}_{s+1} \ldots d\mathbf{q}_{s+i},$$

we arrive at the result

$$\gamma_s^i = \int \mu_s \mu_{s+1} \ldots \mu_{s+i-1} d\mathbf{q}_{s+1} \ldots d\mathbf{q}_{s+i}. \tag{32}$$

The above reasoning serves as a proof by induction of the correctness of the expression (30) for the "principal" parts of the particular solutions $\overline{F}_s^i$. $\overline{F}_s^i$ differs from the exact solution F_s^i only that in the latter additional terms are contained, while the structure of the solution $\overline{F}_s^i$ coincides with the structure of F_s^i, in the sense in which we are interested. To determine the exact solution F_s^i it is always necessary to operate not with the "principal" parts of the particular solutions, but with the complete solutions, i.e., with expressions of the type of Eq. (27), in which the constants b_s^k are chosen with reference to the boundary conditions on the relaxation of the correlations, as was done in obtaining Eq. (28). Use of the general solutions does not essentially complicate the proof of the stated theorem and leads only to more cumbersome expressions. We note, by the way, that the procedure considered by us also gives another method for the construction of the results of the theory of Ursell and Maier for the coordinate part of the distribution function. All the reasoning presented above holds also in the presence of external conservative forces; in this case one must understand H_s the energy of the corresponding group of particles including these external forces.

We shall make a few remarks regarding the dependence of the applicability of the above considerations on the character of the interaction law $\Phi(r)$ between the particles and on the density of the system. If the interaction potential falls off with increasing r faster than r^{-3}, and if the density of the system is sufficiently small, the multiplicative structure of the equilibrium solution is that derived above, where in the second part of the theorem an essential role is played by the assumption of the adiabatic character of the transition to the limit in the condition of the relaxation of the correlations, in the sense indicated earlier.

But if the potential Φ falls off with increasing r as r^{-3} or more slowly, then the series representing the part of the solution depending on the coordinates diverges, and an additional investigation is necessary. For a potential of the form

$$\Phi(r) = -ar^{-n} + br^{-m},$$

$$(m > n) \quad \text{or} \quad \Phi(r) = -ar^{-n} + Ae^{-\alpha r}$$

the reasoning presented is correct for $n > 3$, i.e., for a "sphere of influence" of the intermolecular forces which is finite, but not necessarily infinitely small (small on the microscopic scale).

Consequently, while in the considerations of Green[1] the smallness of the density of the system is not used, but rather the microscopic smallness of the "sphere of influence" is used (in essence a mechanism of instantaneous collisions is assumed), in the theorem presented above the opposite situation obtains: the smallness of the density of the system is explicitly used, while the microscopic smallness of the sphere of influence of intermolecular forces is not required; the size of the sphere of influence is limited simply by the formal requirement of convergence of the integrals appearing as coefficients of a series in powers of the density. In order to relax the requirements referring to the density of the system and to the character of the interaction, as mentioned above, an additional investigation is necessary.

In conclusion, let us consider the example of a system on which, besides the conservative internal forces, external forces depending only on the velocities of the particles are also acting.

The basic equation for the distribution function f_s in the most general case of arbitrary forces has the form:

$$\frac{\partial f_s}{\partial t} + \sum_{i=1}^{s} \frac{\partial f_s}{\partial \mathbf{x}_i} \mathbf{y}_i + \frac{1}{m} \times \sum_{i=1}^{s} \frac{\partial}{\partial \mathbf{y}_i} (f_s \mathbf{F}_i) + \frac{1}{m} \sum_{i,\, j} \frac{\partial}{\partial \mathbf{y}_i} (f_s \mathbf{F}_{ij}) + \frac{1}{m} \iint \sum_{i=1}^{s} \frac{\partial}{\partial \mathbf{y}_i} (\mathbf{F}_{i,\, s+1} f_{s+1})\, d\mathbf{x}_{s+1} d\mathbf{y}_{s+1} = 0, \tag{33}$$

where we have used the same notation as in Ref. 3 and in Eqs. (1)-(5) of this article. Setting

$$\partial f_s / \partial t = 0, \quad f_s = \psi_s(\mathbf{x})\, \varphi_s(\mathbf{y}),$$

for the equilibrium solution, we obtain

$$\sum_{i=1}^{s} \Big\{ \frac{\partial \psi_s}{\partial \mathbf{x}_i} \varphi_s \mathbf{y}_i + \frac{1}{m} \frac{\partial \varphi_s}{\partial \mathbf{y}_i} \Big(\sum_{j=1}^{s} \psi_s \mathbf{F}_{ij} + \int \psi_{s+1} \mathbf{F}_{i,\, s+1} d\mathbf{x}_{s+1} \Big) + \frac{\psi_s}{m} \Big(\frac{\partial \varphi_s}{\partial \mathbf{y}_i} \mathbf{F}_i + \frac{\partial \mathbf{F}_i}{\partial \mathbf{y}_i} \varphi_s \Big) \Big\} = 0. \tag{34}$$

A necessary condition for the realization of an equilibrium solution, existing in the absence of external forces of the type considered above, is the simultaneous fulfillment of the following equations

$$\frac{\partial\varphi_s}{\partial \mathbf{y}_i}\frac{\alpha}{m} = \varphi_s\mathbf{y}_i;\quad \frac{\partial\varphi_s}{\partial \mathbf{y}_i}\mathbf{F}_i + \frac{\partial \mathbf{F}_i}{\partial \mathbf{y}_i}\varphi_s = 0 \tag{35}$$

and, consequently, the forces $\mathbf{F}_i$ must satisfy the condition

$$(m/\alpha)(\mathbf{y}_i\mathbf{F}_i) + \partial\mathbf{F}_i/\partial\mathbf{y}_i = 0. \tag{36}$$

In particular it follows from (36) that gyroscopic forces will not disturb the equilibrium condition of the system. In an analogous way one can treat the more general conditions mentioned in Ref. 2.

I take this occasion to express my gratitude to N. N. Bogoliubov for proposing the problem, and also to M. A. Leontovich and Ia. B. Lopatinskii for discussion of questions connected with the work.

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Electron Energy Spectrum in a Crystal Located in a Magnetic Field

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It is shown that the discrete energy levels of an electron in a crystal are widened into narrow bands in strong magnetic fields. The structure of the energy zone in a crystal located in a magnetic field is studied. The possible influence of the broadening on the de Haas-van Alphen effect is discussed.

A FREE electron in a uniform magnetic field performs a finite motion[1] which corresponds to a classical revolution (at least in the direction perpendicular to the magnetic field H). The minimum quantum-mechanical "radius" of this revolution is

$$\alpha_0 = \sqrt{\hbar c/eH}. \tag{1}$$

In addition to the magnetic field the electron is acted upon in the crystal by a periodic electric field (the lattice constant will be designated by a), and for all real fields H.

$$\varepsilon = a/\alpha_0 \ll 1 \tag{2}$$

(thus, for example, with $H \sim 10^4$ oersteds and $a = 2.5 \times 10^{-8}$ cm we have $\epsilon = 10^{-2}$).

In the theory of electron motion in a crystal placed in a magnetic field the only terms that are retained (except in Ref. 2) are those which remain finite when $\epsilon \to 0$. As a result, the energy levels of the electron in a magnetic field are degenerate and depend, just as in the case of free electrons, on only two quantum numbers (see, for example, Refs. 3 and 4). If terms that vanish together with ϵ are retained the degeneracy is removed and the character of the spectrum is changed.

1. In the absence of the periodic field the energy levels of the motion of a free electron in the plane ⊥ H are expressed by the equation $E_n = \mu H(n + 1/2)$ and the eigenfunctions are

$$\psi_{k,n} = e^{ik_1x}\varphi_n\left(\frac{y+\alpha_0^2k_1}{\alpha_0}\right),$$

where the φ_n are Chebyshev-Hermite functions. The energy is independent of the quantum number k_1 which determines the position of the "center of oscillation" of the electrons $y_0 = -\alpha_0^2 k_1$, since all y_0 are equivalent in free space. In a periodic field this equivalence disappears and the degeneracy is removed. In the approximation of weakly bound electrons (when the periodic field can be considered a small perturbation) the following dispersion law holds for small n (at the bottom of the energy zone):

$$E_{k_1n} = \mu^* H\left(n + \frac{1}{2}\right) + \sum_{p=1}^{\infty} A_p(n, H)\cos\left(\frac{4\pi p\alpha_0^2 k_1}{a}\right), \tag{3}$$

where $A_1(n, H) = (-1)^n n(\mu H)^2 V^2 E_0^{-3}\pi^{-4}$; $E^0 = \hbar^2/ma^2$ and V is a quantity of the order of the first Fourier coefficients of the potential.

For large n (close to the middle of the energy zone) the ordinary perturbation theory of a degenerate state can be used, resulting in the dispersion law

$$E_{h_1n} = \mu H(n + {}^1/_2) + VJ_{nn}\left(\frac{2\pi}{\varepsilon}\right)\left(\cos\frac{2\pi y_0}{a} + \cos\frac{\pi}{p+1}m\right), \tag{4}$$

$$J_{nm}\left(\frac{2\pi}{\varepsilon}\right) = \int_{-\infty}^{+\infty} \varphi_n(x)\varphi_m(x)e^{2\pi ix/\varepsilon}dx.$$

in (4) p designates the rank of the secular determinant and m is any integer $m \leq p$. As $p \to \infty$ the argument $\pi m/(p+1)$ changes continuously from 0 to π. The integral $J_{nn}(2\pi/\epsilon)$ is always small for small n $(N \equiv \sqrt{2n+1} \ll \pi/\epsilon)$. Therefore, for small n a second-order correction of the energy must be made (which is given by (3)]. For $N > \pi/\epsilon$ $(\epsilon N - \pi \equiv \epsilon N_1 \ll \pi)$,

$$J_{nn}(2\pi/\varepsilon) = (-1)^{n+1}\varepsilon 2^{-1/4}\pi^{-5/4}(\varepsilon N_1)^{-1/4} \times \sin[2\pi\varepsilon^{-2}(\varepsilon N_1)^{3/2} - {}^3/_4\pi]$$

and thus $J_{nn} \sim \epsilon$.

It can be seen from (3) and (4) that to each n there corresponds not a level but a narrow zone or band, whereas for free electrons the levels are very sharp.

The relative level width (the width compared with the distance μH between levels) is for small n:

$$\delta(n,H) = \varepsilon^2 n 2\pi^{-4}V^2E_0^{-2}, \tag{5}$$

and for large n:

$$\delta(n,H) = 8VJ_{nn}(2\pi/\varepsilon)(\mu H)^{-1}. \tag{6}$$

The level width vanishes when $a \to 0$ or $V \to 0$.

From (5) and (6) it can be seen that in the approximation of weakly bound electrons the relative level width at the bottom of the zone is $\sim\epsilon^2$ and close to the middle of the zone it is $\sim\epsilon$. The levels broaden towards the middle of the zone.

2. The broadening of discrete levels into narrow bands also occurs in the approximation of strongly bound electrons. In this approximation and in the absence of a magnetic field the electron energy spectrum is known to be composed of allowed and forbidden zones. We shall examine the structure of an allowed energy zone in a magnetic field (the edges of the zone are almost unshifted).

The problem consists of the approximate solution of the equation

$$\frac{1}{2m}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}\right)^2\psi + V_p\psi = E\psi, \tag{7}$$

$$(A_x = -H_y,\ A_y = A_z = 0).$$

For our approximation we represent ψ by the series

$$\psi = \sum_{\mathbf{n}} a_{\mathbf{n}} \exp(-in_2x/\alpha_0^2)\varphi_0(\mathbf{r} - \mathbf{n}), \tag{8}$$

where n is the radius-vector of a lattice site, the functions in the sum satisfy the equation

$$\frac{1}{2m}\left(\hat{p} - \frac{e}{c}A\right)^2\psi + V(\mathbf{r} - \mathbf{n})\psi = E_0\psi, \tag{9}$$

and $V(\mathbf{r})$ is the potential function in an atom at the point $\mathbf{n} = 0$.

A difference equation is obtained for the coefficients $a_{\mathbf{n}}$. Separating the variables which refer to motion along the magnetic field, we obtain for the motion in a plane perpendicular to **H**

$$\lambda_\perp a(n_1,n_2) = a(n_1+1,n_2) + a(n_1-1,n_2) + a(n_1,n_2+1)e^{-i\varepsilon^2 n_1} + a(n_1,n_2-1)e^{i\varepsilon^2 n_1}, \tag{10}$$

where $\lambda_\perp$ is the energy of motion in this plane. This equation also follows from the polar model of a metal[5], where it determines the energy of motion of a quasi-particle (an excitation of the crystal, bearing an electric charge). In this paper it is shown that from (10) under the most general assumptions, there follows the functional equation

$$(\pm\lambda_\perp - 2\cos\varepsilon x)\varphi(x) = \varphi(x+\varepsilon) + \varphi(x-\varepsilon). \tag{11}$$

The signs + and – correspond to the lower and upper halves of the energy zone, while $\lambda_\perp = 0$ corresponds to the middle of the zone (± will be omitted hereafter).

Introducing the operator $\hat{K} = i\partial/\partial x$ we can write (11) as

$$(2\cos\varepsilon\hat{K} + 2\cos\varepsilon x)\,\varphi(x) = \lambda_\perp\,\varphi(x).$$

Since the operator in the left-hand side of the equation is invariant with respect to the substitution $\epsilon x \to \epsilon x + 2\pi$, the solution of this equation by the Flock-Bloch theorem is

$$\varphi(x) = e^{ik\varepsilon x}\,U_k(x)$$

and can be written as

$$\varphi(x) = \sum_{m=-\infty}^{+\infty} C(m\varepsilon + k\varepsilon)\,e^{i(m\varepsilon+k\varepsilon)x}. \qquad (12)$$

We expand the coefficients C in a series of Chebyshev-Hermite functions:

$$C(m\varepsilon + k\varepsilon) = \sum_{r=0}^{\infty} b_r\,\varphi_r(m\varepsilon + k\varepsilon).$$

Substituting this in (12) and, furthermore, employing Poisson's sum formula and a Fourier integral for the Chebyshev-Hermite functions:

$$\varphi_r(x) = \frac{1}{\sqrt{2\pi}\,i^r}\int_{-\infty}^{+\infty} \varphi_r(t)\,e^{itx}\,dx,$$

we write the solution as

$$\varphi(x) = \sum_{p=-\infty}^{+\infty} e^{-2\pi iph} \sum_{r=\infty}^{\infty} i^r b_r \varphi_r\left(x + \frac{2\pi p}{\varepsilon}\right). \qquad (13)$$

Substituting (13) in (11) and multiplying by $\varphi_n(x + 2\pi q/\epsilon)$, we integrate along the entire axis Ox and, using the relationships which exist for the functions φ_n [2]:

$$\Phi_{mn}(\alpha) \equiv \int_{-\infty}^{+\infty} e^{i\alpha x}\,\varphi_n(x)\,\varphi_m(x)\,dx$$

$$= e^{i(n-m)\pi/2}\int_{-\infty}^{+\infty} \varphi_m(x-\alpha)\,\varphi_n(x)\,dx$$

$$= e^{i(m-n)\pi/2}\int_{-\infty}^{+\infty} \varphi_m(x+\alpha)\,\varphi_n(x)\,dx,$$

we obtain

$$\begin{aligned} & b_n\lambda - 4\sum_q b_{n+4q}\,\Phi_{n,n+4q}(\varepsilon) \\ & + \sum_{r=0}^{\infty} i^{r-n} b_r\left[e^{2\pi ik} + (-1)^{r-n} e^{-2\pi ik}\right] \\ & \times\left\{\lambda\Phi_{rn}\left(\frac{2\pi}{\varepsilon}\right) - \int_{-\infty}^{+\infty} 2\cos\varepsilon x\,\varphi_r\left(x - \frac{2\pi}{\varepsilon}\right)\varphi_n(x)\,dx\right. \\ & - \int_{-\infty}^{+\infty} \varphi_r\left(x - \frac{2\pi}{\varepsilon} + \varepsilon\right)\varphi_n(x)\,dx \\ & \left. - \int_{-\infty}^{+\infty} \varphi_r\left(x - \frac{2\pi}{\varepsilon} - \varepsilon\right)\varphi_n(x)\,dx\right\} = 0. \end{aligned} \qquad (14)$$

Here we have dropped terms containing $\Phi_{rn}(4\pi/\epsilon)$, etc., which are of an order degree of exponential smallness than those which have been retained in (14).

In order to solve (14) in first approximation we also neglect these terms (i.e., the second summation). The coefficients $\Phi_{n,n+4q}(\epsilon)$ diminish rapidly with increasing q (as $1/(4q)!$]. It can be assumed that the coefficients b_{n+4q} change much more slowly (as will be confirmed later). Then we obtain

$$\lambda_\perp = 4\sum_q \Phi_{n,n+4q}(\varepsilon), \qquad (15)$$

where the summation actually extends only to $|q| \sim 2 - 3$. This assumption can be checked by setting $b_{n+2q} \approx b_n(-1)^q$ in a definite small interval of change of q and writing (13) with these b_{n+2q}.

The function which results agrees with the fundamental Eq. (11), where $\lambda_\perp$ agrees with (15). For small n, Eq. (15) or (16) gives $\lambda_\perp = 4 - \epsilon^2 N^2$, ($N^2 \equiv 2n + 1$), which is a solution that was obtained earlier[5].

The integrals $\Phi_{n,n+4q}(\epsilon)$ are expressed in terms of the Chebyshev-Laguerre polynomials $L_n^{(4q)}(\epsilon^2/2)$. By using the asymptotic expression for these polynomials in terms of Bessel functions[6], we obtain $\Phi_{n,n+4q}(\epsilon) \approx J_{4q}(N\epsilon)$. From the theory of Bessel functions it is known that

$$2J_0(x) + 4\sum_{q=1}^{\infty} J_{4q}(x) = 1 + \cos x,$$

whence we obtain for $\lambda_\perp$

$$\lambda_{\perp} = \pm 2(1 + \cos \varepsilon N). \tag{16}$$

This equation determines the energy levels of the particle as a function of the level number n. The width of the zone is 8 as in the absence of a magnetic field when $\lambda_{\perp}$ is expressed by $\lambda_{\perp} = 2 \times (\cos k_1 a + \cos k_2 a)$.

In second approximation we take the neglected terms into consideration, using for b_r the values obtained in first approximation [for the integrals $\Phi_{n,n+p}(2\pi/\epsilon)$, $q \ll n$ it is possible to use other asymptotic representations of the Chebyshev-Laguerre polynomials which fit the case $N < \pi/\epsilon$]. We then obtain instead of (16)

$$\lambda_{\perp} = \pm 2(1 + \cos \varepsilon N)/[1 + J_n(2\pi/\varepsilon)\cos 2\pi k], \tag{17}$$

where

$$J_n\left(\frac{2\pi}{\varepsilon}\right) = {}^1/_2(-1)^n \sqrt{\varepsilon N}\left[\left(\frac{\pi}{\varepsilon N}\right)^2 - 1\right]^{-1/4} N^{-1/2} \times \exp\left\{-\frac{4}{3}N^2\left(\frac{\pi}{\varepsilon N} - 1\right)^{3/2}\right\} \tag{18}$$

[(17) is valid when $J_n(2\pi/\epsilon) \ll 1$].

Thus even in the approximation of strongly bound electrons each n corresponds to a narrow band rather than to a single level. The width of this band, as can be seen from (18), depends strongly on n in the present approximation. Below and above the zone (for $\epsilon N \ll 1$) the levels are actually quite sharp; in the middle of the zone ($\epsilon N \lesssim \pi$) the levels broaden to distinct even though narrow bands.

Since $J_n(2\pi/\epsilon)$ is small, (17) can be written as

$$\lambda_{\perp} = \pm 2(1 + \cos \varepsilon N) \times [1 - J_n(2\pi/\varepsilon)\cos 2\pi k]. \tag{19}$$

Band widths in the approximation of weakly bound electrons are greater than in the strong binding approximation, but the general characteristics of the bands are identical. Thus, in both approximations if ϵN (i.e., the energy) is fixed and ϵ is decreased the width of the bands is diminished. If ϵ is fixed and N is increased (i.e., for levels closer to the middle of a zone) the band width increases.

It is seen from (16) that the structure of a zone is preserved in a magnetic field and that its edge is not shifted. For $H \to 0$, $\epsilon \to 0$ the number of levels within a zone increases and the width of each zone approaches the vanishing point. In the limit all levels fuse into a continuous zone.

3. If the broadening of levels is sufficiently large it must influence the de Haas-van Alphen effect. That is, the oscillations of magnetic susceptibility and of other physical quantities can in some cases possess the character of more or less strongly marked beats. In order to prove this let us assume that

$$E_{nk_3}(k_1) = f(n, k_3) + \Delta(k_1, n, k_3)$$

[k_1 is understood to stand for two continuously changing quantum numbers in (4)]. The number of electron states

$$Z(E, H) \sim \int dk_1 \sum_n k_3(n, E) = \int dk_1 \sum_{p=-\infty}^{+\infty} \int e^{2\pi i p n} k_3(n, E)\, dn.$$

The integrals in the summation can be calculated by the method of steepest descents, after which for the oscillating terms that enter into the thermodynamical potential Ω we obtain the following expression:

$$\cos\left(\frac{2\pi pc}{h\,|\,e\,|\,H} S_{\max} - \gamma\right) \int dk_1 \times \exp\left\{2\pi i p \frac{c}{h\,|\,e\,|\,H} \frac{\partial S_{\max}}{\partial \zeta} \Delta_{\max}(k_1, \zeta)\right\} \tag{20}$$

instead of the cosine which is obtained when the broadening is neglected. Here $S_{\max}$ is the extremal area of the section of the surface E=const cut off by the plane perpendicular to H[3], and ζ is the Fermi energy.

The effect of the additional factor in (20) can be studied by using the example of almost-free electrons, for which the integral in (20) is calculated as

$$J_0^2(2\pi p \Delta_{\max}/\mu H),$$

(J_0 is the zero-order Bessel function). Here the quantity $\Delta_{\max}/\mu H = \delta_{\max}$ in (6) oscillates as H changes [with considerably lower frequency than the cosine in (20)]. This leads to small fluctuations of J_0 without a change of sign (when the width of the levels is not large). Such modulated oscillations (beats) of magnetic susceptibility have actually been observed[7] in some metals. From the form of the argument of J_0 it can be seen that beats in small fields must be clearer than in larger fields, as is fully confirmed for beryllium[7], in which the beats were investigated in the range $3 \times 10^3 - 2 \times 10^4$ oersteds.

We note in conclusion that the broadening of the levels is strongly dependent on the approximation that is assumed and the the broadening must therefore be calculated outside the framework of the approximations for weakly or strongly bound electrons. It is easily shown that broadening occurs in the general case. In any periodic field V_p the state of an electron is described by the same quantum numbers k_1, n, k_3 by which we describe a free electron in a magnetic field. In fact, if in the Schrödinger equation[7] we put $A_x = -Hy$, $A_y = A_z = 0$, then because of the symmetry of the Hamiltonian (which remains periodic in the Ox and Oz directions) the general solution must be of the form

$$\psi_{k_1 n k_3} = e^{i(k_1 x + k_3 z)} u_{k_1 n k_3}(x, y, z), \tag{21}$$

where the functions $u_{k_1 n k_3}(x, y, z)$ is periodic with the period of the lattice along Ox and Oz and decreases as $\exp(-y^2/2\alpha_0^2)$ at infinity. We see from the equation for $u_{k_1 n k_3}$ that the energy of the electron $E_{n k_3}(k_1)$ is a periodic function of k_1 with the period a/α_0^2.

The author wishes to express his gratitude to I. M. Lifshitz for discussions of the present article.

Note added in proof: Attention has recently[8] been called to the broadening of levels and, in part, to the influence which this broadening has on the de Haas-van Alphen effect. The author is apparently unfamiliar with Ref. 2, which was published in 1952.

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Possible States of a Two-Graviton System

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Eigenfunctions are obtained for the angular momentum operator of one and two gravitons. The states of a single graviton are separated, according to their parity, into quasielectric and quasimagnetic states. The possible states of a two-graviton system are investigated. The Clebsch-Gordan coefficients are calculated for $S = 3$ and $S = 4$.

An investigation by Landau[1] concerning the angular momentum of a system of two photons led to the discovery of certain general selection rules limiting the possible values of angular momentum and parity for such a system. These rules appear as the consequence of specific characteristics of photons, associated with their zero rest mass.

Since gravitons, like photons, have zero rest mass, it would be of interest to look into the analogous problem of determining the limitations on the possible values of angular momentum and parity for a two-graviton system. By using generalized spherical vectors, as in earlier work, it is possible to obtain eigenfunctions of the angular momentum operator for one and two gravitons. The rules prohibiting certain states, characterized by definite values of total angular momentum and parity, follow from the apparent form of these functions.

In the linear approximation of the general theory of relativity, the tensor of the gravitational field $g_{\mu\nu}$, coincident with the metric tensor, departs little from its Galilean values $\delta_{\mu\nu} : g_{\mu\nu} = \delta_{\mu\nu} = h_{\mu\nu}$. Investigating this equation for a particle with zero rest mass and spin 2, Fierz[2] showed that in a gauge transformation of the field

$$h_{\mu\nu} = h'_{\mu\nu} + \partial\Lambda_\mu / \partial x_\nu + \partial\Lambda_\nu / \partial x_\mu,$$

where Λ_μ is an arbitray vector, only two components of the tensor $h_{\mu\nu}$ can remain different from zero, in accordance with the two possible values of polarization. Thus a weak graviational field is described by a three-dimensional tensor of second rank, satisfying the condition of orthogonality, with a trace equal to zero.

The operator of the total angular momentum $\hat{M}_{ij}$ of a single graviton is defined as a transformation of the tensor function by an infinitesimal three-dimensional rotation

$$(kl \mid \hat{M}_{ij} \mid rs) = \frac{1}{i}\delta_{kr}\delta_{ls}\left(p_i \frac{\partial}{\partial p_j} - p_j \frac{\partial}{\partial p_i}\right) - \frac{1}{i}(\delta_{jk}\delta_{ir}\delta_{ls} - \delta_{ik}\delta_{jr}\delta_{ls} + \delta_{jl}\delta_{is}\delta_{kr} - \delta_{il}\delta_{js}\delta_{kr}) = (kl \mid \hat{L}_{ij} + \hat{S}_{ij} \mid rs), \tag{1}$$

where p_i is the momentum of a graviton, and kl, rs are tensor indices taking the values 1, 2, 3. In configuration space the angular momentum operator (1) operates on the single-graviton wave function $\psi_{rs}(p)$.

The division of the total angular momentum into an orbital part $\hat{L}_{ij}$ and a spin part $\hat{S}_{ij}$ has limited physical meaning, since there is no rest system for a graviton, and therefore the usual definition of spin is not applicable. Furthermore, a state with a definite value of orbital and spin angular momentum does not satisfy the condition of orthogonality, so that only certain superpositions of these states have physical meaning. However, this separation enables us to construct eigenfunctions of the total angular momentum from more simple eigenfunctions of the orbital and spin angular momenta. The spherical harmonics $\Phi_{Lm} = aY_{Lm}(n)$ are eigenfunctions of the operators $\hat{L}^2_{ij}$ and $\hat{L}_z$, where L is the value of the orbital angular momentum, m is its projection on the z axis, and $\mathbf{n} = \mathbf{p}/|\mathbf{p}|$.

$$\int Y^*_{Lm} Y_{L'm'}\, d\omega = \delta_{LL'}\,\delta_{mm'}; \qquad \int_{-\infty}^{\infty} a^2 n^2\, dn = 1.$$

From the equations

$$(kl \mid {}^1/_2 \hat{S}^2_{ij} \mid rs)\,\chi_{\mu,\,rs} = S(S+1)\,\chi_{\mu,\,kl} = 6\chi_{\mu,\,kl}, \tag{2}$$

$$(kl \mid \hat{S}_z \mid rs)\,\chi_{\mu,\,rs} = \mu\chi_{\mu,\,kl}, \qquad \mu = 0, \pm 1, \pm 2.$$

(where μ is the projection of the spin $S = 2$ on the z-axis) the following eigenfunctions of the spin operators are obtained:

$$\chi_{0,\,kl} = \frac{1}{\sqrt{6}}\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -2\end{pmatrix}; \tag{3}$$

$$\chi_{\pm 1,\,kl} = \mp\frac{1}{2}\begin{pmatrix}0 & 0 & 1\\ 0 & 0 & \pm i\\ 1 & + i & 0\end{pmatrix};$$

$$\chi_{\pm 2,\,kl} = -\frac{1}{2}\begin{pmatrix}1 & \pm i & 0\\ \pm i & -1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

normalized so that $\chi^*_{\mu,kl}\,\chi_{\mu',\,kl} = \delta_{\mu\mu'}$. We write the wave function of a single graviton in compliance with the Clebsch-Gordan formula for the separation of the direct product of the representations $D_L \times D_S$ into irreducible parts:

$$\psi_{kl}^{JLM} = a\sum_{\mu} C_{JM}^{L,\,M-\mu;\,2,\,\mu}\,Y_{L,\,M-\mu}\,\chi_{\mu,\,kl}, \tag{4}$$

where J is the value of the total angular momentum, M is its projection on the z-axis, L is the value of the orbital angular momentum and the $C_{J,M}^{L,M-\mu;\,2,\mu}$ are the Clebsch-Gordan coefficients, given, for example, in Ref. 3.

Taking advantage of the definition of the generalized spherical S-vectors of type λ[4]

$$(Y^S_{J,\,J+\lambda,\,M})_\mu \tag{5}$$
$$= (-1)^\lambda\left[\frac{2J+1}{2J+2\lambda+1}\right]^{1/2} C^{J,\,M;\,S,\,\mu}_{J+\lambda,\,M+\mu}\,Y_{J+\lambda,\,M+\mu},$$

we rewrite the eigenfunction of a single graviton in the form

$$\psi_{kl}^{JLM} = a\sum_{\mu}(Y^2_{JLM})^{\mu}\,\chi_{\mu,\,kl}, \tag{6}$$
$$(Y^2_{JLM})^{\mu} = (-1)^{\mu}\,(Y^2_{JLM})_{-\mu}.$$

From the functions ψ_{kl}^{JLM} it is possible to construct two linearly independent combinations, satisfying the orthogonality condition

$$n_k\,\psi_{kl}^{JM} = 0;\quad \psi_{kl}^{JM} = a\sum_{\mu}(Y^{JM})^{\mu}\,\chi_{\nu,\,kl}; \tag{7}$$
$$Y^{JM} = \sum_L \rho_L\,Y_{JLM};$$
$$Y^{(1)\,JM} = \left[\frac{J+2}{2J+1}\right]^{1/2} Y_{J,\,J-1,\,M} + \left[\frac{J-1}{2J+1}\right]^{1/2} Y_{J,\,J+1,\,M};$$
$$Y^{(2)\,JM} = \left[\frac{(J+1)(J+2)}{(2J+1)(2J-1)}\right]^{1/2} Y_{J,\,J-2,\,M} + \left[\frac{6(J-1)(J+2)}{(2J-1)(2J+3)}\right]^{1/2} Y_{J,\,J,\,M} + \left[\frac{J(J-1)}{(2J+1)(2J+3)}\right]^{1/2} Y_{J,\,J+2M}.$$

The two solutions (7) differ in parity. For tensor fields the inversion operator I is defined in the following way:

$$I\,\psi_{kl}(\mathbf{n}) = \psi_{kl}(-\mathbf{n}).$$

Noting the relationship $Y_{Lm}(-\mathbf{n}) = (-1)^L Y_{Lm}(\mathbf{n})$, we obtain

$$IY^{(1)\,JM} = (-1)^{J+1}\,Y^{(1)\,JM},$$
$$IY^{(2)\,JM} = (-1)^{J}\,Y^{(2)\,JM}.$$

By analogy with the solutions for a single photon[5] we call $Y^{(1)\,JM}$ a state of quasimagnetic type, $Y^{(2)JM}$ a state of quasielectric type.

As was to be expected, both solutions (7) are meaningful only for values of $J \geq 2$, that is, the total angular momentum carried away by a graviton cannot be less than two. This follows from the quadrupole character of graviton emission.

Two free gravitons are desribed by a tensor of fourth rank

$$\psi_{klmn}(\mathbf{p}_1\mathbf{p}_2) = \psi_{mnkl}(\mathbf{p}_2\mathbf{p}_1).$$

In the center-of-mass system $\mathbf{p}_1 + \mathbf{p}_2 = 0$ the wave function will depend only on the relative momentum $\mathbf{p}_1 - \mathbf{p}_2 = 2\mathbf{p}$, and the orbital angular momentum can be represented in the form

$$\hat{L}_{ij} = -i\left(p_i\frac{\partial}{\partial p_j} - p_j\frac{\partial}{\partial p_i}\right). \tag{8}$$

We obtain the following expression for the spin operator of two gravitons:

$$(klmn\,|\,\hat{S}_{ij}\,|\,pq\,rs) \tag{9}$$
$$= (kl\,|\,\hat{S}_{ij}\,|\,pq)\,\delta_{mr}\,\delta_{ns} + (mn\,|\,\hat{S}_{ij}\,|\,rs)\,\delta_{kp}\,\delta_{lq}$$
$$= -i\,(\delta_{jp}\,\delta_{ik}\,\delta_{ql} - \delta_{ip}\,\delta_{jk}\,\delta_{ql} - \delta_{iq}\,\delta_{pk}\,\delta_{jl} + \delta_{jq}\,\delta_{pk}\,\delta_{il})\,\delta_{rm}\,\delta_{sn}$$
$$-\,i\,(\delta_{jr}\,\delta_{im}\,\delta_{sn} - \delta_{ir}\,\delta_{jm}\,\delta_{sn} - \delta_{is}\,\delta_{rm}\,\delta_{jn} + \delta_{js}\,\delta_{rm}\,\delta_{in})\,\delta_{pk}\,\delta_{ql}.$$

Taking advantage of the fact that the eigenfunctions of the orbital angular momentum are the spherical harmonics Y_{Lm}, and that those of the spin angular momentum are the quadratic combinations $\chi_{\mu_1,kl}\,\chi_{\mu_2,\,mn}$, we construct solutions for two gravitons from the Clebsch-Gordan formula:

$$\psi_{klmn}^{JLMS} = a\sum_{\mu} C_{JM}^{L,\,M-\mu;\,S,\,\mu}\,Y_{L,\,M-\mu}\,\chi^S_{\mu,\,klmn}, \tag{10}$$
$$\chi^S_{\mu,\,klmn} = \sum_{\mu_2} C^{2,\,\mu-\mu_2;\,2\mu_2}_{S,\,\mu}\,\chi_{\mu-\mu_2,\,kl}\,\chi_{\mu_2,\,mn}.$$

Here J is the total angular momentum of the two gravitons, L is the orbital angular momentum, M is the projection of J on the z-axis, S is the spin of the two-graviton system and μ is the projection of S on the z-axis. In view of the normalization properties

$$\chi^{*S}_{\mu,\, hlmn}\, \chi^{S'}_{\mu',\, hlmn} = \delta_{SS'}\, \delta_{\mu\mu'},$$

$$\int (Y^{*S}_{JLM})^{\mu}\, (Y^{S}_{J'L'M'})^{\mu}\, d\omega = \delta_{JJ'}\, \delta_{LL'}\, \delta_{MM'},$$

it can be shown that the functions (10) are normalized in the following way:

$$\int \psi^{*JLMS}_{hlmn}\, \psi^{J'L'M'S'}_{hlmn}\, dp = \delta_{JJ'}\, \delta_{LL'}\, \delta_{MM'}\, \delta_{SS'}.$$

If we take into account the orthogonality requirement

$$\sum n_h \psi^{JM}_{hlmn} = 0; \quad \psi^{JM}_{hlmn} = \sum_{L,\,S} \rho_{L,\,S}\, \psi^{JLMS}_{hlmn} \tag{11}$$

$$= a \sum_{\mu,S} (Y^{JMS})^{\mu}\, \chi^{S}_{\mu,\, hlmn},\; Y^{JMS} = \sum_{L} \rho_{L,\,S}\, (Y^{S}_{JLM})$$

we can determine the apparent form of all 25 coefficients $\rho_{L,S}$ for the five possible values of L : J, $J \pm 1$, $J \pm 2$, and the five values of S: 0, 1, 2, 3, 4. It should be pointed out that for two identical particles, the parity of the orbital angular momentum conforms with that of the total spin.

For determining the coefficients $\rho_{L,S}$ we require the Clebsch-Gordan coefficients for $S = 3$, calculated in Ref. 6 for $S = 4$, which can be calculated by the usual methods from the formulas (3.110) and (3.111) in Ref. 3. By comparing the coefficients of identical spherical harmonics with zero, we obtain from the requirement (11) the result that, for odd states with $L = 2k + 1$, and for odd total angular momenta J,

$$\rho_{J,\,1} = \rho_{J,\,3} = \rho_{J \pm 2,3} = 0.$$

Thus the state $L = 2k + 1$, $J = 2n + 1$ is forbidden for any J.

In an anlogous manner we obtain the coefficients $\rho_{L,S}$ for odd states with even total angular momentum, that is, we determine the following functions, normalized to unity:

$$\begin{aligned} Y^{IJ,\, M,\, 3} & \qquad\qquad (12) \\ = & \left[\frac{3J(J-1)(J+1)}{4(2J-3)(2J+1)(2J+3)}\right]^{1/2} Y^{3}_{J,\, J-1,\, M} \\ & - \left[\frac{3J(J+1)(J+2)}{4(2J-1)(2J+1)(2J+5)}\right]^{1/2} Y^{3}_{J,\, J+1,\, M} \\ & - \left[\frac{5J(J-1)(J-2)}{4(2J-3)(2J-1)(2J+1)}\right]^{1/2} \\ & \times Y^{3}_{J,\, J-3,\, M} \\ & + \left[\frac{5(J+1)(J+2)(J+3)}{4(2J+1)(2J+3)(2J+5)}\right]^{1/2} Y^{3}_{J,\, J+3,\, M}. \end{aligned}$$

This solution admits odd states with even angular momentum for any J.

Even states with odd total angular momentum can be described by the following functions, normalized to unity:

$$\begin{aligned} Y^{\mathrm{II}\, J,\, M,\, 4} = & \left[\frac{7(J-3)(J+3)(J+4)}{2(2J+3)(2J-3)(2J+1)}\right]^{1/2} Y^{4}_{J,\, J-1,\, M} \\ & + \left[\frac{7(J-3)(J-2)(J+4)}{2(2J+5)(2J+1)(2J-1)}\right]^{1/2} Y^{4}_{J,\, J+1,\, M} \\ & + \left[\frac{(J+4)(J+3)(J+2)}{2(2J-1)(2J-3)(2J+1)}\right]^{1/2} \\ & \times Y^{4}_{J,\, J-3,\, M} \qquad\qquad (13) \\ & + \left[\frac{(J-3)(J-2)(J-1)}{2(2J+5)(2J+3)(2J+1)}\right]^{1/2} Y^{4}_{J,\, J+3,\, M}. \end{aligned}$$

There will be no state here with $S = 2$, since $\rho_{J \pm 1,2} = 0$. For $J = 1$ the rules for the addition of angular momentum admit $L = 4$, but for $J = 1$, $\rho_{J \pm 3,4} = 0$; Similarly, for $J = 3$

$$\rho_{J-1,4} = \rho_{J+1,4} = \rho_{J+3,4} = 0,$$

that is, even states of two gravitons with $J = 1$ and $J = 3$ are forbidden.

For even states with even total angular momenta the solution $\rho_{J,2} = \rho_{J \pm 2,2} = 0$ is obtained:

$$\begin{aligned} Y^{III\, J,M,4} & \qquad\qquad (14) \\ = & \left[\frac{(J+1)(J+2)(J+3)(J+4)}{4(2J+1)(2J-1)(2J-3)(2J-5)}\right]^{1/2} Y^{4}_{J,J-4,M} \\ & + \left[\frac{7(J-3)(J+2)(J+3)(J+4)}{(2J+3)(2J+1)(2J-1)(2J-5)}\right]^{1/2} Y^{4}_{J,J-2,\, M} \\ & + \left[\frac{35(J-3)(J-2)(J+3)(J+4)}{2(2J+5)(2J+3)(2J-1)(2J-3)}\right]^{1/2} Y^{4}_{J,J,\, M} \\ & + \left[\frac{7(J-3)(J-2)(J-1)(J+4)}{(2J+7)(2J+3)(2J+1)(2J-1)}\right]^{1/2} Y^{4}_{J,J+2,M} \\ & + \left[\frac{J(J-1)(J-2)(J-3)}{4(2J+7)(2J+5)(2J+3)(2J+1)}\right]^{1/2} Y^{4}_{J,J+4,M}. \end{aligned}$$

For the case under discussion $S = 4$: for $J = 0$, L can be equal only to 4, and the corresponding $\rho_{J+4,4}$ is found to be equal to zero; for $J = 2$, $\rho_{J,4} = \rho_{J+2,4} = \rho_{J+4,4} = 0$.

It can be easily verified that even states with even total angular momentum can be described by the following equation, which satisfies the orthogonality condition (11):

$$\psi^{JVJM}_{klmn} = \frac{1}{2\sqrt{6}} Y_{JM} (\eta^0_{knlm} + \eta^0_{kmln} - \eta^0_{klmn}) = \psi_0 \chi^0_{klmn} = \psi^0_{klmn}, \tag{15}$$

where

$$\eta^0_{klmn} = (\delta_{kl} - p_k p_l / p^2)(\delta_{mn} - p_m p_n / p^2).$$

Substituting $\varphi_0 \chi^0_{klmn}$ into the equation for the eigenfunction of the square of the angular momentum operator:

$$\begin{aligned} M^2 \psi^0_{klmn} = {} & ({}^1\!/_2 \hat{\mathscr{M}}^2_{ij} + 8) \psi^0_{klmn} \\ & + 2i (\hat{\mathscr{M}}_{kp} \psi_{plmn} + \hat{\mathscr{M}}_{lq} \psi^0_{kqmn} \\ & + \hat{\mathscr{M}}_{mr} \psi^0_{klrn} + \hat{\mathscr{M}}_{ns} \psi^0_{klms}) \\ & + 2 (2\psi^0_{mlkn} + 2\psi^0_{nlmk} + 2\psi^0_{klmn} \\ & - \delta_{km} \psi^0_{plpn} - \delta_{kn} \psi^0_{plmp} - \delta_{lm} \psi^0_{kppn} \\ & - \delta_{ln} \psi^0_{kpmp} - \delta_{mn} \psi^0_{klpp} - \delta_{kl} \psi^0_{ppmn}), \end{aligned} \tag{16}$$

we obtain an equation for ψ_0,

$$M^2 \psi^0 = {}^1\!/_2 \mathscr{M}^2_{ij} \psi^0, \tag{17}$$

that is, $M^2 = J(J+1)$; $\psi_0 = P_{JM} e^{iM\varphi} = Y_{JM}$, a spherical harmonic. From the symmetry condition for the two-graviton solution

$$\psi^0_{klmn}(p) = \psi^0_{mnkl}(-p),$$

noting that $\chi^0_{klmn} = \chi^0_{mnkl}$, we obtain

$$Y_{JM}(\mathbf{p}) = Y_{JM}(-\mathbf{p}) = (-1)^J Y_{JM}(\mathbf{p}),$$

that is, $J = 2n$.

All results concerning the number of states are brought together in the following table:

J	Even States	Odd States
0	1	1
1	0	0
2	1	1
3	0	0
$2n$	2	1
$2n+1$	1	0

Here it is obvious that for two gravitons, the appearance of states with total angular momentum $J = 1$ and $J = 3$, as well as the appearance of odd states with odd total angular momentum, is strictly forbidden.

These results are in complete agreement with the rules ascertained by Shapiro[7], by which it is forbidden for a particle to disintegrate into two identical bosons of zero rest mass and spin S if 1) the spin of the disintegrating particle is odd and less than $2S$, or if 2) the spin of the disintegrating particle is odd and its wave function is odd.

The method used here makes it possible to determine the number of states with predetermined angular momentum and parity, and also to obtain eigenfunctions for one and two gravitons.

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Properties of Energy Surfaces of Heavy Nuclei

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A detailed investigation is carried out of the properties of the energy surface of heavy nuclei for each of the four types of nuclear parity. Along with a more precise formulation of known empirical laws and parameters of the energy surface, it is shown that for given mass numbers A, the masses of the nuclei reach a minimum value for several different values of Z depending on the parity of Z. The curvature of the isobar parabolas does not depend on the parity of Z and is apparently several times larger for nuclei with even A. The character of the shell $N = 126$ and $Z = 82$ is made clear. The results are compared with the usual formulas for the binding energy.

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1. INTRODUCTION

ACCORDING to statistical theory, the masses of the nuclear isobars $M(A, Z)$ reach a minimum value M^* for certain values $Z = Z^*$ for which $\partial M(A, Z)/\partial Z = 0$[1]. This conclusion is found in qualitative agreement with the empirical rule of Mattauch-Shchukarev, according to which there is only one β-stable nucleus among isobaric nuclei of odd mass. The statistical theory also points to the parabolic character of the cross section of the energy surface, at least in the neighborhood of $Z = Z^*$. As the systematics of nuclear mass show (which have been given in a number of works[2-8]), the parabolic character of the isobaric energy cross sections in general agrees with experiment. There is also established empirically the dependence of the energy of the nucleus on the type of parity. Under otherwise equal conditions (one and the same A and Z) the masses of even-even nuclei (Z_e, N_e) are less than the masses of nuclei with odd mass numbers, which in turn have smaller masses than the odd-odd nuclei (Z_o,N_o). This difference of nuclear masses, which is connected with their parity, is taken into account by the well-known correction to the parity[1]. There is also evidence on the existence of small differences in the mass of nuclei with odd mass numbers, depending on the parity of Z[9-11]. Finally, a sharp change has been discovered in the energy of nuclei in the neighborhood of places of filling of the neutron and proton shells[3-8].

Thus, the principal properties of the energy surface of nuclei are sufficiently well understood at the present time. However, many details still need to be made more exact. In particular, it would be desirable to determine whether the real isobaric curves are exactly quadratic parabolas or, whether one can consider the parabolas to be some sort of approximation. Moreover, there is interest in making more precise the parameters of the energy surfaces, especially taking into account all possible corrections to the parity, the role of the shells, and other possible individual peculiarities of the nuclei. Here there is known a significant number of total energies of isobaric transitions, in part measured experimentally, in part calculated from the energy of α - β cycles[7,8]. At present, in the interval $210 < A < 240$, significant appearances of any shells or subshells[3,8,13,19,23] which would alter the general regularities have not been discovered.

In the present work the energies of β-transitions of heavy nuclei ($A > 210$) have been used for purposes of empirical analysis. Here no previous assumptions are made on the character of the mutual position of the energy surfaces in dependence on the parity of A or Z. It is only noted that there exist no more than 4 different types of nuclei, connected with the parity of A and Z, the masses of which, under otherwise equal conditions, can be different.

2. SYSTEMATICS OF THE ENERGY OF β-TRANSITIONS

The necessary energies of β-transitions Q_β were taken from tables[14] which contain, along with the experimental values, values computed from the energetics of α - β cycles. We also have used the data on the systematics of the energy of α-decay[9,12,13] which permit us to calculate a certain number of energies Q_β in addition. For confirmation, the latter values were computed in independent fashion from $n\beta$- and $n\alpha$-cycles. For this purpose, the necessary experimental values of the binding energy of the neutrons were taken from Refs. 15-17.

For convenience in the subsequent analysis, we systematized the energies of the β-transitions in a fashion somewhat different than was done in

Refs. 3, 7 and 8. In the graph of the dependence of the energy on Z, the energies $E_{\beta-}$ for β^--transitions and the energies E_{β^+} for β^+-transitions (K capture) of isobars (A = const) of a given parity (for example, odd-odd) are plotted in Fig. 1. For convenience, we used for the energies of the β^-- and β^+-transitions, not the quantities Q_{β^-} and Q_{β^+}, but

$$E_{\beta^-} = Q_{\beta^-} + 0.51 \text{ mev and}$$

$$E_{\beta^+} = Q_{\beta^+} - 0.51 \text{ mev.}$$

E_{β^-} (and also E_{β^+}) represent the differences of masses of nuclear isobars between which β^- (β^+) transitions occur (in contrast to Q_{β^-} and Q_{β^+} which correspond to mass differences of the neutral atoms). In those cases, when the isobar X_Z^A was shown to be stable, for example, in the behavior of β^--decay, the energy E_{β^+} with negative sign is plotted on the graph for a given Z. This energy is that with which the nucleus X_{Z+1}^A is converted into the given nucleus X_Z^A. The same was achieved when the given isobar X_Z^A was shown to be stable relative to conversion into the isobar X_{Z-1}^A. The points of β^--transitions and those of β^+-transitions (for A = const) are separately joined together on the graph. As is seen from Fig. 1, the lines of the β^-- and β^+-transitions for each of the values of A represent very nearly straight lines, which intersect crosswise (at a point which we shall denote by Z^0). The slopes of each of the branches are independent of the mass numbers A within the limits of experimental error.

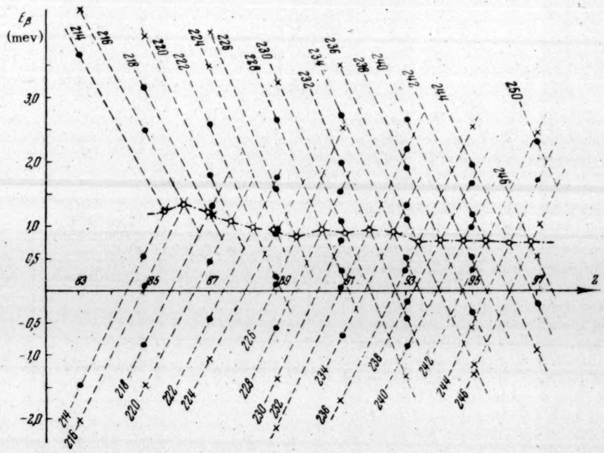

FIG. 1. Energies of β-transitions of odd-odd nuclei. ●—experimental values also obtained from α-β cycles, ×—values found with the help of extrapolated values of E_α, o—points of intersection of β^- and β^+ branches.

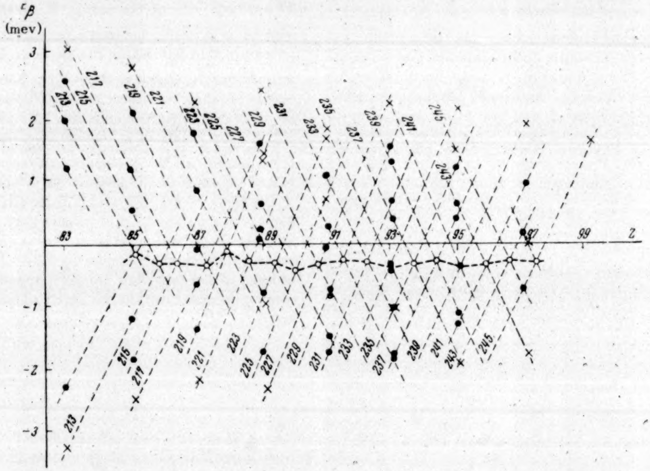

FIG. 1a. Energies of β-transitions of nuclei with A odd. ●--experimental values also found from α-β cycles, ×--values in the finding of which the data of α-decay were used, o--intersection points of β^- and β^+ branches.

In order to establish this, we combine in one graph the curves which pertain to different A, by means of a plane parallel shift such that the points Z^0 coincide. As is seen from Fig. 2, where such a shifting has been carried out, all the points lie on two crosswise intersecting straight lines.

If, in place of the odd-odd nuclei considered here, one chooses a nucleus of different parity (see Fig. 1*a*), then an analogous picture is obtained. We note one special circumstance, which was discovered upon consideration of Fig. 2 (and others analogous to it for other types of parity), in which the slopes of the right and left branches of the "cross" are different according to the following Table I.

3. ANALYSIS OF THE EMPIRICAL DATA

Making use of the results of the preceding section, we can draw concrete conclusions about the property of the energy surface in the region of heavy nuclei.

Let us consider a typical case of the intersection of the branches of β^- and β^+-decay for any fixed value of A (Fig. 3). The equation of the line AB will be

TABLE I

Type of Parity		Tangent of the angle of slope
Left Branch	Right Branch	
$Z_o, N_e, (A_o)$	$Z_e, N_o, (A_o)$	1.18
$Z_o, N_o, (A_e)$	$Z_e, N_e, (A_e)$	1.14
$Z_e, N_o, (A_o)$	$Z_o, N_e, (A_o)$	1.08
$Z_e, N_e, (A_e)$	$Z_o, N_o, (A_e)$	1.06

$$E_{\beta^-}(A_e, Z_o) = D_{oo} - \operatorname{tg}\alpha_{oo}(Z - Z^0_{oo}) \qquad (1)$$

(the subscripts *nn* are inserted to show that in the given case we are considering β^--decay of nuclei of the type Z_o, N_o). Similarly, for the branch corresponding to β^+-transitions, $Z_o \rightarrow (Z-1)_e$ (the line CD in Fig. 3):

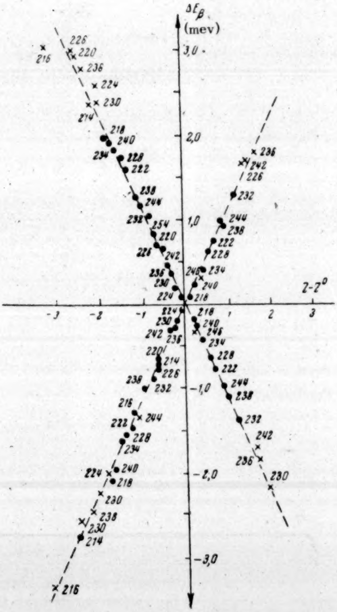

FIG. 2. Reduced energies of β-transitions.

$$E_{\beta^+}(A_e, Z_o) = D_{oo} + \mathrm{tg}\,\beta_{oo}(Z - Z^0_{oo}). \quad (1a)$$

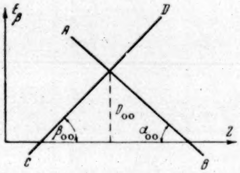

FIG. 3

Moreover, the following relations are valid:

$$E_{\beta^-}(A_e, Z_o) = -E_{\beta^+}(A_e, Z_o + 1);$$

$$E_{\beta^+}(A_e, Z_o) = -E_{\beta^-}(A_e, Z_o - 1).$$

On the other hand, $E_{\beta^-}(A_e, Z_o)$ must be equal to the difference of the masses of the initial $M(A_e, Z_o)$ and the final $M(A_e, Z_o + 1)$ of the nuclei:

$$E_{\beta^-}(A_e, Z_o) = M(A_e, Z_o) - M(A_e, Z_o + 1)$$

$$= D_{oo} - \mathrm{tg}\,\alpha(Z - Z^0_{oo}). \quad (2)$$

Further, in similar fashion,

$$M(A_e, Z_o + 1) - M(A_e, Z_o + 2) \quad (2a)$$

$$= -D_{oo} - \mathrm{tg}\,\beta_{oo}(Z + 2 - Z^0_{oo}).$$

Combining (2) and (2a), we find

$$M(A_e, Z_o) - M(A_e, Z_o + 2)$$

$$= -(\mathrm{tg}\,\alpha_{oo} + \mathrm{tg}\,\beta_{oo})(Z - Z^0_{oo}) - 2\,\mathrm{tg}\,\beta_{oo}.$$

Since $Z^0_{oo} = f(A)$, then we can write in place of the foregoing:

$$M(A_e, X_o) - M(A_e, X_o + 2) \quad (3)$$

$$= -\lambda_{oo}X_o - \mu_{oo},$$

where

$$X_o = Z - Z^0_{oo};$$

$$\lambda_{oo} = \mathrm{tg}\,\alpha_{oo} + \mathrm{tg}\,\beta_{oo};$$

$$\mu_{oo} = 2\,\mathrm{tg}\,\beta_{oo}.$$

The functional Eq. (3) has the following solution:

$$M(A_e, Z_o) = a_{oo}(A) + {}^1/_4\,\lambda_{oo}X_o^2 + {}^1/_2\,\nu_{oo}X, \quad (4)$$

where $\nu_{oo} = \tan\beta_{oo} - \tan\alpha_{oo}$. Thus,

$$M(A_e, Z_o)$$

$$= a_{oo}(A) + {}^1/_4\lambda_{oo}(Z - Z^0_{oo})^2 + {}^1/_2\nu_{oo}(Z - Z^0_{oo}).$$

Consequently,

$$M(A_e, Z_e)$$

$$= a_{oo}(A) + {}^1/_4\lambda_{oo}(Z - Z^0_{oo})^2 - D_{oo} - {}^1/_4\lambda_{oo}.$$

From the condition $(\partial M_{oo}(A, Z)/\partial Z)_A = 0$ we find

$$Z^*_{oo} = Z^0_{oo} - \nu_{oo}/\lambda_{oo}, \quad (Z^*_{ee} = Z^0_{oo}).$$

Therefore, $M(A_e, Z_o)$ can be written in the form

$$M(A_e, Z_o)$$

$$= a_{oo}(A) + k_{oo}(Z - Z^*_{oo})^2 + (\nu_{oo})^2/16k_{oo},$$

where $k_{oo} = 1/4\,\lambda_{oo}$ represents the curvature of the isobar parabolas. One can determine the correction to the parity $2\delta^{ee}_{oo}$ between nuclei of the type Z_e, N_e and nuclei of the type Z_o, N_o as the difference of the energy between the minimum corresponding parabolas. In such a case,

$$\delta^{ee}_{oo} = {}^1/_2\{M(A_e, Z^*_{oo}) - M(A_e, Z^*_{ee})\}$$

$$= {}^1/_2\{D_{oo} + k_{oo} + \nu^2_{oo}/16k_{oo}\}. \quad (5)$$

Since $\nu_{oo} \ll k_{oo}$, then

$$\delta^{ee}_{oo} \approx {}^1/_2(D_{oo} + k_{oo}). \quad (5a)$$

Thus we can write

$$M(A_e, Z_o) \approx b_{oo}(A) + k_{oo}(Z - Z^*_{oo})^2 - \delta^{ee}_{oo}, \quad (6)$$
$$M(A_e, Z_e) \approx b_{oo}(A) + k_{oo}(Z - Z^0_{oo})^2 + \delta^{ee}_{oo},$$

where

$$b_{oo} = a_{oo} + (\nu^2_{oo}/16k_{oo}) - \delta^{ee}_{oo}.$$

Completely analogous to the foregoing, for the case of odd mass numbers A_0 we get the following dependencies of the masses of the nuclei $M(A, Z)$ on Z and A:

$$M(A_o, Z_o) = b_{oe}(A) + k_{oe}(Z - Z^*_{oe})^2 + \delta^{eo}_{oe},$$
$$M(A_o, Z_e) = b_{oe}(A) + k_{oe}(Z - Z^0_{oe})^2 - \delta^{eo}_{oe} \quad (7)$$

(oe denotes an odd number of protons, even number of neutrons; eo, an even number of protons, odd number of neutrons). We note that in Eq. (7) the correction δ^{eo}_{oe} appears, which takes into account the dependence of the mass of odd nuclei (A-odd) on the parity of Z. The mean value of such a correction in the interval of mass numbers under consideration is about 0.15 mev, in agreement with the estimates given in Refs. 9-11 and 19. This fact shows that the coupling energy of the protons is somewhat larger than the coupling energy of the neutrons. An analogous correction δ^{ee}_{oo} between even-even and odd-odd Z and N is shown to be equal in mean to 0.75 mev. The dependence of δ^{eo}_{oe} and δ^{ee}_{oo} on A, obtained from Eq. (5a) upon consideration of values of D and k found empirically (see Fig. 1), is plotted in Fig. 4. There is also shown the dependence of the radius of curvature of the isobar parabolas k on the mass number A for the case of even and odd values of A.

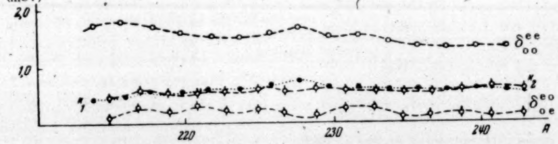

FIG. 4. Corrections to the parity δ and coefficient k as functions of A. k_1 are the curvatures of the isobar parabolas for A even, k_2 for A odd.

As Figs. 4 and 1 show, k does not change with increase in A, even though it does experience small fluctuations. The mean values of k are equal to 0.56 and 0.54 for the case of even and odd mass numbers, respectively, and do not depend on the parity of A.

In accord with the above, the values of Z^* determined from the condition $\partial M/\partial Z = 0$ are equal for all four types of parity, and thus, instead of one curve $Z^* = f(A)$, we have in fact a curve for nuclei of each of the four types (4 curves in all). However, these curves are very close together. One can represent approximately the curve for Z^*[11,19,22,23] (not taking into account the small deviations from this curve resulting from the individual properties of the nuclei: parity, etc.) in the range of mass numbers under consideration ($214 < A < 244$) by the following empirical formula

$$Z^*_{av} = 0{,}356\,A + 9{,}0. \quad (8)$$

4. THE CHARACTER OF THE NUCLEAR SHELLS $N = 126$ AND $Z = 82$

For the calculation of the jumps or "anomalies" in the energies of β-decay, associated with nuclear shells, one must determine the difference between the experimental energies of β-transitions, E_β and the values of E_β which do not take into consideration the effect of the shells, and which can be calculated by Eqs. (1) and (2). Equal energy differences, E_{β^-} are different from zero in the following cases (all values in mev):

$Tl^{210}(+1.5)$, $Tl^{209}(+1.6)$, $Tl^{208}(+2.1)$, $Tl^{206}(-0.5)$,
$Tl^{205}(-1.2)$, $Pb^{214}(-0.4)$, $Pb^{213}(-0.7)$, $Pb^{212}(-0.5)$,
$Pb^{211}(-0.6)$, $Pb^{210}(-0.6)$, $Pb^{209}(-0.3)$, $Bi^{208}(+2.2)$,
$Bi^{215}(-0.3)$, $Bi^{214}(-0.4)$, $Bi^{213}(-0.2)$, $Bi^{212}(-0.4)$,
$Bi^{211}(-0.5)$, $Bi^{210}(-0.5)$, $Po^{209}(+2.3?)$.

In order to make it clear whether these peculiarities are connected with the relative increase in the binding energy of the initial nucleus, or with the decrease in the binding energy of the final nucleus, between which nuclei β-transitions occur, it is quite proper to use the energy of addition of neutrons and protons, as was done in Refs. 4-6. However, such energies are rather difficult to compare amongst themselves, and it is most difficult to make a reliable separation of the effect of the shells. Using the results of the previous investigation, we can, however, bring all the isobaric nuclei to a homogeneous condition. For this purpose, we calculate the addition energy of the neutron $E^*_{n\gamma}$ which the nucleus with (A, Z^*_A) ought to have, i.e., the nucleus located on the stability curve of Z^*.

It follows from the energy cycle that

$$E^*_{n\gamma} = E_{n\gamma} + E_1 - E_2,$$

where E_1 is the mass difference for nuclei of like parity (A, Z) and (A, Z^*_A); E_2 is the mass difference for nuclei $(A + 1, Z)$ and $(A + 1, Z^*_A)$. E_1 and E_2 can be calculated easily if we make use of the empirical smoothed values of Z^* and k.

The reduced energies $E^*_{n\gamma}$ found for isobars of one type of parity ought to coincide exactly if only the masses of the nuclei are given by Eqs. (6) and (7). Deviations ought to appear only in places of anomalously high or low binding energies upon filling of the nuclear shells. If $b_{oo}(A)$ and $b_{oe}(A)$ [see Eqs. (6) and (7)] are smooth functions of A (and do not, in particular, depend on the parity of A) and $b_{oo}(A) = b_{oe}(A)$, then all the reduced energies $E^*_{n\gamma}$ ought to be distributed along two smooth curves. The distance between these curves must, in accordance with Eqs. (6) and (7), be equal to

$$\Delta E^*_{n\gamma} \approx 2\,(\delta^{ee}_{oo} - \delta^{eo}_{oe}).$$

As is seen from Fig. 5, where the dependence of $E^*_{n\gamma}$ on A is shown, the laws referred to are actually demonstrated. Moreover, the anomalously high addition energies of neutrons, if their number is less than 126 (in comparison with the addition energy in the region $N > 126$) are also shown.

For a complete analysis from p-n – β-cycles, the binding energies of protons $E^*_{p\gamma}$ were also calculated. The values of the deviation of $E^*_{n\gamma}$ and $E^*_{p\gamma}$ from the mean values (from the curves in Fig. 5) are plotted in Fig. 6.

Consideration of this scheme of "anomalies" shows that (a) the binding energies of neutrons in the region $N < 126$ are raised on the average by 1.5 mev (in comparison with the mean values) and by ~ 2.1 mev relative to nuclei with $N = 127, 128$; (b) the binding energies of protons in the region $Z < 82$ are higher, relative to the mean, by ~1.4 mev, and the nuclei with $Z = 83, 84$ have energies $E^*_{p\gamma}$ that are lower by ~0.5 mev[13, 4]. The character of the neutron shell $N = 126$ agrees well with the assumptions made on it from its spin-orbit behavior. Actually, the conclusions drawn above, from the point of view of the one particle model, can be interpreted in the following way. For $N < 126$, the neutron single particle levels are placed comparatively close to one another, with approximately equal spacing. The level which corresponds to $N = 127$ and the neutron one-particle levels following it are greatly increased relative to the preceding levels. The separation of the levels $N = 126$ and 127 is larger by ~2.1 mev than that between neighboring levels. The character of the proton levels of $Z = 82$ is quite similar.

5. COMPARISON WITH THE BETHE-WEIZSÄCKER FORMULA

For a comparison of the results of our analysis with the Bethe-Weizsäcker formula, we rewrite the latter in the form[19]

$$E_{\text{bind}} = \varepsilon A - \gamma A^{2/3} - \alpha A \left\{\left(\frac{I}{A}\right)^2 + \frac{A^{2/3}}{\beta}\left(1 - \frac{I}{A}\right)^2\right\} + \delta, \tag{9}$$

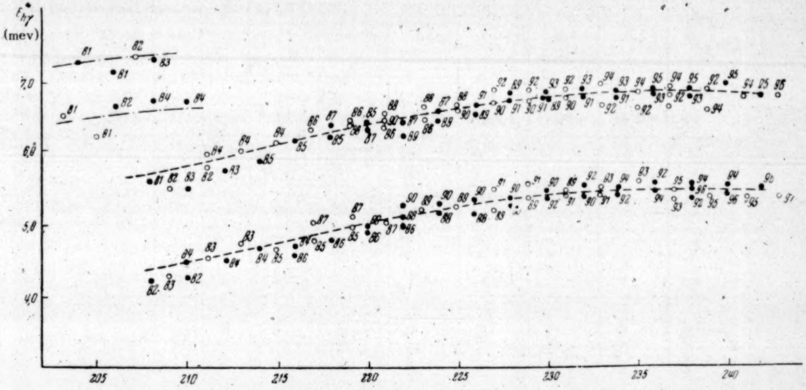

FIG. 5. Reduced energies of addition of neutrons. ●–even A, o–odd A. The numbers by each of the points denote the number of protons.

where the first term characterizes the volume energy, the second the surface, the third the symmetric, the fourth the Coulomb; ϵ, γ, β, α are numerical coefficients. The term δ characterizes the correction to the parity, which, in accord with the foregoing analysis, can be written for the given mass number interval in the following form:

$$\delta = \begin{cases} +\delta_{oe}^{eo} \approx +0.1 \quad \text{for} \quad A_o,\ Z_e, & (10) \\ -\delta_{oe}^{eo} \approx -0.1 \quad \text{for} \quad A_o,\ Z_o, \\ +\delta_{oo}^{ee} \approx +(2.77 - 0.0088\,A) \quad \text{for} \quad A_e,\ Z_e, \\ -\delta_{oo}^{ee} \approx -(2.27 - 0.0088\,A) \quad \text{for} \quad A_e,\ Z_e. \end{cases}$$

Equation (9) can be rewritten in the form[19]

$$E_{\text{bind}} = \varepsilon A - \gamma A^{2/3} - \alpha I_A - k_I (I - I_A)^2 + \delta, \quad (11)$$

where I_A is determined by the condition $(\partial E_{\text{bind}}/\partial I)_A = 0$, whence $I_A = A/(1 + \beta A^{-2/3})$, in which I_A is related to Z^* by the following:

$$I_A = I^* + 0.78/k = A - 2Z^* + 0.78/k, \quad (12)$$

$$k_I = k/4 = -{}^{1}/_{2}\,(\partial^2 E_{\text{CB}}/\partial I^2)_A$$
$$= \alpha/A\,(1 + A^{2/3}/\beta). \quad (13)$$

Comparing the empirical values of Z^* and k with Eqs. (12) and (13), we find the following dependence of the coefficients α and β on A:

$$\alpha \approx 0.1\ A + 2.32, \quad (14)$$
$$\alpha/\beta \approx 0.03 + 0.00067\ A,$$

i.e., the coefficients α for the term of symmetric energy, and α/β for the Coulomb energy term, do not remain constant, but increase with increase in the mass number A. One can undertake to connect this fact with the fact that in the range considered the effective radii increase somewhat more slowly than according to the law $A^{1/3}$. Actually, a decrease in the radius leads to an increase in the coefficient for the Coulomb energy term, and also for the symmetric energy term, inasmuch as the kinetic energy which makes the essential contribution to the term increases upon compression of the nucleus. The mean values of the coefficients for Coulomb and symmetric terms are close to the mean values of the same quantities found by other methods[1,2,20,21].

CONCLUSION

The study of the properties of the energy surface in the region of heavy nuclei $210 < A < 244$

Tl	+2.2	+1.8	+1.55	−0.3	−0.1				Z=81
	+1.2	+1.55	+1.8	+1.7	+1.3	+1.15			
Pb	+2.4	+2.3	+1.8	−0.4	−0.3	−0.1	0	+0.2	Z=82
		−0.7	−0.7	−0.8	−0.7	−0.6	−0.5	−0.9	
Bi			+1.6	−0.5	−0.2	+0.1	+0.1	−0.2	Z=83
		−0.7	−0.8	−0.6	−0.4	−0.2	−0.4	−0.3	
Po			+1.5	−0.3	0	+0.2	−0.1	−0.1	Z=84
N=123	N=124	N=125	N=126	N=127	N=128	N=129	N=130	N=131	

FIG. 6. Scheme of "anomalies" in addition energies of neutrons and protons.

permits us to make the following conclusions:

1. The isobaric cross sections of the surface of the masses are, with sufficient accuracy, parabolas of second order for the nuclei of each of the types of parity [at least, for the values $|Z - Z^*| < (4\text{-}5)$]. Their curvatures are not dependent on the parity of Z, and evidently depend only slightly on the parity of A ($k = 0.56$ for A_e and 0.54 for A_o);

2. Minimal isobaric cross sections are achieved for nuclei of different parities for particular (distinct) values $Z = Z^*$. In this case the curves $Z^* = f(A)$ are higher for nuclei of the type Z_o, N_o and Z_o, N_e (which possibly coincide) and are lower (by ~ 0.1 in units of Z) for nuclei of the type Z_e, N_e or Z_e, N_o (which may also coincide);

3. The correction to the parity δ for even-even nuclei is equal in mean to +0.75 mev, for odd-odd, to −0.75 mev, for even-odd, +0.1 mev, for odd-even, −0.1 mev;

4. The character of the shells $N = 126$ and $Z = 82$ agrees with the hypothesis of spin-orbit interaction. The effect of both shells reaches a maximum at the nucleus P^{208};

5. Comparison of the empirical data with the customary formula for the binding energy shows that in the mass interval under consideration its coefficients do not remain constant, but change somewhat; the character of their change agrees with the assumption on the certain volume (in comparison with the law $A^{1/3}$) of the decrease of the effective radii of heavy nuclei.

In conclusion, I take this opportunity to thank Professor D. D. Ivanenko for his constant interest in the work, and also S. I. Larin and N. A. Bekeshko for a series of valued suggestions.

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Scattering of Electrons in a Liquid due to Violation of Long Range Order

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Further development of the zone theory of liquids of the author is given by the investigation of the motion of electrons in a liquid in terms of cartesian coordinates. The assumed zero approximation wave functions are similar to the Bloch functions for crystals and satisfy equations which differ from the Schrödinger equation by small terms. Scattering of the electrons in a liquid due to violation of long range order in the atomic arrangement has been calculated and the dependence of the corresponding electron mean free path on wave number has been determined for small values of the wave number.

1. INTRODUCTION

IN earlier papers of the author[1,2] it was shown theoretically that, for slight violation of local order in a crystal and disappearance of long range order, i.e., upon melting of the crystal, the energy spectrum of the electrons possesses a zone structure. The calculation was carried out in a distorted coordinate system ξ_1, ξ_2, ξ_3 such that in it the self-consistent potential of the electron in the liquid $V(\xi_1, \xi_2, \xi_3)$ is a periodic function of the cartesian coordinates x_1, x_2, x_3. It was shown that, in the system of coordinates ξ, the solution of the wave equation could be constructed from Bloch wave functions of the type

$$\psi_\varkappa = G^{-3/2} u_\varkappa(\xi_1, \xi_2, \xi_3) \exp\left\{ i \sum_{\alpha=1}^{3} \varkappa_\alpha \xi_\alpha \right\}, \qquad (1)$$

where $u_\varkappa(\xi_1, \xi_2, \xi_3)$ is a function of the same periodicity as $V(\xi_1, \xi_2, \xi_3)$. The factor $G^{-3/2}$ appears because, for conveneince in later calculations, the functions $\psi_\varkappa$ are normalized for the base region, which consists of G^3 elementary cells, and the function $u_\varkappa$ is normalized for a single elementary cell.

The stationary solution is a wave packet which consists of functions (1) and which moves progressively. Thus the disappearance of long range order has an unimportant effect on the energy spectrum of the electrons in the body and on the character of their motion. However, the disappearance of strict periodicity in the position of the atoms ought to lead to additional scattering of electrons, to a decrease in their mean free path, and to an increase in resistance. As was noted by Wilson[3], the resistance of a liquid with electronic conductivity will be determined in part by the disorder of the regular positions of the atoms and in part from the thermal vibrations of the lattice. We call the first component of resistance the "liquid" resistance, and the second component the "phonon" resistance.

Wilson[3] obtained the ratio for the conductivity in the solid and liquid phases close to the melting point for a series of metals, and tried to explain the higher resistance upon liquefaction by a change in the Debye temperature θ which enters into the expression for the mean free path, assuming that the other quantities do not change appreciably during the melting process. For several metals, the agreement between the ratio of conductivites calculated by such a method and those obtained experimentally were satisfactory. However, for mercury the experimentally measured ratio was 4.1, the computed value, 2.23. Consequently, there is here an appreciable amount of an additional mechanism for the scattering of the electrons--the "liquid" resistance. It is not accidental that this resistance was specifically observed in mercury; actually, mercury liquefies at a low temperature, at which point the lattice vibrations are not very intense. One can assume that in other metals, which liquefy at much higher temperature, the "liquid" resistance is masked by the phonon resistance.

In the present work it is proposed to estimate the length of the mean free path of the electron in a liquid which is characterized by violations of long range order, i.e., by the "liquid" scattering of the electrons. The value of this length of mean free path allows us to compute, as is customary in a theory of electrical conductivity, the "liquid" resistance which ought to play a fundamental role at low temperatures in liquids with electronic conductivity. At high temperatures, this resistance is perhaps only a small correction to the ordinary phonon resistance; nevertheless, the existence of "liquid" resistance has a value in principle.

In carrying out the calculation, we do not take into account the thermal vibrations of the lattice and its local distortions, assuming that the length of the free path is determined only by the violation of long range order.

Computation of the mean path length encounters the difficulty that in cartesian coordinates x the deviation from a periodic potential is large, and it is therefore not possible to use perturbation theory, employing the ordinary Bloch functions for the zeroth approximation. In the system of coordinates ξ where, as was shown in Refs. 1 and 2, the perturbation is small, it is difficult to describe the motion of the electron in the external field, since the homogeneous electrical field in the system ξ has varying components. Therefore, we solve the problem of the motion of the electron in the liquid, i.e., in a quasi-periodic potential, in rectangular coordinates, but as a zeroth approximation, we use, not the ordinary Bloch functions, but the quasi-Bloch functions of the form

$$\psi_{\mathbf{k}} = G^{-3/2} u_{\mathbf{k}}(\xi_1, \xi_2, \xi_3)\, e^{i\mathbf{kr}}, \tag{2}$$

where $u_{\mathbf{k}}$ is a quasi-periodic function of the deformed coordinates ξ_1, ξ_2, ξ_3. We note that the functions (2) are essentially different from (1) since

$$\sum_{\alpha=1}^{3} \varkappa_\alpha \xi_\alpha \neq \mathbf{kr},$$

because the coordinates ξ_α are not cartesian.

2. CONSIDERATION OF THE MOTION OF AN ELECTRON IN A LIQUID IN CARTESIAN COORDINATES

We shall show that the quasi-Bloch functions (2) satisfy an equation which differs from the Schrödinger equation only in small terms of order ϵ --the degree of violation of local order in the melting of the crystal. For simplicity we first turn to the one-dimensional case, where the quasi-Bloch function has the form

$$\psi_k = G^{-1/2} u_k(\xi)\, e^{ikx}, \tag{3}$$

where, in the one-dimensional case, the modulating function u_k is a strictly periodic function of ξ.

Substitution of the ordinary Bloch function $\psi_k = u_k(x)\, e^{ikx}$ in the Schrödinger equation for the crystal, i.e., for periodic $V(x)$, gives the following equation for the function $u_k(x)$ which is periodic in x:

$$d^2u_k / dx^2 + 2ik\, du_k / dx - (k^2 + \widetilde{V} - \widetilde{E})\, u_k = 0, \tag{4}$$

where, for brevity, we write

$$\widetilde{V} = 2mV / \hbar^2; \quad \widetilde{E} = 2mE / \hbar^2; \tag{5}$$

E is the total energy of the electron. It can be shown that for a periodic $V(\xi)$, the periodic function $u_k(\xi)$ satisfies the equation

$$d^2u_k / d\xi^2 + 2ik\, du_k / d\xi - (k^2 + \widetilde{V} - \widetilde{E})\, u_k = 0. \tag{6}$$

However, since ξ is not a cartesian coordinate, Eq. (6), in contrast to (4), is not an exact consequence of the Schrödinger equation.

We transform in Eq. (6) from ξ to x, setting, as in Ref. 1,

$$dx / d\xi = 1 + \varepsilon\gamma, \tag{7}$$

where ϵ is a small quantity (the degree of violation of local order), and γ is an arbitrary function of the coordinates, of order of magnitude unity. As a result, we obtain

$$(1 + \varepsilon\gamma)^2 \frac{d^2u_k}{dx^2} + (1 + \varepsilon\gamma)\left(2ik + \varepsilon\frac{d\gamma}{dx}\right)\frac{du_k}{dx} - (k^2 + \widetilde{V} - \widetilde{E})\, u_k = 0. \tag{8}$$

Multiplying Eqs. (4) and (8) by $(\hbar^2/2m)G^{-1/2}e^{ikx}$, we get the expressions $\hat{H}\psi_k = E_k\psi_k$ and $\hat{H}_0^k\psi_k = E_k\psi_k$, respectively. Here, comparing (4) and (8), one can be convinced that

$$\hat{H}\psi_k = \hat{H}_0^k\psi_k + \varepsilon\hat{W}^k\psi_k + \varepsilon^2\hat{w}^k\psi_k; \tag{9}$$

$$\hat{H}_0^k\psi_k = \frac{\hbar^2}{2m\sqrt{G}}\left[(1 + \varepsilon\gamma)^2\frac{d^2u_k}{dx^2} + (1 + \varepsilon\gamma)\left(2ik + \varepsilon\frac{d\gamma}{dx}\right)\frac{du_k}{dx} - (k^2 + \widetilde{V})\, u_k\right]e^{ikx}; \tag{10}$$

$$\hat{W}^k\psi_k = -\frac{\hbar^2}{2m\sqrt{G}}\left[2\gamma\frac{d^2u_k}{dx^2} + \left(2ik\gamma + \frac{d\gamma}{dx}\right)\frac{du_k}{dx}\right]e^{ikx}; \tag{11}$$

$$\hat{w}^k\psi_k = -\frac{\hbar^2}{2m\sqrt{G}}\left[\gamma^2\frac{d^2u_k}{dx^2} + \gamma\frac{d\gamma}{dx}\frac{du_k}{dx}\right]e^{ikx}. \tag{12}$$

We see that the actual functions (3) satisfy equations which differ from the Schrödinger equation only by small terms. The equation $\hat{H}_0^k\psi_k = E_k\psi_k$ can be considered as the equation of zeroth approximation, and the operators $\epsilon\hat{W}^k$ and $\epsilon^2\hat{w}^k$ as the perturbation. Integrating by parts the products $(d^2u_k/dx^2)e^{ikx}$ and $(du_k/dx)e^{ikx}$ in Eqs. (10)-(12), we can obtain expressions for the same operators; however, in what follows the expressions (10)-(12) are required directly.

The three-dimensional case can be similarly treated. The following equation for the periodic function $u_k(x_1, x_2, x_3)$ can be obtained from the Schrödinger equation:

$$\sum_{\alpha=1}^{3}\frac{\partial^2 u_k}{\partial x_\alpha^2}+2i\mathbf{k}\cdot\sum_{\alpha=1}^{3}\mathbf{i}_\alpha\frac{\partial u_k}{\partial x_\alpha}-(k^2+\tilde{V}-\tilde{E})\,u_k=0, \tag{13}$$

where i_α is the position of the coordinate axis x_α. Substituting x_α for ξ_α here, we get an equation which is entirely analogous to the one-dimensional case Eq. (6):

$$\sum_{\alpha=1}^{3}\frac{\partial^2 u_k}{\partial \xi_\alpha^2}+2i\mathbf{k}\cdot\sum_{\alpha=1}^{3}\mathbf{e}_\alpha\frac{\partial u_k}{\partial \xi_\alpha} \tag{14}$$
$$-(k^2+\tilde{V}-E)\,u_k=0,$$

where e_α is the site tangent to the coordinate line ξ_α. But, in contrast to the one-dimensional case, the solutions of Eq. (14) will not be strictly periodic functions of ξ_α. Actually, while the vector **k** has a constant direction, the vectors $\mathbf{e}_\alpha$ have different directions in the various elementary cells. However, the direction of $\mathbf{e}_\alpha$ changes slowly from cell to cell; therefore, the u_k will be quasi-periodic functions of ξ_α in the sense that the values of u_k at corresponding points of neighboring cells will be equal to one another with accuracy to small order of ϵ. But even in cells that are distant from one another, the values of u_k will differ only slightly. It follows from Eq. (14) that they are as much different as the u_k in the ordinary Bloch functions, which correspond to identical absolute values, but to different directions of the vector **k**. This difference is usually neglected in the theory of metals (see, for example, Bethe and Sommerfeld, Ref. 4, p .188).

We transform in Eq. (14) from the coordinates ξ to the coordinates x: after simple calculation, we obtain

$$\sum_{\beta=1}^{3}\left\{\sum_{\gamma=1}^{3}\frac{\partial^2 u_k}{\partial x_\beta\,\partial x_\gamma}\sum_{\alpha=1}^{3}\frac{\partial x_\beta}{\partial\xi_\alpha}\frac{\partial x_\gamma}{\partial\xi_\alpha}+\frac{\partial u_k}{\partial x_\beta}\left[\sum_{\alpha=1}^{3}\frac{\partial^2 x_\beta}{\partial\xi_\alpha^2}\right.\right.$$
$$\left.\left.+\,2i\sum_{\gamma=1}^{3}k_\gamma\sum_{\alpha=1}^{3}\left[\sum_{\varkappa=1}^{3}\left(\frac{\partial x_\varkappa}{\partial\xi_\alpha}\right)^2\right]^{-1/2}\frac{\partial x_\beta}{\partial\xi_\alpha}\frac{\partial x_\gamma}{\partial\xi_\alpha}\right]\right\} \tag{15}$$
$$-(k^2+\tilde{V}-E)\,u_k=0.$$

If we neglect quantities of order ϵ, then in the limits of each elementary cell we can regard the system of coordinates ξ as a cartesian system, rotated relative to the system x. But then

$$\partial x_\beta/\partial\xi_\alpha=\cos(x_\beta,\xi_\alpha);\qquad \partial^2 x_\beta/\partial\xi_\alpha^2=0;$$

$$\sum_{\alpha=1}^{3}\frac{\partial x_\beta}{\partial\xi_\alpha}\frac{\partial x_\gamma}{\partial\xi_\alpha}=\cos(x_\beta,x_\gamma)=\delta_{\beta\gamma};\qquad \sum_{\varkappa=1}^{3}\left(\frac{\partial x_\varkappa}{\partial\xi_\alpha}\right)^2=1.$$

Taking into account terms of order ϵ, we can set

$$\sum_{\alpha=1}^{3}\frac{\partial x_\beta}{\partial\xi_\alpha}\frac{\partial x_\gamma}{\partial\xi_\alpha}=\delta_{\beta\gamma}+\varepsilon\gamma_{\beta\gamma};\qquad \sum_{\alpha=1}^{3}\frac{\partial^2 x_\beta}{\partial\xi_\alpha^2}=\frac{\varepsilon\gamma_\beta}{a}; \tag{16}$$

$$\sum_{\alpha=1}^{3}\left[\sum_{\varkappa=1}^{3}\left(\frac{\partial x_\varkappa}{\partial\xi_\alpha}\right)^2\right]^{-1/2}\frac{\partial x_\beta}{\partial\xi_\alpha}\frac{\partial x_\gamma}{\partial\xi_\alpha}=\delta_{\beta\gamma}+\varepsilon\Gamma_{\beta\gamma}.$$

Here $\gamma_{\beta\gamma}$, γ_β and $\Gamma_{\beta\gamma}$ are arbitrary functions of the coordinates, which have order unity. They are determined somewhat differently than the functions $\gamma_{\beta\gamma}$ in Ref. 2, but possess the same properties, with the exception that here the mean values of $\gamma_{\beta\gamma}$ and $\Gamma_{\beta\gamma}$ cannot be simultaneously equated to zero; a is the lattice constant.

Substitution of Eq. (16) in Eq. (15) allows us to obtain our equation in the form

$$\hat{H}_0^k\psi_{\mathbf{k}}=E_{\mathbf{k}}\psi_{\mathbf{k}}, \tag{17}$$

$$\hat{H}_0^{\mathbf{k}}\psi_{\mathbf{k}} \tag{18}$$
$$=\frac{\hbar^2}{2mG^{3/2}}\left[\sum_{\beta=1}^{3}\left(\frac{\partial^2 u_{\mathbf{k}}}{\partial x_\beta^2}+2ik_\beta\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}\right)-(\tilde{V}+k^2)\,u_{\mathbf{k}}\right.$$
$$+\,\varepsilon\sum_{\beta=1}^{3}\sum_{\gamma=1}^{3}\left(\frac{\partial^2 u_{\mathbf{k}}}{\partial x_\beta\,\partial x_\gamma}\gamma_{\beta\gamma}+2ik_\gamma\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}\Gamma_{\beta\gamma}\right)$$
$$\left.+\,\frac{\varepsilon}{a}\sum_{\beta=1}^{3}\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}\gamma_\beta\right]e^{i\mathbf{kr}}.$$

Comparing this with the actual expression for $\hat{H}\psi_{\mathbf{k}}$, we conclude that

$$\hat{H}\psi_{\mathbf{k}}=\hat{H}_0^{\mathbf{k}}\psi_{\mathbf{k}}+\varepsilon\hat{W}^{\mathbf{k}}\psi_{\mathbf{k}}; \tag{19}$$

$$\hat{W}^{\mathbf{k}}\psi_{\mathbf{k}} = -\frac{\hbar^2}{2mG^{3/2}}\left[\sum_{\beta=1}^{3}\sum_{\gamma=1}^{3}\left(\frac{\partial^2 u_{\mathbf{k}}}{\partial x_\beta\,\partial x_\gamma}\gamma_{\beta\gamma} + 2ik_\gamma\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}\Gamma_{\beta\gamma}\right) + \frac{1}{a}\sum_{\beta=1}^{3}\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}\gamma_\beta\right]e^{i\mathbf{kr}}. \tag{20}$$

In contrast to the one-dimensional case, the operator proportional to ϵ^2 is absent here. It is included as a factor in the operator $\epsilon\hat{W}^{\mathbf{k}}$ and is obtained explicitly if we express $\Gamma_{\beta\gamma}$ by $\gamma_{\beta\gamma}$, which would appear to be an unnecessary complication. As in the one-dimensional case, we can, upon integrating by parts, obtain an expression for the operators $\hat{H}_0^{\mathbf{k}}$ and $\hat{W}^{\mathbf{k}}$, but we shall not do this, inasmuch as the expressions $\hat{H}_0^{\mathbf{k}}\psi_{\mathbf{k}}$ and $\hat{W}^{\mathbf{k}}\psi_{\mathbf{k}}$ are used directly in what follows. We note only that the operators $\hat{H}_0^{\mathbf{k}}$ and $\hat{W}^{\mathbf{k}}$ depend on **k**.

It is important to note that the functions (2) describe the motion of quasi-free electrons in a liquid in entirely the same way as the Bloch functions describe the quasi-free electrons in the crystal. The acceleration of the wave packet in an external field is computed in the same way, and one can use the method of the effective mass. Actually, the relation between the velocity and the energy of the electron, and also between the acceleration and the actual force, can be obtained without using the periodic function $u_{\mathbf{k}}$, but only its property that they change slowly in their dependence on **k** (Seitz[5], pp. 332-336). But, by virtue of the identity of the Eqs. (13) and (14), the dependence on **k** of the quasi-periodic functions $u_{\mathbf{k}}$ that we have introduced is the same as for ordinary periodic functions. Consequently, all the positions of the Bloch scheme are used in the case under consideration.

Since the operators $\hat{H}_0^{\mathbf{k}}$ are not self-adjoint and depend on **k**, the problem of the orthogonality of their eigenfunctions $\psi_{\mathbf{k}}$ must be considered. For this purpose, as usual, we write down Eq. (17) for the functions $\psi_{\mathbf{k}}$ and $\psi^*_{\mathbf{k}'}$, multiplying the first of these equations by $\psi^*_{\mathbf{k}'}$ and the second by $\psi_{\mathbf{k}}$, carry out integration over all space and subtract one from the other. As a result, we obtain

$$\int(\psi^*_{\mathbf{k}'}\hat{H}_0^{\mathbf{k}}\psi_{\mathbf{k}} - \psi_{\mathbf{k}}\hat{H}_0^{\mathbf{k}'^*}\psi^*_{\mathbf{k}'})\,d\tau = (E_{\mathbf{k}} - E_{\mathbf{k}'})\int\psi^*_{\mathbf{k}'}\psi_{\mathbf{k}}\,d\tau. \tag{21}$$

Substituting $\hat{H}_0^{\mathbf{k}} = \hat{H} - \epsilon\hat{W}^{\mathbf{k}}$ here, and making use of the self-adjoint property of the operator $\hat{H}$, we get

$$(E_{\mathbf{k}} - E_{\mathbf{k}'})\int\psi^*_{\mathbf{k}'}\psi_{\mathbf{k}}d\tau = -\varepsilon\int(\psi^*_{\mathbf{k}'}W^{\mathbf{k}}\psi_{\mathbf{k}} - \psi_{\mathbf{k}}\hat{W}^{\mathbf{k}'^*}\psi^*_{\mathbf{k}'})\,d\tau. \tag{22}$$

In view of the presence of the multipliers $\gamma_{\beta\gamma}$ and $\Gamma_{\beta\gamma}$ in W and the factor $e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}}$, the expression under the integral sign on the right side of Eq. (22) is an arbitrary sign-varying function of the coordinates. Integrals of such a type were estimated in Refs. 1 and 2, their mean square values being proportional to $G^{-3/2}$, where G^3 is the number of elementary cells in the base region. In this way, the right side of Eq. (22) has the order $\epsilon G^{-3/2}$, i.e., it is practically equal to zero. This means that if $E_{\mathbf{k}} \neq E_{\mathbf{k}'}$, then

$$\int\psi^*_{\mathbf{k}'}\psi_{\mathbf{k}}d\tau \sim \varepsilon G^{-3/2} \approx 0, \tag{23}$$

i.e., the functions $\psi_{\mathbf{k}}$ are practically orthogonal.

3. SOLUTION OF THE KINETIC EQUATION FOR THE ELECTRON IN THE LIQUID

In view of the fact that the operators $\hat{H}_0^{\mathbf{k}}$ depend on **k**, the derivation of the equation must be carried out anew from the theory of quantum processes. In particular, there remains an open question of the completeness of the set of functions $\psi_{\mathbf{k}}$. Nevertheless, based on the analogy between $\psi_{\mathbf{k}}$ and the usual Bloch functions, and taking into account the large number G^3 of the functions $\psi_{\mathbf{k}}$, we make the assumption that the solution of the time dependent Schrödinger equation

$$\hbar i\,\partial\psi/\partial t = \hat{H}\psi \tag{24}$$

can, with a sufficient degree of accuracy, be represented in the form of a series

$$\psi(\mathbf{r}, t) = \sum_{\mathbf{k}} c_{\mathbf{k}}(t)\,\psi_{\mathbf{k}}(\mathbf{r})\,e^{iE_{\mathbf{k}}t/\hbar}. \tag{25}$$

Substituting Eq. (25) in (24), expanding each term of the sum on the right side of $\hat{H}\psi_{\mathbf{k}}$ into $\hat{H}_0^{\mathbf{k}}\psi_{\mathbf{k}}$ and $\epsilon\hat{W}^{\mathbf{k}}\psi_{\mathbf{k}}$, integrating over the entire volume and taking into accout (17) and (23), we arrive at the usual equation of the theory of quantum processes:

$$i\hbar\dot{c}(\mathbf{k}') = \varepsilon W_{\mathbf{k}'\mathbf{k}}\exp i\{(E_{\mathbf{k}'} - E_{\mathbf{k}})t/\hbar\}, \tag{26}$$

where

$$W_{\mathbf{k}'\mathbf{k}} = \int\psi^*_{\mathbf{k}'}\hat{W}^{\mathbf{k}}\psi_{\mathbf{k}}d\tau = \frac{\hbar^2}{2mG^{3/2}a^2}I_{\mathbf{k}'\mathbf{k}}, \tag{27}$$

$$I_{\mathbf{k}'\mathbf{k}}=\frac{a^2}{G^{3/2}}\int u^*_{\mathbf{k}'}\left[\sum_{\beta=1}^{3}\sum_{\gamma=1}^{3}\left(\frac{\partial^2 u_{\mathbf{k}}}{\partial x_\beta\,\partial x_\gamma}\gamma_{\beta\gamma}+2ik_\gamma\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}\Gamma_{\beta\gamma}\right)\right.$$
$$\left.+\frac{1}{a}\sum_{\beta=1}^{3}\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}\gamma_\beta\right]e^{i(\mathbf{k}-\mathbf{k}')\mathbf{r}}\,d\tau. \tag{28}$$

Subsequent calculation differs little from the usual theory of electrical conductivity (see, for example, Ref. 4, Sec. 34-35). Integration of Eq. (26) over the time gives

$$|c(\mathbf{k}')|^2=\varepsilon^2|W_{\mathbf{k}'\mathbf{k}}|^2\,\Omega(E_{\mathbf{k}'}-E_{\mathbf{k}}) \tag{29}$$
$$=\frac{\varepsilon^2\hbar^4}{4m^2G^3a^4}|I_{\mathbf{k}'\mathbf{k}}|^2\,\Omega(E_{\mathbf{k}'}-E_{\mathbf{k}}),$$

$$\Omega(\eta)=2\,\frac{1-\cos(\eta t/\hbar)}{(\eta/\hbar)^2}. \tag{30}$$

In order to obtain the total change of the distribution function $f(\mathbf{k}', t)$ as a result of liquid scattering, we must multiply Eq. (29) by the probability of the initial state $f(\mathbf{k})$, sum over all possible values of $\mathbf{k}$, multiply (taking account of the Pauli principle) by the probability of nonoccupation of the state $\mathbf{k}'$ -- by $1-f(\mathbf{k}')$, and consider the inverse transitions from state $\mathbf{k}'$ to state $\mathbf{k}$. As a result, we obtain

$$f(\mathbf{k}',t)-f(\mathbf{k}',0) \tag{31}$$
$$=(\varepsilon^2\hbar^4/4m^2G^3a^4)\sum_{\mathbf{k}}\Omega(E_{\mathbf{k}'}-E_{\mathbf{k}})$$
$$\times\{|I_{\mathbf{k}'\mathbf{k}}|^2 f(\mathbf{k})[1-f(\mathbf{k}')]$$
$$-|I_{\mathbf{k}\mathbf{k}'}|^2 f(\mathbf{k}')[1-f(\mathbf{k})]\}.$$

We note that here the usual relations between the matrix elements of the forward and reverse processes of the transitions $W_{\mathbf{k}'\mathbf{k}}=W^*_{\mathbf{k}\mathbf{k}'}$ does not occur. However, if we limit ourselves to the isotropic approximation, employed in the classical theory of electrical conduction, then, as we shall see below, in Eq. (31)

$$|I_{\mathbf{k}'\mathbf{k}}|^2\approx|I_{\mathbf{k}\mathbf{k}'}|^2. \tag{32}$$

Taking (32) into account, making appropriate reductions and differentiating (31) with respect to time, we get the change in the distribution function as a result of the "liquid" scattering:

$$\left(\frac{\partial f}{\partial t}\right)_{\text{ж}}=\frac{\varepsilon^2\hbar^4}{4m^2G^3a^4} \tag{33}$$
$$\times\frac{\partial}{\partial t}\sum_{\mathbf{k}}|I_{\mathbf{k}'\mathbf{k}}|^2\,\Omega(E_{\mathbf{k}'}-E_{\mathbf{k}})[f(\mathbf{k})-f(\mathbf{k}')].$$

We consider that the energy of the electron depends only on $\mathbf{k}$, but not on the direction of $\mathbf{k}$. We introduce in $\mathbf{k}$-space the spherical system of coordinates $\mathbf{k}$, θ, φ, the axis of which has the direction $\mathbf{k}'$. In place of the sum over $\mathbf{k}$ we can write the integral

$$\sum_{\mathbf{k}}F(\mathbf{k})=\Omega_0\left(\frac{G}{2\pi}\right)^3\int k^2F(\mathbf{k})\sin\theta\,d\varphi\,d\theta\,dk; \tag{34}$$

Ω_0 is the volume of the elementary cell. The function $\Omega(\eta)$ has a sharp maximum for $\eta=0$; therefore, upon integration over k on the right side of (33) over all factors except Ω, we can set $k=k'$ and carry out these multiplications under the integral sign over k. The remaining integral is equal to

$$\int_0^\infty\Omega(E_{\mathbf{k}'}-E_{\mathbf{k}})\,dk\approx\frac{1}{dE/dk} \tag{35}$$
$$\times\int_{-\infty}^{\infty}\Omega(\eta)\,d\eta=\frac{2\pi\hbar t}{dE/dk}$$

and, consequently,

$$\left(\frac{\partial f}{\partial t}\right)_{\text{ж}}=\frac{\varepsilon^2\hbar^3\Omega_0k'^2}{16m^2a^4\pi^2(dE/dk)} \tag{36}$$
$$\times\int_0^\pi d\theta\int_0^{2\pi}d\varphi\,|I_{\mathbf{k}'\mathbf{k}}|^2\sin\theta\,[f(\mathbf{k})-f(\mathbf{k}')].$$

The kinetic equation for the electrons in the liquid in the stationary case has the form

$$\partial f/\partial t=(\partial f/\partial t)_{\text{ж}}+(\partial f/\partial t)_{\nabla\varphi,\,\nabla\mu,\,\nabla T}=0, \tag{37}$$

where the last term describes the change in the distribution function because of the presence of a potential gradient of the concentration or temperature. For simplicity, we shall consider that the concentrations of electrons and the temperature in the liquid are constants and that a homogeneous field E_x is placed along the x-axis. In this case, as is well known [Ref. 4, Eq. (33.18)],

$$(\partial f/\partial t)_{\nabla\varphi}=-(eE_x/\hbar)\,\partial f/\partial x \tag{38}$$

and if the field is not too large, the distribution function is sought in the form

$$f=f_0(E)+f_1(E)\,k_x, \tag{39}$$

where the second term is a small correction.

We now substitute Eqs. (36) and (38) in (37); in Eq. (36) we put f_1k_x in place of f, since the terms with f_0 are eliminated, and we can replace f with f_0 in Eq. (38), neglecting the small correction. In total, we get

$$\frac{\varepsilon^2\hbar^3\Omega_0 k'^2}{16m^2a^4\pi^2\,(dE/dk)}\int_0^\pi d\theta \tag{40}$$

$$\int_0^{2\pi} d\varphi\,|I_{k'k}|^2 \sin\theta f_1\,(k_x - k'_x) = \frac{eE_x}{\hbar}\frac{\partial f_0}{\partial k'_x}.$$

Let ϑ or ϑ' be, respectively, the angles between x and **k** or **k**′. Consequently, $k_x = k\cos\vartheta$; $k'_x = k'\cos\vartheta'$, whence $\cos\vartheta = \cos\vartheta'\cos\theta + \sin\theta\sin\vartheta'\sin\varphi$. Since (within the framework of the isotropic approximation) $I_{\mathbf{k}'\mathbf{k}}$ does not depend on φ, then, on integration over φ, the second term of this expression vanishes. For the remaining terms, integration over φ gives the factor 2π. On the other hand,

$$\frac{\partial f_0}{\partial k'_x} = \frac{df_0}{dE}\frac{dE}{dk'}\frac{k'_x}{k'}, \tag{41}$$

therefore, the factor $k'_x = k'\cos\vartheta'$ vanishes on the left and right sides of Eq. (40). Solving this equation with respect to f_1, denoting by I

$$I = \int_0^\pi |I_{k'k}|^2\,(1 - \cos\theta)\sin\theta d\theta \tag{42}$$

and dropping the prime from k, we obtain

$$f_1 = -\frac{8\pi m^2 a^4}{\varepsilon^2\hbar^3\Omega_0 k^3 I}\frac{eE_x}{\hbar}\frac{df_0}{dE}\left(\frac{dE}{dk}\right)^2. \tag{43}$$

Comparing this expression with the distribution function for free electrons:

$$f = f_0 - (df_0/dE)\,eE_x\,lv_x/v \tag{44}$$

and assuming

$$v_x/v = k_x/k, \quad \Omega_0 \approx a^3,$$

we get the free path length of the electron in the liquid:

$$l = \frac{8\pi m^2}{\hbar^4 I}\frac{1}{k^2}\left(\frac{dE}{dk}\right)^2\frac{a}{\varepsilon^2}. \tag{45}$$

If we represent the energy of the electron in the form $E = \hbar^2k^2/2m^*$, which can be done in each case for electrons in the conduction zones of a semiconductor, Eq. (45) is simplified and yields

$$l = (8\pi/I)\,(a/\varepsilon^2)\,(m/m^*)^2, \tag{46}$$

where m^* is the effective mass of the electron in the zone.

4. CALCULATION OF THE MATRIX ELEMENTS AND ESTIMATE OF THE ELECTRON MEAN FREE PATH

In order to estimate the mean free path l and to establish its dependence on k, i.e., on the energy of the electron, we must compute the coefficient $I_{\mathbf{k}'\mathbf{k}}$ in the expression for the matrix element of transition.

In analogy to what was done in the estimation of the matrix elements in Refs. 1 and 2, we decompose the integral in (28) into a set of integrals over the elementary cells. The factors $\gamma_{\beta\gamma}$, $\Gamma_{\beta\gamma}$ and γ_β remain nearly constant in the limits of each elementary cell and can be taken outside the integration over the cell. These quantities are arbitrary functions of the coordinates; therefore, summation of both the separated terms of Eq. (28) in the integral over a single elementary cell, as well as of the integrals over all G^3 elementary cells, are carried out according to the law of summation of random quantities, i.e., the squares are summed. Since the various cells are oriented in all possible ways with respect to the direction of **k**, integrals over the individual cells, which are obtained after removing the factors $\gamma_{\beta\gamma}$, $\Gamma_{\beta\gamma}$ and γ_β from under the integral sign, are also random quantities, and therefore independent of the removed factors. But the average of the product of two independent random variables is equal to the product of the mean. Therefore, summation over G^3 elementary cells in the expression for the of the square modulus $I_{\mathbf{k}'\mathbf{k}}$ gives the product of the mean values of the squares of $\gamma_{\beta\gamma}$, $\Gamma_{\beta\gamma}$ and γ_β the mean square of the integral over one cell and by the factor G^3[which reduces to the factor $(G^{-3/2})^2$ which follows from Eq. (28)]. As a result, we obtain

$$|I_{\mathbf{k}'\mathbf{k}}|^2 \approx \sum_{\beta=1}^{3}\sum_{\gamma=1}^{3}\Big(\overline{\gamma^2_{\beta\gamma}}\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta\gamma}|^2} \tag{47}$$

$$+ 4k^2{}_\gamma a^2\overline{\Gamma^2_{\beta\gamma}}\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta}|^2}\Big) + \sum_{\beta=1}^{3}\overline{\gamma^2_\beta}\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_\beta|^2}.$$

Here,

$$\chi^{\mathbf{k}'\mathbf{k}}_{\beta\gamma} = a^2\int u^*_{\mathbf{k}'}\frac{\partial^2 u_{\mathbf{k}}}{\partial x_\beta\,\partial x_\gamma}e^{i\mathbf{q}\mathbf{r}}\,d\tau_0, \tag{48}$$

$$\chi^{\mathbf{k}'\mathbf{k}}_\beta = a\int u^*_{\mathbf{k}'}\frac{\partial u_{\mathbf{k}}}{\partial x_\beta}e^{i\mathbf{q}\mathbf{r}}\,d\tau_0, \quad \mathbf{q} = \mathbf{k} - \mathbf{k}'.$$

The bars over the symbols indicate averaging over all elementary cells, the integrals (48) extend over a single elementary cell.

In the isotropic approximation, and for cubic crystals,

$$\overline{\gamma^2_{11}} = \overline{\gamma^2_{22}} = \overline{\gamma^2_{33}}; \tag{49}$$

$$\overline{\gamma^2_{12}} = \overline{\gamma^2_{23}} = \overline{\gamma^2_{13}}; \quad \overline{\gamma^2_{1}} = \overline{\gamma^2_{2}} = \overline{\gamma^2_{3}};$$

$$|\chi^{\mathbf{k}'\mathbf{k}}_{11}|^2 = |\chi^{\mathbf{k}'\mathbf{k}}_{22}|^2 = |\chi^{\mathbf{k}'\mathbf{k}}_{33}|^2\text{, etc.}$$

Similar conditions exist for Γ. Consequently, Eq. (47) can be rewritten in the form

$$|I_{\mathbf{k}'\mathbf{k}}|^2 \approx 3\overline{\gamma^2_{\beta\beta}}\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta\beta}|^2} + 6\overline{\gamma^2_{\beta\gamma}}\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta\gamma}|^2} + 4k^2a^2\,(\overline{\gamma^2_{\beta\beta}} + 2\overline{\gamma^2_{\beta\gamma}})\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta}|^2} + 3\overline{\gamma^2_{\beta}}\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta}|^2}, \tag{50}$$

where the indices β and γ are not equal. Since $k = k'$, then $u_{\mathbf{k}'} = u_{\mathbf{k}}$ in the isotropic approximation, and condition (32) follows from Eqs. (48) and (50).

To estimate the order of the quantities, we set

$$\overline{\gamma^2_{\beta\beta}} = \overline{\gamma^2_{\beta\gamma}} = \overline{\Gamma^2_{\beta\beta}} = \overline{\Gamma^2_{\beta\gamma}} = \overline{\gamma^2_{\gamma}} \simeq 1 \tag{51}$$

and then

$$|I_{\mathbf{k}'\mathbf{k}}|^2 \approx 3\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta\beta}|^2} + 6\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta\gamma}|^2} + 3\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta}|^2} + 12k^2a^2\,\overline{|\chi^{\mathbf{k}'\mathbf{k}}_{\beta}|^2}. \tag{52}$$

The integrals χ are of the order unity but depend on k and on the angle θ between $\mathbf{k}$ and $\mathbf{k}'$. To explain this dependence, we expand the functions $u_{\mathbf{k}}$ in a triple Fourier series in the limits of one elementary cell:

$$u_{\mathbf{k}} = \sum_{\nu_1=-\infty}^{\infty} \sum_{\nu_2=-\infty}^{\infty} \sum_{\nu_3=-\infty}^{\infty} A^k_{\nu_1\nu_2\nu_3} \exp\left\{\frac{2\pi i}{a}\sum_{\alpha=1}^{3}\nu_\alpha\xi_\alpha\right\}. \tag{53}$$

It is assumed here that the elementary cell is a cube of edge a and the coordinates ξ, which are cartesian within the cell, are laid out along the edges of the cube. All this is valid, of course, to accuracy of a small order of ϵ. We neglect the difference in $u_{\mathbf{k}}$ in the various elementary cells.

Substituting (53) in (48) and carrying out the integration, we get

$$|\chi^{\mathbf{k}'\mathbf{k}}_{\beta\gamma}| = b^k_{\beta\gamma}\prod_{\alpha=1}^{3}\frac{\sin(aq_\alpha/2)}{aq_\alpha/2} + B^k_{\beta\gamma}\prod_{\alpha=1}^{3}\sin\frac{aq_\alpha}{2} \tag{54}$$

and similar expressions for $|\chi^{\mathbf{k}'\mathbf{k}}_{\beta}|$. Here

$$b^k_{\beta\gamma} = 4\pi^2a^3\sum_{\nu_1}\sum_{\nu_2}\sum_{\nu_3}|A^k_{\nu_1\nu_2\nu_3}|^2\,\nu_\beta\,\nu_\gamma; \tag{55}$$

$$B^k_{\delta\gamma} = 4\pi^2\sum_{\nu_1\nu_2\nu_3}\sum_{\mu_1\mu_2\mu_3}{}' \frac{A^{k*}_{\nu_1\nu_2\nu_3}\,A^k_{\mu_1\mu_2\mu_3}\,\mu_\beta\,\mu_\gamma}{\prod_{\alpha=1}^{3}(\pi/a)(\mu_\alpha-\nu_\alpha)}, \tag{56}$$

q_α is the component of the vector $\mathbf{q}$ along the axis ξ_α. The prime on the summation sign signifies that the summation over μ_1, μ_2, μ_3 is carried out for $\mu_1 \neq \nu_1 \ldots$. In the denominator of (56) we have neglected the term $\mathbf{q}_\alpha$ in comparison with $(\pi/a)(\mu_\alpha - \nu_\alpha)$. The coefficients $b^{\mathbf{k}}_\beta$ and $B^{\mathbf{k}}_\beta$ in the expressions for $|\chi^{\mathbf{k}'\mathbf{k}}_\beta|$ are obtained similarly. We note that, if (53) converges sufficiently rapidly, then the coefficients $B^k_{\beta\gamma}$ are small in comparison with $b^k_{\beta\gamma}$.

Investigation of Eq. (54) in the general form is difficult; we therefore assume that $aq_\alpha/2 \ll 1$. This is undoubtedly valid for electrons in the conduction band of a semiconductor, but can also be assumed for a metal, since there is little probability that all three numbers q_1, q_2, and q_3 have values close to the maximum. But in this approximation, terms of second order can be neglected in Eq. (54). Expanding the sine terms in series and neglecting terms of second order with respect to aq_α, we get

$$|\chi^{\mathbf{k}'\mathbf{k}}_{\beta\gamma}| \approx b^k_{\beta\gamma}\left(1 - \frac{a^2}{24}\sum_{\alpha=1}^{3}q^2_\alpha\right) = b^k_{\beta\gamma}\left(1 - \frac{a^2q^2}{24}\right) = b^k_{\beta\gamma}\left[1 - \frac{a^2k^2}{12}(1-\cos\theta)\right]. \tag{57}$$

Similar expressions can be obtained for $\chi^{\mathbf{k}'\mathbf{k}}_\beta$.

Substitution of Eq. (57) in Eq. (50) gives, after we neglect fourth degree terms in ak,

$$|I_{\mathbf{k}'\mathbf{k}}|^2 \approx b^k\left[1 - \frac{a^2k^2}{6}(1-\cos\theta)\right] + b^k_0\,k^2a^2, \tag{58}$$

where

$$b^k = 3\overline{\gamma^2_{\beta\beta}}\,(b^k_{\beta\beta})^2 + 6\overline{\gamma^2_{\beta\gamma}}\,(b^k_{\beta\gamma})^2 + 3\overline{\gamma^2_{\beta}}\,(b^k_{\beta})^2; \tag{59}$$

$$b^k_0 = 4\overline{\Gamma}^2_{\beta\beta}\,(b^k_\beta)^2 + 8\overline{\Gamma}^2_{\beta\gamma}\,(b^k_\beta)^2. \tag{60}$$

The coefficients b^k and b^k_0 depend very weakly on k (only through the dependence of u_k on k) and in the first approximation, we can neglect this dependence. Substituting (58) in (42), and carrying out the integration, we get

$$I \approx c + c_1 a^2 k^2, \qquad (61)$$

$$c = 2b^k, \quad c_1 = 2b_0^k - {}^4/_9\, b^k. \qquad (62)$$

In accord with (46), the dependence of the length of the mean free path on the wave number has the form

$$l = \frac{8\pi}{c + c_1 a^2 k^2} \frac{a}{\varepsilon^2} \left(\frac{m}{m^*}\right)^2. \qquad (63)$$

The coefficients c and c_1 are best obtained from experiment, since their calculation from theoretical formulas requires the knowledge of the coefficients of the expansion of the functions u_k in Fourier series and these theoretical formulas are very approximate.

It is evident that these calculations are applicable not only to liquids but also to amorphous solids, in particular, to amorphous semiconductors. In contrast to the liquids, solid amorphous bodies can exist at low temperatures, when "liquid" scattering exists; therefore, the theoretical laws obtained for the length of the mean free path of the electron are best compared with experimental data for amorphous semiconductors.

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Renormalizability of Pseudoscalar meson Theory with Pseudovector coupling

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It is shown that the convergence of a pseudoscalar theory with pseudovector coupling is not improved even when corrections for the propagation function for the mesons (which are connected with the vacuum polarization) are taken into account.

AS is well known, the program of renormalization, which has been successfully applied in quantum electrodynamics, is not sufficient to obtain convergent results that have physical meaning in most types of meson theories.[1] The hypothesis has been advanced (Ref. 2,* see also Ref. 3) that part of the divergences which take place in meson theories results only from the inapplicability of perturbation theory and that the convergence of pseudoscalar meson theory with pseudovector coupling can be improved if one were to use a true propagation function for the meson (i.e., to consider for the propagation function the energy corrections connected with the polarization of the nucleonic vacuum). Calculation of this function reduces to the calculation of the "polarization operator."

The authors of the works mentioned above make use of the method suggested by Feynman[4] for consideration of the vacuum polarization in quantum electrodynamics, which is practically identical with the regularization of Pauli-Villars. In momentum representation, the propagation function of the free meson is, as is known, equal to $(k^2+\mu^2)^{-1}$ (k is the external momentum). Improvement of the convergence of the theory is obtained as a consequence of the fact that the polarization operator computed by the authors mentioned varies as k^4 for $k \to \infty$. Making use of several general properties of the theory, it is possible to show that the polarization operator varies as k^2 as $k \to \infty$. Consequently, the result obtained in Refs. 2, 3 is based on a loose application of the Feynman method to the problem under consideration.

Thus, use of a true propagation function does not improve the convergence of the given theory. This is in agreement with the general results obtained in Ref. 5; weighty objections against the work of Ref. 2 were also put forward by Feldman.[6] Thus there is left in principle only the possibility of improvement of the convergence of the theory by use of a true "vertex part" (see Ref. 7).

1. We shall show that for the pseudoscalar mesonfield with pseudovector coupling, a certain relation is valid which we shall for brevity call the "theorem of equivalence", in a certain sense analogous to the law of continuity in quantum electrodynamics.**

Following Schwinger,[9] we define the Green's function of the spinor field:

$$G_{\alpha\beta}(x, x') = i\langle T(\psi_\alpha(x)\bar{\psi}_\beta(x'))\rangle.$$

Making use of the equations for ψ and $\bar{\psi}$, we obtain the following equation for $G(x,x')$:

$$\begin{aligned}&\Big\{\gamma_\mu \frac{\partial}{\partial x_\mu} - ig\gamma_5\gamma_\mu\langle\frac{\partial\varphi}{\partial x_\mu}\rangle + m \\ &- g\gamma_5\gamma_\mu\left[\frac{\partial}{\partial z_\mu}\frac{\delta}{\delta J(z)}\right]_{z\to x}\Big\} G(x, x') = \delta(x-x'),\end{aligned} \tag{1}$$

$$\begin{aligned}&G(x, x')\Big\{-\gamma_\mu \frac{\partial}{\partial x'_\mu} + ig\langle\frac{\partial\varphi}{\partial x'_\mu}\rangle\gamma_\mu\gamma_5 + m \\ &+ g\left[\frac{\partial}{\partial z_\mu}\frac{\delta}{\delta J(z)}\right]_{z\to x'}\gamma_\mu\gamma_5\Big\} = \delta(x-x'),\end{aligned}$$

where J is the external source of the meson field. Making obvious transformations, we get

$$\begin{aligned}&\mathrm{Sp}\left\{\gamma_5\gamma_\mu\frac{\partial}{\partial x_\mu}G(x, x)\right\} \\ &+ 2m\,\mathrm{Sp}\{\gamma_5 G(x, x)\} = 0.\end{aligned} \tag{2}$$

In the momentum representation this relation has the form

$$\begin{aligned}&\int(dp)\,\mathrm{Sp}\{\gamma_5 i\hat{k}G(p, p-k)\} \\ &+ 2m\int(dp)\,\mathrm{Sp}\{\gamma_5 G(p, p-k)\} = 0,\end{aligned} \tag{3}$$

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*It should be noted that charge renormalization is carried out incorrectly in Ref. 2.

**This relation is an immediate generalization of a similar relation obtained by Schwinger[8] without account of the proper radiation field.

where

$$(dp) = (2\pi)^{-2}\, d^4p; \quad G(p, p') = \int e^{-ipx}\, G(x, x')\, e^{ip'x'}\, (dx)\,(dx'). \tag{4}$$

The equation for the Green's function of the meson field has the form:

$$\left(\frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x_\mu} - \mu^2\right) D(x, x') + g\,\frac{\partial}{\partial x_\mu}\operatorname{Sp}\left\{\gamma_5\gamma_\mu \frac{\delta G(x, x)}{\delta J(x')}\bigg|_{J=0}\right\} = -\delta(x - x'), \tag{5}$$

where

$$D(x, x') = \delta\varphi(x) / \delta J(x')\,|_{J=0}.$$

The polarization operator is defined by the following relation:

$$\int P(x, y)\, D(y, x')\, d^4y = -g\,\frac{\partial}{\partial x_\mu}\operatorname{Sp}\left\{\gamma_5\gamma_\mu \frac{\delta G(x, x)}{\delta J(x')}\bigg|_{J=0}\right\}. \tag{6}$$

From this it follows that

$$P(x, x') = \frac{\partial}{\partial x_\mu}\frac{\partial}{\partial x'_\nu} P_{\mu\nu}(x, x'), \tag{7}$$

where

$$\begin{gathered} P_{\mu\nu}(x, x') = -ig^2 \int d^4y\, d^4y'\, \operatorname{Sp}\{\gamma_5\gamma_\mu G(x, y)\,\gamma_\nu\Gamma_5(y, x', y')\, G(y', x)\}, \\ -i\gamma_\mu(\partial / \partial x_\mu)\,\Gamma_5(t, x, y) = \delta G^{-1}(t, y) / \delta g\varphi(x). \end{gathered} \tag{7a}$$

Taking the equivalence relation (2) into account, we find that $P(x,x')$ is

$$P(x, x') = \frac{\partial}{\partial x'_\nu}\left\{2img^2 \int d^4y\, d^4y' \times \operatorname{Sp}[\gamma_5 G(xy)\,\gamma_\nu\Gamma_5(y, x', y')\, G(y', x)]\right\}. \tag{8}$$

2. In the first approximation of perturbation theory,

$$\Gamma_5(tsy) = \gamma_5\delta(t - s)\,\delta(t - y).$$

In accord with Eqs. (7) and (7a),

$$\begin{aligned} P(k) &= \frac{ig^2}{(2\pi)^4}\int d^4p\, \operatorname{Sp}\left\{\gamma_5\hat{k}\,\frac{1}{i(\hat{p} + \hat{k}) + m}\,\gamma_5\hat{k}\,\frac{1}{i\hat{p} + m}\right\} \\ &= \frac{-8im^2g^2}{(2\pi)^4}\int d^4p\, \frac{k^2}{(p^2 + m^2)(p - k)^2 + m^2} \\ &\quad + \frac{ig^2}{(2\pi)^4}\int d^4p\, \left\{\hat{k}\,\frac{1}{i\hat{p} + m}\,\hat{k}\,\frac{1}{i(\hat{p} - \hat{k}) + m}\right\}. \end{aligned} \tag{9}$$

It follows from Eq. (8) that

$$P(k) = -\frac{8im^2g^2k^2}{(2\pi)^4}\int \frac{d^4p}{(p^2 + m^2)(p - k)^2 + m^2}. \tag{9a}$$

Comparing Eqs. (9) and (9a), we find, in the first approximation of perturbation theory, the following consequence of the theorem of "equivalence;"

$$\int d^4p\, \operatorname{Sp}\left\{\hat{k}\,\frac{i\hat{p} - m}{p^2 + m^2}\,\hat{k}\,\frac{i(\hat{p} - \hat{k}) - m}{(p - k)^2 + m^2}\right\} = 0. \tag{10}$$

This integral also figures in quantum electrodynamics, where it is equal to zero because of the law of continuity.

However, one can verify this fact by direct calculations only with the aid of artificial procedures (the regulators of Pauli-Villars, the "s-parametization" of Schwinger[8]). This is connected with the fact that all methods of calculating divergent integrals that are applied in quantum field theory (introduction of a shape factor, integration over an invariant finite region) are not invariant to a change of variable, or more precisely, to its "displacement", since $\int f(p-k)\, d^4p$ and $\int f(p)\, d^4p$ differ by some finite quantity in the case when the integrals are quadratic. Meanwhile, repeating the argument of the "theorem of equivalence" in momentum space, it is easy to convince oneself that the possibility of such a displacement of variable of integration is substantial.*

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*The necessity of a displacement of the variable to reduce the integral of Eq. (10) to zero is also seen from the following:

$$\begin{aligned} &\int \operatorname{Sp}\left\{\gamma_\mu\,\frac{1}{i\hat{p} + m}\,\hat{k}\,\frac{1}{i(\hat{p} - \hat{k}) + m}\right\} d^4p \\ &= -i\int \operatorname{Sp}\left\{\gamma_\mu\,\frac{1}{i\hat{p} + m}\,[i\hat{p} + m - i(\hat{p} - \hat{k}) - m] \times \frac{1}{i(\hat{p} - \hat{k}) + m}\right\} d^4p \\ &= -i\int \operatorname{Sp}\left\{\gamma_\mu\,\frac{1}{i(\hat{p} - \hat{k}) + m} - \gamma_\mu\,\frac{1}{i\hat{p} + m}\right\} d^4p = 0, \end{aligned}$$

if we replace p by $p + k$ in the first term of the difference.

We shall start out from Eq. (9a) for the polarization operator. We subtract from the integral in this expression its value for $k = 0$:

$$\int \frac{d^4p}{(p^2+m^2)\,[(p-k)^2+m^2]} - \frac{d^4p}{(p^2+m^2)^2} \tag{11}$$
$$= \int d^4p \, \frac{(2pk-k^2)}{(p^2+m^2)^2\,[(p-k)^2+m^2]}$$
$$= 2\pi^2 i\,\{1-\theta \operatorname{ctg}\theta\},$$

where we have set $k^2 = -4m^2 \sin^2 \theta$. Consequently,

$$P(k) = 16\pi^2 (2\pi)^{-4} m^2 g^2 k^2 \tag{11a}$$
$$\times (1-\theta \operatorname{ctg}\theta) + k^2 A,$$

where $A = -8i\,(2\pi)^{-4} m^2 g^2 \int (p^2+m^2)^{-2}\, d^4p$ is a logarithmically diverging constant.

After renormalization, (9k) takes the form:

$$P'(k) = P(k) - P(k)\big|_{k^2=-\mu^2} - (k^2+\mu^2)\,\frac{\partial P}{\partial k^2}\bigg|_{k^2=-\mu^2} \tag{12}$$
$$= -\frac{g^2 m^6 (k^2+\mu^2)^2}{(2\pi)^2}\int_0^1 \left\{ v^2 dv \Big/ \left[m^2 + \frac{k^2}{4}(1-v^2)\right]\left[m^2 - \frac{\mu^2}{4}(1-v^2)\right]^2 \right\}$$
$$= \frac{4m^2 g^2}{(2\pi)^2}\left\{ k^2(\theta_0 \operatorname{ctg}\theta_0 - \theta \operatorname{ctg}\theta) + (k^2+\mu^2)\left(\frac{1}{2} - \frac{\theta_0}{\sin 2\theta_0}\right)\right\},$$

where $\mu^2 = 4m^2 \sin^2 \theta_0$.

3. Upon calculation of the polarization operator by Feynman's method, we get

$$P(k) = -8i\,(2\pi)^{-4} g^2 k^2 2\pi^2 i$$
$$\times \{1 - \theta \operatorname{ctg}\theta + k^2/12m^2\} + k^2 C,$$

where C is a logarithmically diverging constant. The difference from our result (11a) is associated with the fact that in the Feynman method*

$$\int m^2 f(p)\, d^4p \neq m^2 \int f(p)\, d^4p.$$

In electrodynamics, this does not lead to nonsingle-valuedness in the calculation of the finite part of the polarization operator, since it manifests itself only in the change of the renormalization constant. In our case also, because of the fact that $P(k)$ has a positive factor k^2 in comparison with electrodynamics, this non-single-valuedness is not removed by the renormalization.

The usual program of renormalization consists of the separation from the divergent expression of the part which contains some first powers of the external momentum, while the remainder no longer contains divergences, and the diverging parts are removed as a consequence of the renormalization of mass and charge. For an expression which diverges logarithmically, this program can be carried out effectively. But in the case of the proper mass of the photon or meson, the initial expression diverges quadratically. A stronger divergence leads to an impossibility of separating the divergent part without introduction or artificial procedure, and the available natural methods violate the general properties of the theory (such as the law of continuity in quantum electrodynamics, or the "theorem of equivalence" in the case of pseudoscalar meson theory with pseudovector coupling).

We have shown that, using the "theorem of equivalence", it is possible to represent the polarization operator in the form of an expression which diverges logarithmically, for which the program of renormalization can be effectively carried out without use of any artificial prodedures.

The reason for the difficulty in the calculation of the vacuum polarization both in quantum electrodynamics and in our case lies in the fact that, in the determination of the polarization operators (see Ref. 5), there enters $G(x,x)$, understood as the limit

$$G(x,x) = \lim_{\xi\to 0} {}^1/_2\,\{G(x, x+\xi) + G(x+\xi, x)\}.$$

However, this limit is insufficiently defined, since the direction has not been indicated along which ξ approaches zero. As a unique method of definition of this process ($\xi \to 0$) there is the requirement that the limit process not violate the general properties which follow from the basic equation of the theory. The condition of

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*Our result corresponds to the fact that the regularization of Feynman is applied to

$$\int \frac{d^4p}{(p^2+m^2)\,[(p-k)^2+m^2]},$$

and not to $\displaystyle\int \frac{m^2 d^4p}{(p^2+m^2)\,[(p-k)^2+m^2]}$.

gauge-invariance in quantum electrodynamics and the "theorem of equivalence" in our case will, as we shall show, be satisfied if the limit process satisfies the condition $k\,\xi = 0$, where k is the external momentum of the photon or meson.

In fact, let us consider the polarization operator which corresponds to an "open loop" of the meson eigen energy:

$$P\left(x+\frac{\xi}{2};\ x-\frac{\xi}{2};\ x'\right)$$

$$= g^2\frac{\partial}{\partial x_\mu}\,\mathrm{Sp}\left\{\gamma_5\gamma_\mu\frac{\delta G\,(x+\xi/2,\, x-\xi/2)}{\delta g\varphi\,(x')}\right\}.$$

In the momentum representation,

$$P(k,\xi) = \frac{ig^2}{(2\pi)^4}\int e^{ip\xi}\,d^4p$$

$$\times\,\mathrm{Sp}\left\{\gamma_5\hat{k}\,\frac{1}{i\,(\hat{p}+\hat{k}/2)+m}\,\gamma_5\hat{k}\,\frac{1}{i\,(\hat{p}-\hat{k}/2)+m}\right\}.$$

Here the polarization operator of interest to us, $P\,(k)$, is the limit of $P\,(k,\ \xi)$ as $\xi\to 0$. Just as in Eq. (9) we divide $P\,(k,\ \xi)$ into two parts:

$$P(k,\xi) = P_1(k,\xi) + P_2(k,\xi);$$

$$P_1(k,\xi) = \frac{ig^2}{(2\pi)^4}\int d^4p\,e^{ip\xi}\,\mathrm{Sp}\left\{\hat{k}\,\frac{1}{i\,(\hat{p}+\hat{k}/2)+m}\,\hat{k}\,\frac{1}{i\,(\hat{p}-\hat{k}/2)+m}\right\};$$

$$P_2(k,\xi) = -\frac{8im^2g^2k^2}{(2\pi)^4}\int d^4p\,e^{ip\xi}\,\frac{1}{[(p+k/2)^2+m^2]\,[(p-k/2)^2+m^2]}\,.$$

The integral which enters into $P_1(k,\ \xi)$ is computed in Ref. 10 and leads to

$$P_1(k,\xi) = -\,4ig^2$$

$$\times\left\{k_\mu\frac{\partial}{\partial\xi_\mu}\Delta+(\xi)\int_{-1}^{1}dv\,(k,\xi)\,e^{iv\,(k,\xi)/2}\right\}.$$

If we superimpose on the limiting process $\xi\to 0$ the condition $k\,\xi = 0$, then $P_1\ (k,\ \xi)$ reduces to zero in correspondence with the theorem of equivalence.*

In analogy with (11), we remove from the integral entering into $P_2\ (k,\ \xi)$ its value for $k = 0$:

$$\int d^4p\,\frac{e^{ip\xi}}{[(p+k/2)^2+m^2]\,[(p-k/2)^2+m^2]}-\int d^4p\,\frac{e^{ip\xi}}{(p^2+m^2)^2}$$

$$=\int d^4p\,\frac{e^{ip\xi}\,(k^2/2)\,(p^2-m^2-k^2/8)}{[(p+k/2)^2+m^2]\,[(p-k/2)^2+m^2]\,(p^2+m^2)}\,.$$

This integral has no singularities for $\xi = 0$ and is equal to $2\,\pi^2\,i\,(1-\theta\,\mathrm{ctg}\,\theta)$ in this case. After renormalization, we again obtain the value (12) for $P\,(k)$.

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*This is also seen from the following:

$$\int d^4p\,e^{ip\xi}\,\mathrm{Sp}\left\{\hat{k}\,\frac{1}{i\,(\hat{p}+\hat{k}/2)+m}\,\hat{k}\,\frac{1}{i\,(\hat{p}-\hat{k}/2)+m}\right\}$$

$$=\int d^4p\,e^{ip\xi}\,\mathrm{Sp}\left\{\hat{k}\,\frac{1}{i\,(\hat{p}+\hat{k}/2)+m}\right.$$

$$\left.-\,\hat{k}\,\frac{1}{i\,(\hat{p}+\hat{k}/2)+m}\right\}$$

$$=\int d^4p\,e^{i\,(p\xi+k\xi/2)}\,\mathrm{Sp}\left\{\hat{k}\,\frac{1}{i\hat{p}+m}\right\}$$

$$-\int d^4p\,e^{i\,(p\xi+h\xi/2)}\,\mathrm{Sp}\left\{\hat{k}\,\frac{1}{i\hat{p}+m}\right\}=0,$$

if $k\,\xi = 0$. The validity of the change of variable here is not called into question since all the integrals converge.

Translated by R. T. Beyer
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The Polarization-Direction Correlation of Successive Gamma-Ray Quanta From Co^{60} and Na^{24}

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Apparatus for observing the correlation of polarization and directions of propagation of successive gamma-ray quanta is described. The polarization sensitivity of this apparatus was determined experimentally and measurements were carried out for gamma-ray quanta of Co^{60} and Na^{24}. The first two excited states of Ni^{60} and Mg^{24} are shown to have even parity.

INTRODUCTION

By measurements of the correlation of polarization and directions of propagation of gamma-ray quanta emitted by atomic nuclei in successive radiative transitions, it is possible to determine the multipolarity of the corresponding transitions and also to separate electric transitions from magnetic[1]. As a result, one succeeds in obtaining both the spins of the nuclei in the excited states and the parity of the wave function in those states. The dependence of the magnitude of polarization of the gamma-rays emitted in a cascade on the angle between their propagation directions was calculated by Falkoff[2] and Hamilton[3].

The first measurements of correlation of the polarization and the propagation direction of gamma-rays were made in 1950 by Metzger and Deutsch[4] and Williams and Wiedenbeck[5]. In recent years still several other experiments were reported[6-8] in which a similar method was used.

It should be noted that the multipolarity of radiative transitions can be determined from measurements of the angular correlation of gamma-rays emitted in cascade[1]. Furthermore, the theory of angular correlation of nuclear radiation is worked out satisfactorily[9], and the technique of measuring angular correlation of gamma-rays is considerably simpler than the technique for measuring polarization-direction correlation of gamma-rays. In those cases where the multipole order of radiative transitions has been determined from the measurements of angular correlation of gamma-rays, as for the transitions considered in the present work, it only remains to determine the parity of the corresponding excited states by a measurement of polarization-direction correlation. To this end it is not necessary, as in the initial experiments of Metzger and Deutsch[4], to measure the degree of polarization for various angles θ between the directions of propagation of the emitted gamma-rays. Sufficiently complete data on the parity of the excited states are obtained by measurements of the degree of polarization for a given angle θ. For this purpose the selected angle θ should be the one exhibiting the maximum degree of polarization of the gamma-rays.

DESCRIPTION OF APPARATUS

The apparatus used in the present work consisted of gamma-ray detectors insensitive to the direction of polarization and a detector (polarimeter) sensitive to the degree of polarization of gamma-rays. As gamma-ray detectors insensitive to the degree of polarization we used two scintillation counters with stilbene crystals. These counters (A and A' in Fig. 1) were 22 mm thick and placed symmetrically about the central axis of the apparatus SS' passing through the source S. The polarimeter consisted of three scintillation crystal counters: the central counter B was used as a scatterer and the side counters C and C' registered the radiation scattered from B. Tolane crystals

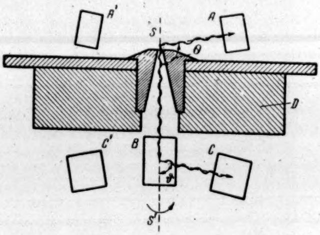

FIG. 1. Principal view of the apparatus: SS' -- symmetry axis of the instrument, S -- the source, A and A' -- scintillation crystals of detectors insensitive to the polarization of photons, B -- the central crystal of polarimeter, C and C' -- side crystals of polarimeter, D-- the shield.

28 mm thick were used in the scintillation counters C and C', stilbene crystals were used in the counter B. The crystals C and C' were shielded by lead from the direct beam of gamma-rays emitted by the source.

The action of the polarimeter is based on the dependence of the differential cross section for Compton scattering on the angle δ between the direction of polarization of the incident photon and the plane of scattering. In that respect the described polarimeter is similar to others described previously[4,6]. The polarimeter can be rotated about the symmetry axis SS', which allows placing of crystals C and C' at any angle δ to the scattering plane of gamma-rays (see Fig. 1). Simultaneously, the counter B rotates about the axis of the apparatus. The ratio of the coincidence counting rates in crystals B and C at the position $\delta=0$ to the coincidence counting rates for $\delta = \pi/2$ depends on the degree of polarization of gamma-rays incident on the counter B. The mean angle θ at which the photons were scattered from crystal B to crystals C or C' was equal to 80° (see Fig. 1).

Photomultiplier tubes FEU-19 were used in the scintillation counters. The selection of nonuniform distribution of potentials on the multiplier dynodes allowed an increase of the total potential supplied to the divider of the multiplier tube to 2200-2500 v. With this arrangement the positive pulses due to the gamma-rays emitted by Zn^{65} taken off the multiplier collector were of the order of 40-50 v and the noise level less than 1 v. The efficiency of detection of 1 mev gamma radiation was of the order of 25%, of 0.3 mev gamma-rays of the order of 50%.

The $0.5\,\mu$sec long pulses from FEU-19 were fed by means of a cascade cathode follower into the coincidence circuits (Fig. 2). In each channel of the coincidence circuits the noise was chopped off and the pulses were formed in a univibrator in a 6H15P tube with inductance $L \approx 100\,\mu$ henries, shunted by DGC-4 as a plate load. The pulses at the output of the univibrator had the form of a half of a sine wave with amplitude 12 v and duration of base $0.3\,\mu$sec. These pulses were fed into double coincidence channels through 6Zh2P mixing tubes with resolving time $\tau_{AC} = 2.8 \times 10^{-8}$ sec, $\tau_{BC} = 3.6 \times 10^{-8}$ sec and $\tau_{AB} = 5.6 \times 10^{-8}$ sec. The triple coincidence of pulses in channels A, B and C were selected by counting the coincidences AC and BC. The resolving time of the triple coincidence counters was determined by the resolution of the double coincidence stages AC and BC. The pulse rates of the photomultiplier tubes and the double coincidence rates must be known to take into account the accidental triple coincidences.

In our measurements of the correlation of polarization and directions of propagation of gamma-rays the accidental triple coincidences N^{acc}_{ABC} composed an important part of the total number of triple coincidences. N^{acc}_{ABC} was computed from the measured photomultiplier pulse rate, double coincidence rate and τ of the double coincidence circuits. Therefore, the stability of the resolving time of the double coincidence circuits was very important. Experiments showed that variations in τ of the circuits used in the present work did not exceed ±5% during a day of experiments.

POLARIZATION SENSITIVITY OF POLARIMETER

The polarization sensitivity R is determined by the ratio of the effective cross sections of the Compton scattering in the plane $\delta = 0$ and the plane $\delta = \pi/2$. Thus R depends on the photon energy according to the Klein-Nishina formula. Because of the finite dimensions of the gamma-ray detectors, the measurements are made in several angular intervals $\Delta\delta$ and $\Delta\theta$ and R then depends on the geometry of measurements and is determined experimentally.

For a known degree of polarization p of gamma-rays the ratio of the coincidence rates q' in channels B and C for $\delta = 0$ and $\delta = \pi/2$ is equal to

$$q' = (p + R)/(pR + 1). \qquad (1)$$

In the measurements of R we used a lead target instead of a radiation source S. A Co^{60} gamma-ray source of an approximate intensity 500 millicuries was placed in the plane $\delta = 0$. A narrow beam of gamma-rays was directed on the target so that the gamma-rays scattered at an angle α (in the experiments α was chosen to be 90° or 60°) fell on the central crystal B of the polarimeter. Under these conditions the energy and polarization intensity of the gamma-rays incident on the polarimeter are well known (Table I).

In the described experiments we measured a ratio of the double coincidence rates n_{BC} for $\delta = 0$ and $\delta = \pi/2$; $q' = n(0) / n(\pi/2)$. The insertion of a target reduced the double coincidence counting rate to about 15% of the full rate at $\alpha = 90°$ and to 90% at $\alpha = 60°$. The counting rate in these experiments was 1 to 2 coincidences per second. In measurements of the soft gamma-rays in the scintillation counters C and C' the amplification of FEU-19 was changed somewhat because of the change in the magnetic fields (namely

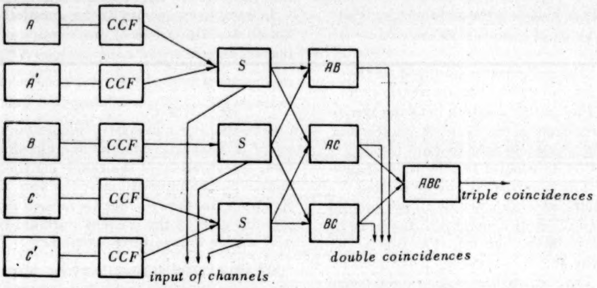

FIG. 2. Block diagram of the detection system: A, A', B, C and C' -- scintillation counters (same symbols as in Fig. 1); CCF -- cascade cathode follower; S -- pulse shaping circuit; AB, AC and BC -- double coincidence circuits, ABC -- triple coincidence circuit.

earth magnetic field) due to the rotation of the polarimeter. A check of the dependence of the double coincidence counting rate on the rotation of the polarimeter (asymmetry of double coincidences) was made with sources of unpolarized gamma-rays Sn^{113}, Cs^{137} and Co^{60} placed at the target location. The measured ratio of the double coincidence rates $n(0)\ /\ n(\pi/2)$ was different from unity only for the soft gamma-rays for Sn^{113} (1.08 ± 0.015). The value of q' obtained in the experiments with polarized gamma-rays, corrected for the asymmetry effect in the double coincidence counting rates, are given in Table I. The values of polarization sensitivity computed from Eq. (1) are also given in Table I. The most accurate measurements were made with scattering angle $\alpha = 60°$ because of a larger coincidence rate. It is evident from Table I that the polarization sensitivity of the above-described polarimeter is not in any way inferior to the polarimeters used in other experiments.

TABLE I. Measurements of the polarization sensitivity of polarimeter.

Scattering Angle α	Energy of the scattering photon, kev	Degree of polarization P	Ratio of coincidence rates q'	Sensitivity of polarimeter R
90°	360	0.46	1.44 ± 0.08	3.1 ± 0.6
60°	560	0.44	1.45 ± 0.02	2.8 ± 0.1

The experimental values of R allow for a calculation of the unknown geometrical factors $\Delta\delta$ and $\Delta\theta$ which are sufficient for calculation of the sensitivity of the polarimeter for gamma-rays of any energy. These calculations are used further in measurements of the polarization-direction correlation of Co^{60} and Na^{24} gamma radiation. Uncertainty in the choice $\Delta\delta$ and $\Delta\theta$ results in an error smaller than the error due to the experimental calibration of the polarization sensitivity of the polarimeter.

MEASUREMENTS ON Co^{60} AND Na^{24}

The gamma radiation accompanying disintegrations of Co^{60} and Na^{24} nuclei is well known. The sequence of emission of the cascade gamma-rays has been established. The measurements of angular correlation of gamma-rays emitted in the studied transitions[10-12], established with certainty the following sequence of spins of the ground state and the two excited states: 0; 2; 4, where both gamma-rays are quadrupoles. Consideration of the various combinations of parities of the excited levels leads to four possible transitions: $E2E2$, $E2M2$, $M2E2$, $M2M2$. The past measurements of the polarization direction correlation of Co^{60} gamma-rays selected one of these transitions as the actual one[4,6]. Therefore, the measurements on Co^{60} discussed below should be considered only as control measurements verifying the entire decay scheme. Using the detailed measurements of the pair conversion coefficients of Na^{24} gamma-rays of Bloom[13] we obtain data on the character of radiative transitions. Measurements of polarization-direction correlation were not made for Na^{24} gamma-rays. In the present work the data on parity of the excited levels of Mg^{24} are obtained by measuring polarization-direction correlation of gamma-rays.

To determine the correlation of polarization and propagation direction of gamma-rays we measured the ratio q of triple coincidence rates in channels A, B and C when crystals C and C' of the polarimeter (cf. Fig. 1) were placed in the plane of scattering of gamma-rays ($\delta = 0$) to the triple coincidence rate then the polarimeter was rotated into the plane $\delta = \pi/2$; $q = N(0) \;/\; N(\pi/2)$. In a general case both gamma-rays γ_1 and γ_2 entering the polarimeter can have various degrees of polarization and also different energy. We shall denote by $(I_{||})$ and $(I_{\perp})$ the intensity of radiation polarized in the plane of scattering of gamma-rays and in the plane perpendicular to the plane of scattering, respectively. Then we shall have for the ratio of the triple coincidence rates

$$q = \frac{(I_{\parallel})_{\gamma_1} a_{\gamma_1} b_{\gamma_2} + (I_{\parallel})_{\gamma_2} a_{\gamma_2} b_{\gamma_1} + R_1 (I_{\perp})_{\gamma_1} a_{\gamma_1} b_{\gamma_2} + R_2 (I_{\perp})_{\gamma_2} a_{\gamma_2} b_{\gamma_1}}{(I_{\perp})_{\gamma_1} a_{\gamma_1} b_{\gamma_2} + (I_{\perp})_{\gamma_2} a_{\gamma_2} b_{\gamma_1} + R_1 (I_{\parallel})_{\gamma_1} a_{\gamma_1} b_{\gamma_2} + R_2 (I_{\parallel})_{\gamma_2} a_{\gamma_2} b_{\gamma_1}}, \quad (2)$$

where a is the polarimeter efficiency, b is the detection efficiency of gamma-rays by channel A and R is the polarimeter sensitivity. The quantities $I_{||}$, $I_{\perp}$, a, b and R are different for the photons γ_1 and γ_2.

The quantities R_1 and R_2 can be calculated from the measurements of polarization sensitivity of the polarimeter as explained in the preceding section. The results of these calculations for gamma-rays of Co^{60} and Na^{24} are given in Table II. The distance of crystal A from the source was taken into account in the calculations of R_1 and R_2. It slightly changed $\Delta\delta$.

TABLE II. The relative efficiency of photon detection.

Isotope	Photon energy mev	Polarimeter sensitivity R	Polarimeter Detection a	Detection Efficiency b
Zn^{65}	1,12	—	1	1
Co^{60} γ_1 . . .	1.17	1,70	0,97	0,98
γ_2 . . .	1.33	1,64	0,87	0.90
Na^{24} γ_2 . . .	1,37	1.67	0.84	0,88
γ_1 . . .	2,75	1,33	0.52	0.54

For the quadrupole transitions that are of interest to us Eq. (2) simplifies into a form similar to Eq. (1),

$$q = (p + \overline{R})/(p\overline{R} + 1). \quad (3)$$

For $E2E2$ or $M2M2$ transitions the polarization intensity is the same for both cascade gamma-rays, $p = p_1 = p_2$ and the average value of the sensitivity $\overline{R}$ is equal to

$$\overline{R} = (R_1 + \eta R_2)/(1 + \eta), \quad (4)$$

where

$$\eta = (a_{\gamma_2}/a_{\gamma_1})\; b_{\gamma_1}/b_{\gamma_2}. \quad (5)$$

For $E2M2$ or $M2E2$ transitions $p = p_1 = 1/p_2$ and

$$\overline{R} = (R_1 + \eta)/(R_2\eta + 1). \quad (6)$$

The polarization intensity p of gamma-rays depends on the angle between the photons γ_1 and γ_2. For quadrupole transitions the maximum polarization intensity is at 90° angle between γ_1 and γ_2. In our measurements this angle was 100° and p was calculated for this angle.

If the multiplier $\eta = 1$, then Eq. (3) does not depend on the efficiency of the gamma detectors. For

TABLE III. Calculated and experimental ratios of triple coincidences $q = N(0)/N(\pi/2)$.

Type of transition	Degree of polarization p	Co^{60}			Na^{24}		
		$\bar{R}$	q calculated	q experimental	$\bar{R}$	q calculated	q experimental
$E2E2$	1,36	1,67	0,92	0,91±0,03	1,50	0,94	0,94±0,03
$E2M2$	1,36	1,02	1,00		0,87	1,02	
$M2E2$	0,73	1,02	1,00		0,87	0,98	
$M2M2$	0,73	1,67	1,08		1,50	1,06	

gamma-rays of different energies R may depend on the detection efficiency of gamma-rays. Therefore we also measured the relative detection efficiencies b and a of gamma-rays by the channel A and the polarimeter respectively. The efficiency of the polarimeter (of double coincidence counting rate n_{BC}) and the counter (pulse rate of channel A) was compared with a known efficiency of a Geiger counter previously calibrated using an ionization chamber[14]. The comparative data obtained with gamma radiation of isotopes Zn^{65}, Co^{60} and Na^{24} allow us to obtain by means of successive extrapolation the relative detection efficiency of monochromatic gamma-rays a and b (cf. Table II). The errors in these measurements are of the order ±0.02. From the data of Table II we obtain the experimental value η for Co^{60} gamma-rays: $\eta = 0.98 \pm 0.04$; for Na^{24} gamma-rays: $\eta = 0.99 \pm 0.04$. Thus $\eta = 1$ within the experimental error. This fact was evident for Co^{60} gamma-rays but not clear for Na^{24}.

The calculations of p, R and q for cascade gamma-rays Co^{60} and Na^{24} are given in Table III. In 1 min long measurements of the polarization-direction correlation of gamma-rays from Co^{60} and Na^{24} approximately 10 triple coincidences ABC were observed, of which 1/3 was accidental. The experimentally determined values of q (Table III) show that the transitions in Co^{60} and Na^{24} are $E2E2$ transitions. Parity and spin of the ground and the first two excited levels of Ni^{60} and Mg^{24} are then: 0^+, 2^+, 4^+. For Co^{60}, these results agree with the data obtained previously. Our conclusions about the parity of the excited states of Mg^{24} verify the results of Bloom[13].

In conclusion, the authors express gratitude to I. S. Shapiro for discussions of the results of this work.

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Yields of Photoneutrons from Intermediate and Heavy Nuclei

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The yields of photoneutrons were measured for various maximum γ-bremsstrahlung energies lying between the threshold of the (γ, n) reactions up to $E_{\max} = 27$ mev. The photoneutron cross section was determined by the "photon difference method" as a function of the photon energy from the yield curves for ten elements (copper, zinc, cadmium, iodine, tantalum, gold, thallium, bismuth, thorium and uranium).

IN the irradiation of nuclei with atomic number $Z \gtrsim 50$ by γ-quanta with energies 10-30 mev, reactions with emission of neutrons [(γ, n), $(\gamma, 2n)$, $(\gamma, 3n)$, . . .] are practically the only reactions associated with photodisintegration of the nuclei. The yield of protons relative to neutrons for nuclei with $Z \approx 50$ does not exceed 3-4%[1] and falls to a fraction of one percent for lead and bismuth[2]. The probability of the reaction (γ, γ') ought not to exceed several percent of the probability of emission of photoneutrons[3]. Thus the total cross section of these photoneutron reactions, with accuracy to within several percent, is equal to the cross section of absorption of γ-quanta, σ_γ, and the radiation yield of photoneutrons gives a series which is characteristic of the nuclear interaction of γ-quanta of the given energy.

In the present work, the yields of photoneutrons were measured for different maximum energies of γ-bremsstrahlung from the threshold of the reaction $(\gamma, n) = E_n$, up to $E_{\max} = 27$ mev for the following ten elements: Cu, Zn, Cd, I, Ta, Au, Tl, Bi, Th and U.

MEASUREMENT OF THE YIELD OF PHOTONEUTRONS AND THE NEUTRON CROSS SECTION

The measurements were carried out on the synchrotron of the Physical Institute of the Academy of Sciences at 20 mev, which gives 150 γ-pulses per sec, each of about 20 μsec duration.

The irradiated samples were placed in the center of a paraffin block and delayed photoneutrons were recorded in the time intervals between the γ-pulses by an ionization chamber filled with BF_3. The absolute yield of the neutrons was calibrated by a standard source (Ra_α + Be). For all maximum energies, a correction was introduced for the different spatial distribution of neutrons in the paraffin and the absorption of γ-rays in the samples.

The number of photons incident per unit time on the irradiated specimen was measured by the ionization in the air space of a thickwalled aluminum dosimeter chamber (the thickness of the front wall was 7.5 cm)[4]. The method of measurement has been suitably described in the literature[5].

The yield curves of photoneutrons have been plotted in Fig. 1. The maximum energy of the γ-bremsstrahlung, $E_{\max}$, is plotted along the abscissa; the ordinate gives the number of neutrons emitted per gram-molecule of sample per sec, for a flux of γ-radiation that gives rise to a current equal to 1 μA in the air space of the dosimeter chamber. Each experimental point on the yield curves represents the mean of five individual series of measurements. The mean square errors are shown (the statistical accuracy was appreciably higher).

The measured integral curves of the neutron yields permitted us to calculate the photoneutron cross section σ_n for different energies of the γ-quanta. The curves of the differential cross sections $\sigma_n(E)$ computed by the "photon difference method"[6], are plotted in Fig. 2. A Schiff spectrum was obtained for the γ-bremsstrahlung[7]. The curves have the typical shape of all photonuclear reactions--the resonance shape associated with the resonant character of the absorption of the γ-quanta by the nuclei.

The basic characteristics of the cross section curves are given in Table I: the cross section maximum $\sigma_{n\max}$; the energy corresponding to the maximum cross section, $E_{\sigma_{n\max}}$; the half width of the curve, and the integral cross section

$$\int_{E_n}^{E_0} \sigma_n \, dE \quad (E_0 = 27 \text{ mev}).$$

The cross section curves plotted in Fig. 2 were obtained from the continuous spectrum of γ-radiation as a result of a complicated calculation and

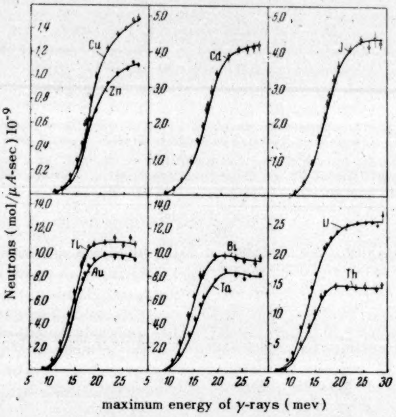

FIG. 1. Yield curves of photoneutrons.

TABLE I. Fundamental characteristics of photoneutron cross sections.

Element	$E_{\sigma_n \max}$ in mev	$\sigma_{n \max}$ in barns	Half width in mev	$\int_{E_n}^{E_0} \sigma_n (E)\, dE$ in mev-barns	$\int_{E_n}^{E_0} \sigma_n (E)\, dE / \sigma_{n \max}$
Copper	17.2	0.126	4.3	0,93	7.4
Zinc	16.3	0,082	6,3	0.66	8,1
Cadmium	16.0	0,270	6.4	2,28	8.4
Iodine	15,5	0.288	6,0	2.35	8.2
Tantalum	14,5	0.452	6,8	3,87	8.6
Gold	14.2	0.571	6 0	4 37	7,6
Thallium	14.6	0.655	5.4	4,99	7,6
Bismuth	13.9	0.537	5,9	3,96	7.4
Thorium	14.5	0.796	5.6	6.33	8.0
Uranium	14.9	1,18	6.8	12.5	10,6

the accuracy of these curves is much less than the accuracy of the measured curves of neutron yields (see Fig. 1). The position of the maximum of the curves is determined with accuracy ± 5 mev. For the cross section maximum and the internal cross section, the errors amount to about 10%. With increase in energy, the error in the determination of the cross section increases, reaching 20-25% at the end of the curves.

The resultant integral cross sections for the photoneutron yields are given in Fig. 3 as a function of the atomic number of the nucleus. We have shown the corresponding maximum cross section $\sigma_{n \max}$ in this same graph. The integral cross section and the maximum cross section both increase with increase in Z ($\sim Z^{1.6}$). In the last column of Table I, the ratio

$$\int_{E_n}^{E_0} \sigma_n \, dE \, / \, \sigma_{n \max}.$$

is plotted.

For all the nuclei investigated, with the exception of uranium, for which the relative fission probability is about 0.3 [5], these ratios ranged from 7.4 to 8.5. One can then roughly consider that the integral cross section expressed in mev-barns is connected with the maximum cross section in barns by the following expression (up to 27 mev):

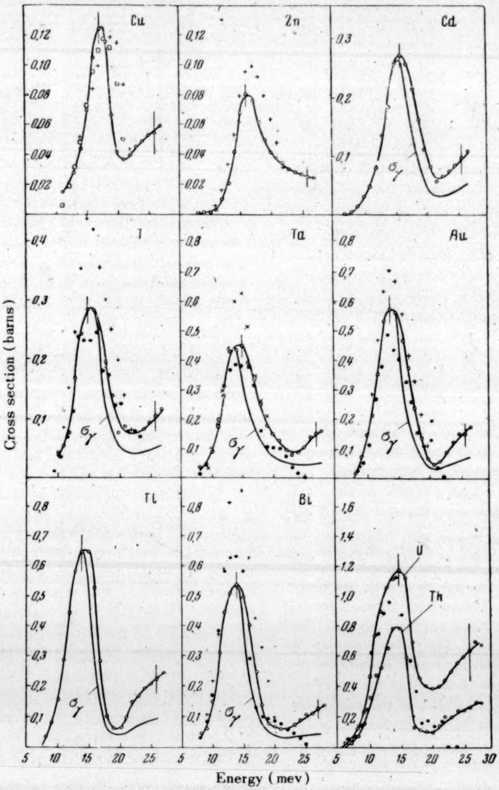

FIG. 2. Photoneutron cross section σ_n, computed from the yield curves by the "photon difference method". + -- cross sections obtained in Ref. 8, ● -- cross sections obtained in Ref. 9, × -- cross section measured in Ref. 10 for tantalum, □ -- cross section of the reaction (γ, n) for copper, obtained by summation of the cross sections of the (γ, n) reaction for Cu^{63} and Cu^{65} (with account taken of isotopic composition), measured in Ref. 6 by the radioactivity of the remaining nuclei. For Cd, I, Ta, Au, Tl and Bi, curves are presented for the cross sections of γ-quanta, computed from the statistical theory of nuclei.

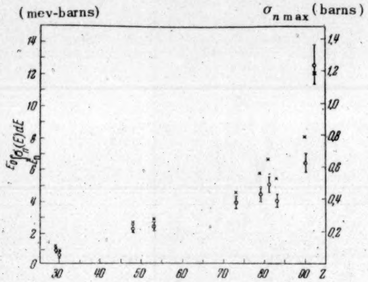

FIG. 3. Integral cross sections of photoneutron yield as functions of atomic numbers. Maximum values $\sigma_{n\,\max}$ are shown by the crosses. The experimental points lie on the curve $Z^{1.6}$.

$$\int_{E_n}^{E_0} \sigma_n\,dE \approx 8\sigma_{n\,\max}.$$

Similar results were obtained in the work of Montalbetti, Katz and Goldemberg[8], for 12 elements with $Z \geq 29$, and in the work of Nathans and Halpern[9] for 9 elements with $Z \geq 29$. In the first work the neutron yields were measured up to $E_{\max} = 22$ mev, in the second, up to $E_{\max} = 25$ mev.

The cross sections $\sigma_n(E)$ obtained by these authors for Cu, Zn, I, Ta, Au, Bi and U, are plotted in Fig. 2. For Cd, Tl and Th, the similar curves in the literature have not been shown. As is evident from the drawing, the cross sections obtained in Refs. 8 and 9 differ rather widely from one another. The curves obtained in the present work lie between them as a rule. In the region of energy above 20 mev, the path of the curves $\sigma_n(E)$ obtained in our research, diverges sharply from those obtained in Ref. 9 for Ta, Au, Bi and U. The decrease of the cross section, almost to zero, observed by Nathans and Halpern, is probably connected with inaccurate consideration of the final points which lie close to the sharp fall-off of the cross section curves. In our case the accuracy of the calculation of these points was controlled by the cross sections at the higher energies, which took on anomalously large values for a decrease in the cross section in the interval 20-30 mev. Different variants of calculations, carried out for the neutron cross section curves, gave appreciable values of the cross section σ_n in all cases (for energies of about 30 mev).

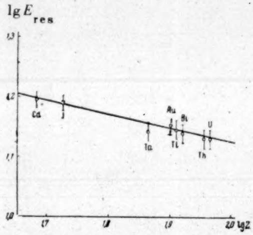

FIG. 4. Dependence of the resonance energy of the absorption of γ-quanta on the atomic number of nuclei. For uranium and thorium, E_{res} are obtained as the results of estimates given in Ref. 5.

ABSORPTION CROSS SECTION OF γ-QUANTA

As was pointed out above, for nuclei with $Z \geq 50$, in energies of the excitation considered, the yield of photoprotons relative to the neutrons was very small. For such nuclei the measured photoneutron cross section was equal to

$$\sigma_n(E) \approx \sigma(\gamma, n) + 2\sigma(\gamma, 2n) + 3\sigma(\gamma, 3n) + \ldots \approx \sigma_\gamma(E)\,\bar{n}(E),$$

where $\bar{n}(E)$ is the mean number of neutrons emitted by the nuclei at the excitation energy E. In accord with statistical theory, we consider the relative probability of the reactions (γ, n), $(\gamma, 2n)$, . . . , and $\bar{n}(E)$ for different excitation energies, and can then obtain curves for the absorption cross section of the γ-quanta, $\sigma_\gamma(E)$.

The relative yields of the reaction $(\gamma, 2n)$, measured experimentally for several isotopes, agree well with the theoretical; therefore, the curves of $\sigma_\gamma(E)$ computed in this fashion ought to be close to the actual ones. Such a calculation was completed for Cd, I, Ta, Au, Tl and Bi. In the case of U and Th, neutrons are also emitted in the photofission of the nuclei; an estimate of $\bar{n}(E)$ was not obtained because of the absence of data on the ν-physical--the number of neutrons emitted immediately in the fission process for various energies of excitation.

In Table II we have plotted the thresholds of photoneutron reactions which are energetically possible, for energies of excitation of nuclei up to 27 mev for the nuclei under consideration. The

TABLE II. Threshold of photoneutron reactions (mev).

Element	(γ, n)	$(\gamma, 2n)$	$(\gamma, 3n)$	$(\gamma, 4n)$
Cadmium	6.7	14.6	23,0	>30
Iodine	9.4	16.2	26.0	32.9
Tantalum	7,6	13.9	21.6	28.2
Gold	8,1	14,9	23.9	$\geqslant 30$
Thallium	7.5	14,0	22	28.8
Bismuth	7.4	14.2	22.5	29.6

binding energies of the neutrons were taken from the table prepared by Kravtsov[11]. For cadmium and thallium, $\bar{n}(E)$ were computed for each isotope separately and then averaged by means of the percent composition of each isotope. According to Weisskopf[12], the constant a in the expression for the density of levels of the remaining nucleus, $\omega = Ce^{\sqrt{aE}}$, was taken equal to $3.35\,(A-40)^{1/3}$. The resultant curves for $\sigma_\gamma(E)$ were plotted in Fig. 2, together with the curves of the cross sections $\sigma_n(E)$. In Table III we have listed the basic characteristics of these curves.

In accordance with the currently available theoretical considerations, for photon energies 10-30 mev, there appears to be a dipole mecahnism of absorption of γ-radiation by the nuclei[13-16]. In different theoretical works which treat this question, the following quantities were computed which characterize the cross section for absorption of the γ-quanta: the resonance energy E_{res}[14,15], the integral cross section

$$\int_0^\infty \sigma_\gamma \, dE$$

(see Ref. 16); the average energy

$$\bar{E} = \int E\sigma_\gamma \, dE \Big/ \int \sigma_\gamma \, dE$$

(see Ref. 16), and the moments

$$\int_0^\infty (\sigma_\gamma / E) \, dE$$

(see Ref. 17) and

$$\int_0^\infty (\sigma_\gamma / E^2) \, dE$$

(see Ref. 13). The dependence of the resonance energy E_{res} on the atomic number of the nucleus is plotted in Fig. 4. The logarithms of the corresponding quantities are plotted along the coordinate axes. Copper and zinc were excluded from consideration, since, because of the high probability of the reaction (γ, p), the position of the maximum of the cross section σ_n could not be identified with E_{res}. The straight line drawn through the experimental points gives the dependence $E_{res} = 35.9\,Z^{-0.215}$ or $E_{res} = 38.0\,A^{-0.188}$. Considering the large errors in the computation of the cross section curves, the resultant expressions agree very well with the expressions for $E_{\sigma_{n\,max}}$ obtained in Ref. 8: $E_{\sigma_{n\,max}} = 37\,A^{-0.186}$, and in Ref. 9: $E_{\sigma_{n\,max}} = 38.5\,A^{-0.186}$. The different models used in Refs. 14 and 15 give the dependencies $A^{-1/3}$ and $A^{-1/6}$.

According to Ref. 16, the integral cross section is

$$\int_0^\infty \sigma_\gamma \, dE = 0.06\,\frac{ZN}{A}\,(1 + 0.8\,x),$$

where x is the exchange part of the potential of the

TABLE III. Characteristics of the cross section of absorption of γ-quanta by nuclei.

Element	E_{res} in mev	$\sigma_\gamma(E = E_{res})$ in barns	half width in mev	$\int_{E_n}^{E_0} \sigma_\gamma \, dE$ in mev-barns	$\int_{E_n}^{E_0} \sigma_\gamma \, dE / 0{,}06 \times (ZN/A)$	$\int_{E_n}^{E_0} (\sigma_\gamma / E) \, dE$	$\int_{E_n}^{E_0} (\sigma_\gamma / E^2) \, dE$	$r_0 \times 10^3$ in cm
Cadmium	15.6	0,263	5,1	1,76	1,06	0,111	0,00745	1,26
Iodine	15,5	0,288	4,9	1,86	1,00	0,117	0,00768	1,16
Tantalum	13.9	0,448	4,5	2,74	1,05	0,190	0,0139	1,15
Gold	14.2	0,571	4,7	3,49	1,23	0,244	0,0182	1,23
Thallium	14.0	0,648	4,6	3,77	1,28	0,266	0,0200	1,25
Bismuth	13,8	0,537	4,8	3,12	1,04	0,230	0,0178	1,16

nuclear forces.

In Table III are plotted the integral cross sections obtained from the curves of

$$\sigma_\gamma, \quad \int_{E_n}^{E_0} \sigma_n \, dE$$

and the ratio

$$\int_{E_n}^{E_0} \sigma_\gamma \, dE \,/\, 0.06 \, \frac{ZN}{A}, \quad (E_0 = 27 \text{ mev}).$$

Inasmuch as the σ_γ are not large, even for energy $E = 27$ mev, the integral cross sections shown in Table III ought to be less than the integral cross sections computed theoretically. The average value of x, obtained from our data, and equal to 0.14, gives a lower limit for the magnitude of the exchange forces. Calculation of the cross section σ_γ for higher energies ought to give higher values for x.

In Refs. 8 and 9, in which the neutron yields were measured for $E_{\max} = 22$ and 25 mev, respectively, the authors computed x, improperly setting

$$\int_{E_n}^{E_{\max}} \sigma_n \, dE = \int_0^\infty \sigma_\gamma \, dE.$$

The value of $x = 0.4$-0.5 obtained by them for heavy nuclei can be shown to be correct only because of the accidental compensation of the higher cross section by the lower upper limit of integration.

The values of the average energies that we obtained were not computed, since the cross sections for energies higher than 27 mev ought to make a relatively large contribution in this case.

The error due to the finite upper limit of the integration is less for the integral $\int \frac{\sigma_\gamma}{E} dE$. According to the calculations of Khokhlov[17],

$$\int_0^\infty \frac{\sigma_\gamma}{E} dE = 4\pi^2 \frac{e^2}{\hbar c} r_0^2 A^{2/3} \left[\left(\frac{N}{A}\right)^2 \varphi(Z) + \left(\frac{Z}{A}\right)^2 \varphi(N) \right].$$

The function $\varphi(n)$ used in the paper was computed for finite and infinite depth of the potential well. Assuming the experimental values of

$$\int_{E_n}^{E_0} \frac{\sigma_\gamma}{E} dE = \int_0^\infty \frac{\sigma_\gamma}{E} dE,$$

one can compute the value of the universal radius r_0. The mean value of r_0, obtained in this fashion, is equal to 1.0×10^{-3} cm for the finite well and 1.2×10^{-13} cm for the infinite well.

The small value of r_0 obtained for the finite well, which is the more realistic case, substantiates the idea that the cross sections σ_γ have a significant value for energies greater than 30 mev.

The smallest effect for neglect of energies higher than 27 mev ought to be that for $\int \frac{\sigma_\gamma}{E^2} dE$. It follows from the work of Migdal[13] that

$$\int_0^\infty \sigma_\gamma \frac{dE}{E^2} = \frac{\pi^3}{20\beta} \frac{e^2}{\hbar c} AR^2 = \frac{\pi^2}{20\beta} \frac{e^2}{\hbar c} r_0^2 A^{4/3},$$

where β is a coefficient characterizing the "affinity" of neutrons and protons in the Weisskopf formula and is approximately 20 mev. The values of r_0 obtained from this formula, if we set the experimental

$$\int_{E_n}^{E_0} \sigma_\gamma \frac{dE}{E^2}$$

equal to

$$\int_0^\infty \sigma_\gamma \frac{dE}{E^2},$$

are given in the last column of the Table. For all six nuclei, r_0 is equal to 1.2×10^{-13} cm, which is in excellent agreement with the value given in recent researches.

This comparison shows qualitative agreement of the experimental results with the data of theoretical calculations obtained under the assumption of a dipole absorption mechanism for the γ-rays. Quantitative comparison with the theoretical expressions, with the exception of the final integral, requires the measurement of the neutron yield at high energies. It is appropriate to carry out such researches in monochromatic measurements.

In conclusion, we take this opportunity to express our gratitude to Iu. K. Khokhlov for fruitful discussions.

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Nuclear Density and the Distribution of Orbital Angular Momentum in Nuclei

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A connection is found between the state of a nucleus, which is characterized by the minimum energy obtained in Ref. 1 and the distribution of the mean square orbital angular momentum of the nucleons in the nucleus. The number of particles in the nucleus for which nucleons in the l-state first appear has been determined.

THE statistical model of a nucleus with nonuniform density distribution of nucleons was investigated in Ref. 1. The parameters of the density function were obtained from the condition of saturation of the binding energy and the density. In the present work, starting out from the particle density obtained in Ref. 1, the distribution of the mean square orbital momentum of the nucleons is calculated and its relation to the state of the nucleus which is characterized by the minimum energy is found. In the statistical Thomas-Fermi model, the mean square of the orbital momentum of the nucleon can be expressed by the following equation (the treatment is carried out only for neutrons, since the same formulas are also valid for protons):

$$\langle L_N^2\rangle_{\mathrm{av}} = \frac{1}{N}\int_0^\infty L^2 n_N(L)\,dL. \tag{1}$$

Here $n_N(L)dL$ is the number of neutrons with orbital momentum between L and $L + dL$ which, according to the Thomas-Fermi model, is equal to

$$n_N(L) = \frac{4L}{\pi\hbar^3}\int_{r_1}^{r_2}\sqrt{r^2 P_n^2(r) - L^2}\,\frac{dr}{r}. \tag{2}$$

N is the total number of neutrons in the nucleus. The limits of integration should be so chosen that the expression under the integral would be real for $r_1 \leq r \leq r_2$.

With the help of (2), Eq. (1) yields[2]

$$\langle L_N^2\rangle_{\mathrm{av}} = \frac{8}{15\pi N\hbar^3}\int_0^\infty \frac{dr}{r}\,[rP_n(r)]^5. \tag{3}$$

The maximum momentum of the neutron $P_n(r)$, according to Eq. (12) of Ref. 1, is connected with the density of neutrons $\rho_n(r)$ by the relation

$$P_n(r) = (3\pi^2)^{1/3}\,\hbar\rho_n^{1/3}(r). \tag{4}$$

Taking (4) into consideration, we express the mean value of (3) by the neutron density distribution function:

$$\langle L_N^2\rangle_{\mathrm{av}} = \frac{8\,(3\pi^2)^{5/3}\hbar^2}{15\pi N}\int_0^\infty r^4\rho_n^{5/3}(r)\,dr. \tag{5}$$

Equation (5), together with the renormalization condition

$$\int \rho_n(r)\,d\tau = N \tag{6}$$

for a known density of particles ρ gives the distribution of the mean square angular momentum of the neutrons (or protons) in the nucleus. On the other hand, we have for the mean value (from the shell model of the nucleus),

$$\langle L_N^2\rangle_{\mathrm{av}} = \frac{1}{N}\sum_{N_i=1}^{N} l_{N_i}(l_{N_i}+1), \tag{7}$$

where L_{N_i} is the orbital quantum number of the N_i the neutron. It is clear from this that a definite requirement follows from the shell model for the probable form of the density distribution of particles in nuclei, viz: only for a choice of a convergent form of the density function of the particles can we obtain results from Eq. (5) which agree with experimental values of (7).

For our case of the density of distribution of particles, determined by Eq. (42) of Ref. 1,

$$\rho_n(r) = {}^1/_2\rho(r) = \rho_{0n}e^{-(r-R_0)/a}, \tag{8}$$

we, making use of Eqs. (6) and (5), obtain after integration

$$\langle L_N^2\rangle_{\mathrm{av}} = \frac{3}{25}2^{-1/3}(9\pi)^{2/3}\,\hbar^2N^{2/3} \times \frac{1+3\varepsilon_0+\frac{36}{5}\varepsilon_0^2+\frac{12\cdot 27}{25}\varepsilon_0^3+\frac{24\cdot 81}{125}\varepsilon_0^4+\frac{243\cdot 24}{625}\varepsilon_0^5}{(1+3\varepsilon_0+6\varepsilon_0^2+6\varepsilon_0^3)^{5/3}}. \tag{9}$$

We get precisely the same equation for protons if we take Eq. (8) for $\rho_p(r)$ and replace N by Z.

Choice of the same function for neutrons and for protons in the problem under consideration is justified by the fact that the experimental data on shells indicate an identical distribution of angular momenta for neutrons and protons. Thus in the case of the density distribution (8), the mean square angular momentum of neutrons and protons in the nucleus is a function of only the single parameter $\epsilon_0 = a/R_0$. One can choose for this parameter only such values which, upon substitution in (9) give results which agree with (7).

If we choose

$$\varepsilon_0 \approx 0.2, \tag{10}$$

we obtain, in the mean, a satisfactory agreement with the shell model. The graph of the function $\langle L^2 \rangle_{\text{av}}$, obtained from Eq. (9), and also from the shell model (7), is plotted in Fig. 1. The solid line in Fig, 1 corresponds to the value of Eq. (10).

It follows from Eq. (10) that the thickness of the surface layer a increases in proportion to the radius R_0 of the rest of the nucleus (with constant density). Inasmuch as $R_0 \sim A^{1/3}$ [see Ref. 1, Eq. (47) and Tables 1 and 2], then $a \sim A^{1/3}$ also.

Consequently, the agreement of the data on the mean square angular momentum of the neutrons and protons in the nucleus (according to the Thomas-Fermi model), with the shell model has as a consequence that, for $A \gtrsim 100$, the thickness of the surface layer ought to be proportional to $A^{1/3}$. Therefore, there is no necessity (as was done in Ref. 2 for the Born-Yang density) to choose the surface layer independent of A.

We establish the connection between the minimum energy, obtained in Ref. 1, and the distribution of the angular momentum in the nucleus. As can be seen from the data of Tables 1 and 2[1], the value of the parameter q, which corresponds to the value of ϵ_0 of Eq. (10), is equal to

$q \approx 3.6$ for the case of Eq. (61) of Ref. 1 (11)

$q \approx 3$ for the case of Eq. (62) of Ref. 1 (12)

Substituting Eqs. (10) and (11) into Eq. (63) of Ref. 1, we obtain for the parameters R_0, a, $\overline{r}_0$ and ρ_0 the values:

$$R_0 = 0.944 \cdot 10^{-13} A^{1/3}; \tag{13}$$

$$a = 0.189 \cdot 10^{-13} A^{1/3};$$

$$\overline{r}_0 = r_0^{*} A^{1/3} = 1.133 \cdot 10^{-13} \ A^{1/3};$$

$$\rho_0 = 0.394\, k_0^3 = 0.915 / (4\pi/3)\, r_0^{*3}.$$

For the pair of values (10 and (12) we get

$$R_0 = 1.133 \cdot 10^{-13} A^{1/3}; \tag{14}$$

$$a = 0.227 \cdot 10^{-13} A^{1/3};$$

$$\overline{r}_0 = 1.36 \cdot 10^{-13} A^{1/3};$$

$$\rho_0 = 0.228\, k_0^3 = 0.915 / (4\pi/3)\, r_0^{*3}.$$

The nuclear density (8), with the values of (13) and (14) for the parameters R_0, a and ρ_0 gives the correct value for the binding energy E/A and the distribution of the mean square orbital momentum, which agrees with the shell model. The values of the parameters (13) which characterize the model of the nucleus (8) are closer to the values found from the experimental data relating to the nuclear scattering of electrons and nucleons of high energy[3] than those of (14). It was shown in Ref. 3 that the theoretically obtained angular distribution of the scattering of electrons (with energy of 15.7 mev) by nuclei of silver and gold with nucleon density distribution of the form (8) agrees with the results of experiments if we assume that

$$R_0 = 0.792\, r_0 A^{1/3}; \ a = 0.208\, r_0 A^{1/3}; \tag{15}$$
$$r_0 \approx 1 \cdot 10^{-13}\, cM; \ \rho_0 = 0.87 / (4\pi/3)\, r_0^3.$$

For the density distribution of nucleons (51) in Ref. 1, we obtain the following expression from Eq. (5):

$$\langle L_N^2 \rangle_{\text{av}} = 0.417\, \hbar^2 N^{4/3}. \tag{16}$$

For constant particle density of Eq. (58), Ref. 1, Eq. (5) gives

$$\langle L_N^2 \rangle_{\text{av}} = 0.885\, \hbar^2 N^{4/3}. \tag{17}$$

The graphs of the functions (16) and (17) are shown in Fig. 1. It is evident from the figure that constant nuclear density gives higher values for the mean square angular momentum (see also Ref. 2), but that the nuclear density of the form of Eq. (51) in Ref. 1 is too low by comparison with experimental data.

2. Let us find the relation between the density distribution (8) and the first appearance of the nucleon with high angular momentum in the nucleus. In the limits of applicability to the

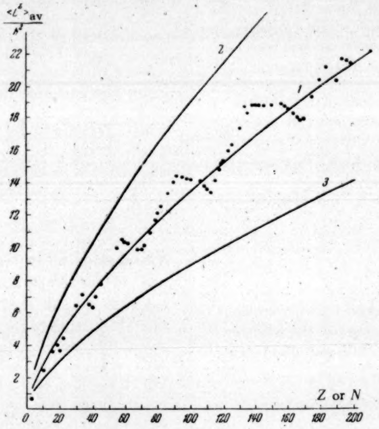

FIG. 1. Comparison of the mean square orbital momentum of nucleons, computed according to the model of Thomas-Fermi, with values computed from the shell model. Points are the data of the shell model: *1* -- for nonuniform density (8); *2* -- for constant density; *3* -- for density of particles of the form of Eq. (51) of Ref. 1.

nucleus of the statistical model of Thomas-Fermi, the following correlation was established between the structure of the shells and the density of particles in the nucleus[4]

$$N_l = \gamma (2l + 1)^3, \tag{18}$$

where

$$\gamma = \frac{1}{24\pi^2}\left[\frac{N}{r^3\rho(r)}\right]_{r=r_m}; \tag{19}$$

$$\frac{1}{r} = -\frac{1}{3}\frac{d}{dr}[\rho(r)/N]. \tag{20}$$

Here N_l is the number of particles which correspond to nuclei which have no nucleons in the l-state; $\rho(r)$ is the density distribution of nucleons in the nucleus; r_m is the root of Eq. (20). Substituting the value of (8) in Eq. (20) in place of $\rho(r)$, we find

$$r_m = 3a. \tag{21}$$

With the help of Eqs. (8), (6) and (21), we obtain N_l and the coefficient γ from Eqs. (18) and (19) as functions of the parameter ϵ_0 in the form

$$N_l = \gamma (2l + 1)^3, \tag{22}$$

where

$$\gamma = (1 / 18\pi\cdot 27) \times (1 + 3\varepsilon_0 + 6\varepsilon_0^2 + 6\varepsilon_0^3)\, \varepsilon_0^{-3} e^{3-1/\varepsilon_0}. \tag{23}$$

The dependence of N_l and γ on ϵ_0, computed by Eqs. (22) and (23) are represented in Fig. 2 (for each l). It is evident from these graphs that for $\epsilon_0 >> 1$, the values of N_l for all l are almost independent of ϵ_0.

For the value

$$\varepsilon_0 = a / R_0 = 0.375 \tag{24}$$

the neutron (or proton) critical numbers computed from Eq. (22) are equal to

$l =$ 0	1	2	3	4	5	6
$N_l =$	1.54	7.12	19.54	41.53	75.83	125.16
Next Integer	2	8	20	42	76	126

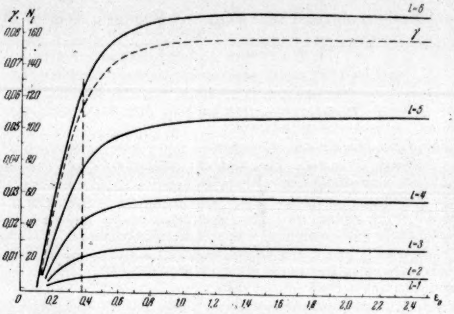

FIG. 2. N_l and γ as functions of $\epsilon_0 = a/R_0$.

These numbers agree well with the values of N_l determined in the quantum-mechanical shell model; these latter are, respectively, equal (square potential well) to 2, 8, 20, 40, 70, 112.

We find from the dependence $q = q(\epsilon_0)$ in Ref. 1 (see Fig. 2) that the value of the parameter q which corresponds to $\epsilon_0 = 0.375$ is equal to

$$q = 3.935 \text{ for case (61) of Ref. 1} \qquad (25)$$

$$q = 3.115 \text{ for case (62) of Ref. 1.} \qquad (26)$$

Substituting (25) and (24) in Eq. (63) of Ref. 1, we find the following values for the parameters:

$$R_0 = 0.718 \cdot 10^{-13} A^{1/3}; \qquad (27)$$

$$a = 0.269 \cdot 10^{-13} A^{1/3};$$

$$\bar{r}_0 = r_0^{*} A^{1/3} = 0.987 \cdot 10^{-13} A^{1/3}; \rho_0$$

$$= 0{,}514\, k_0^3 = 0.792 / (4\pi / 3)\, r_0^{*3}.$$

For the values of Eq. (24) and (26), we have

$$R_0 = 0.907 \cdot 10^{-13} A^{1/3}; \; a = 0.340 \cdot 10^{-13} A^{1/3}; \qquad (28)$$

$$\bar{r}_0 = r_0^{*} A^{1/3} = 1.247 \cdot 10^{-13} A^{1/3};$$

$$\rho_0 = 0.255\, k_0^3 = 0.792 / (4\pi / 3)\, r_0^{*3}.$$

In conclusion, I express my deep gratitude to Prof. A. A. Sokolov for the discussion of the results, and for his interest in the work.

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Nucleomesodynamics in Strong Coupling. III
Translational Motion, Meson-Field Mass and Magnetic Moment of the Nucleon

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We consider an extended nucleon strongly interacting with the meson field. The meson field is assumed pseudoscalar, and the interaction of pseudovector type. This paper is an immediate continuation of the work of the writers[1,2], in which the nucleon was assumed infinitely heavy (stationary). A method is developed which is based on the expansion of the solution in powers of the reciprocal of the nucleon mass. In zeroth approximation the result corresponds to an infinitely heavy nucleon and coincides with that previously found[1,2]. The method is applicable only in the case of sufficiently slow (nonrelativistic) motion of the nucleon. The wave function of the system is calculated to the accuracy of terms of first order in the reciprocal of the nucleon mass. The meson-field mass of the nucleon is calculated to the accuracy of terms quadratic in the reciprocal of the nucleon mass. In the limit of strong coupling of the meson field with the nucleon, the nucleon magnetic moment is calculated in terms of an expansion in powers of the reciprocal of the nucleon mass. The first term of the series corresponds to the approximation of the infinitely heavy nucleon, and the following terms give a correction connected with the translational motion. The results obtained indicate that the difference of the magnetic moments of proton and neutron can be explained, not by taking into account the translational motion of the nucleon, but by renouncing the assumption of the limit of strong coupling.

1. INITIAL EQUATIONS; TRANSFORMATION TO NEW VARIABLES AND SEPARATION OF TRANSLATIONAL MOTION OF SYSTEM

In previous papers[1,2] we considered an extended nucleon strongly interacting with the meson field. The latter was assumed to be pseudoscalar and to contain charged and neutral mesons symmetrically. The interaction of the meson field with the nucleon was taken to be of pseudovector type. The nucleon was assumed infinitely heavy, i.e., stationary (fixed at the origin of coordinates). The approximation method was based on the condition of strong coupling

$$(g/\mu a)^2 \gg 1, \tag{1}$$

where g is a dimensionless constant of interaction of the nucleon with the meson field, μ is the reciprocal of the Compton wavelength of the meson, and a is the effective radius of the extended nucleon.

In the present paper we retain all the initial assumptions of the earlier papers[1,2], with the exception of the assumption of an infinitely heavy nucleon. The mass of the nucleon is here taken as finite, and its translational motion is taken into account. Thus the Hamiltonian of the system has the form

$$\hat{H} = -(1/2\,M_0)\,\Delta_\xi + \hat{H}_0 + \hat{H}'(\xi; q), \tag{2}$$

where

$$\hat{H}_0 = \frac{1}{2}\sum_{\alpha=1}^{3}\int[\pi_\alpha^2 + (\nabla\varphi_\alpha)^2 + \mu^2\varphi_\alpha^2]\,dV = \frac{1}{2}\sum_{\alpha\varkappa}\omega_\varkappa\,[q_{\alpha\varkappa}^2 - \partial^2/\partial q_{\alpha\varkappa}^2] \tag{3}$$

is the energy of the oscillations of the meson field,

$$\hat{H}'(\xi, q) = -\frac{g}{\mu}\sqrt{4\pi} \times \sum_{\alpha=1}^{3}\tau_\alpha\int(\sigma, \nabla\varphi_\alpha)\,U(\mathbf{r}-\xi)\,dV \tag{4}$$

is the energy of interaction of the nucleon with the meson field, $U(\mathbf{r})$ is the form factor of the nucleon, σ and τ are the operators of spin and isotopic spin, respectively, and ξ is the coordinate of the nucleon. M_0 is the mass of the "bare" nucleon, not including the meson-field part of the mass.

The normal coordinates $q_{\alpha\varkappa}$ of the meson field are determined as the coefficients of the expansion of the meson field $\varphi_\alpha(\mathbf{r})$ in a Fourier series of the form

$$\varphi_\alpha(\mathbf{r}) = \sum_\varkappa \frac{q_{\alpha\varkappa}}{\sqrt{\omega_\varkappa}}\chi_\varkappa(\mathbf{r}), \tag{5}$$

$$\chi_\varkappa(\mathbf{r}) = \sqrt{\frac{2}{L^3}}\begin{cases}\cos\boldsymbol{\varkappa}\mathbf{r}. & \text{for } \varkappa_x \leqslant 0,\\ \sin\boldsymbol{\varkappa}\mathbf{r} & \text{for } \varkappa_x > 0,\end{cases} \tag{6}$$

L^3 is the volume of the fundamental region of periodicity, and $\omega_\varkappa = \sqrt{\mu^2 + \varkappa^2}$ ($\omega_\varkappa > 0$). Here, as before[1,2], the natural system of units is used ($\hbar = c = 1$).

It is readily seen that the operator (2) for the energy of the system is invariant with respect to the translational transformation $T_{\mathbf{a}}$, which consists of the simultaneous displacement of both the nucleon and the entire meson field by the vector **a**: $\xi \to \xi + \mathbf{a}$, $\varphi_\alpha(\mathbf{r}) \to \varphi_\alpha(\mathbf{r} - \mathbf{a})$. To such a transformation of the field there corresponds the following transformation of the normal coordinates:

$$q_{\alpha\varkappa} \to q_{\alpha\varkappa} \cos \boldsymbol{\varkappa}\mathbf{a} + q_{\alpha,-\varkappa} \sin \boldsymbol{\varkappa}\mathbf{a}.$$

The translation operator $T_{\mathbf{a}}$ commutes with $\hat{H}$, and is consequently an integral of the motion. By carrying out infinitely small translations in the directions of the three axes of the Cartesian coordinates, one readily convinces onself that the vector

$$\hat{\mathbf{k}} = -i\nabla_{\xi} - i \sum_{\alpha\varkappa} \boldsymbol{\varkappa}\, q_{\alpha,-\varkappa}\, \partial/\partial q_{\alpha\varkappa} \tag{7}$$

is also an integral of the motion. It will be called the total momentum of the system.

In order to separate the translational motion of the system, it is expedient to go from the variables ξ, $q_{\alpha\varkappa}$ to new variables R, $p_{\alpha\varkappa}$ by the formulas

$$\xi = \mathbf{R}; q_{\alpha\varkappa} = p_{\alpha\varkappa} \cos \boldsymbol{\varkappa}\mathbf{R} + p_{\alpha,-\varkappa} \sin \boldsymbol{\varkappa}\mathbf{R}; q_{\alpha,-\varkappa} = p_{\alpha,-\varkappa} \cos \boldsymbol{\varkappa}\mathbf{R} - p_{\alpha\varkappa} \sin \boldsymbol{\varkappa}\mathbf{R}. \tag{8}$$

The new variables $p_{\alpha\varkappa}$ are the coefficients of the expansion of the function $\tilde{\varphi}_\alpha(\mathbf{r}) \equiv \varphi_\alpha(\mathbf{r} + \xi)$ in a series of the form (5)

$$\tilde{\varphi}_\alpha(\mathbf{r}) = \sum_{\varkappa} \omega_\varkappa^{-1/2} p_{\alpha\varkappa} \chi_\varkappa(\mathbf{r}). \tag{9}$$

The transformation from the coordinates $q_{\alpha\varkappa}$ $q_{\alpha,-\varkappa}$ to the coordinates $p_{\alpha\varkappa}$, $p_{\alpha,-\varkappa}$ by the formula (8) is entirely analogous to the rotation of Cartesian coordinates in a plane by the angle $\varkappa$R, and in just the same way leaves invariant the expressions

$$q_{\alpha\varkappa}^2 + q_{\alpha,-\varkappa}^2 = p_{\alpha\varkappa}^2 + p_{\alpha,-\varkappa}^2;$$

$$\frac{\partial^2}{\partial q_{\alpha\varkappa}^2} + \frac{\partial^2}{\partial q_{\alpha,-\varkappa}^2} = \frac{\partial^2}{\partial p_{\alpha\varkappa}^2} + \frac{\partial^2}{\partial p_{\alpha,-\varkappa}^2}.$$

Therefore, in the new variables $\hat{H}_0$ retains its previous form:

$$\hat{H}_0(q) = \hat{H}_0(p) = \frac{1}{2} \sum_{\alpha\varkappa} \omega_\varkappa (p_{\alpha\varkappa}^2 - \partial^2/\partial p_{\alpha\varkappa}^2). \tag{10}$$

Substituting into Eq. (4) the function $\tilde{\varphi}_\alpha(\mathbf{r} - \xi)$ instead of $\varphi_\alpha(\mathbf{r})$, one readily obtains

$$\hat{H}'(\xi, q) = -\frac{g}{\mu}\sqrt{4\pi} \times \sum_{\alpha=1}^{3} \tau_\alpha \int (\sigma, \nabla\tilde{\varphi}_\alpha(\mathbf{r}))\, U(\mathbf{r})\, dV = \hat{H}'(0, p). \tag{11}$$

In the new variables the operator for the energy of the system has the form

$$\hat{H} = \frac{1}{2M_0}(-i\nabla_{\mathbf{R}} + \hat{\Omega})^2 + \hat{H}_0(p) + \hat{H}'(0, p), \tag{12}$$

where

$$\hat{\Omega} = i \sum_{\alpha\varkappa} \boldsymbol{\varkappa} p_{\alpha,-\varkappa}\, \partial / \partial p_{\alpha,\varkappa}. \tag{13}$$

In Eq. (12) the differentiation with respect to **R** is carried out with constant $p_{\alpha\varkappa}$. The momentum operator (7) has, in the new variables, the simple form

$$\hat{\mathbf{k}} = -i\nabla_{\mathbf{R}}. \tag{14}$$

Since the operators (12) and (14) commute, the wave function of the system can be chosen so that it is an eigenfunction of both operators. An eigenfunction of the operator (12) has the form

$$\chi = L^{-3/2} e^{i\mathbf{kR}} \Psi(p). \tag{15}$$

where in the general case $\Psi(p)$ is an arbitrary function of the $p_{\alpha\varkappa}$. In order that (15) be at the same time also an eigenfunction of the operator (12), it is necessary that $\Psi(p)$ satisfy the equation

$$\left[\frac{1}{2M_0}(\mathbf{k} + \hat{\Omega})^2 + \hat{H}_0(p) + \hat{H}'(0,p)\right]\Psi(p) = E\Psi(p). \tag{16}$$

In this equation the coordinate R already does not appear at all. Thus the translational motion of the system separates exactly.

2. THE APPROXIMATION METHOD

In the case of the infinitely heavy nucleon, for $M_0 \to \infty$, the first term of the left member of Eq. (16) vanishes, and the equation goes over into that for a stationary nucleon, which was solved in our earlier papers[1,2]. The difference is only that instead of the coordinates $q_{\alpha\varkappa}$ used earlier[1,2] there now appear the new variables $p_{\alpha\varkappa}$. In the case of finite but sufficiently large M_0 we shall treat the term

$$(1/2M_0)\,(\mathbf{k} + \hat{\Omega})^2$$

of Eq. (16) as a small perturbation. Then applying the ordinary perturbation theory we obtain the expansion of the solution of Eq. (16) in powers of $1/M_0$.

The solution of the unperturbed equation, found previously[1,2], has the form

$$\Psi = \psi_s^v \Phi\,(v, p), \tag{17}$$

where ψ_s^v is a four-component function describing the spin and charge motion of the nucleon and v_i are angle variables. As shown before[1,2], the angles v_i are themselves functions of the $p_{\alpha\varkappa}$, so that in its application to functions of the form (17)

$$\partial/\partial p_{\alpha\varkappa} = \partial/\partial^* p_{\alpha\varkappa} + \sum_i (\partial v_i \,/\, \partial p_{\alpha\varkappa})\, \partial/\partial v_i ,$$

where $\partial/\partial^* p_{\alpha\varkappa}$ denotes differentiation only with respect to the variable $p_{\alpha\varkappa}$ not occurring in the arguments v_i. According to this,

$$\hat{\Omega} = i \sum_{\alpha\varkappa} \varkappa p_{\alpha,-\varkappa} \left[\frac{\partial}{\partial^* p_{\alpha\varkappa}} + \sum_i \frac{\partial v_i}{\partial p_{\alpha\varkappa}} \frac{\partial}{\partial v_i} \right]. \tag{18}$$

As found before[1,2], in the case of strong coupling the field $\tilde{\varphi}_\alpha(\mathbf{r})$ in the neighborhood of the nucleon remains nearly equal to the self-consistent meson field $\tilde{\varphi}_\alpha^v(\mathbf{r})$, which is determined by Eq. (60) of an earlier paper[1],

$$\tilde{\varphi}_\alpha^v(\mathbf{r}) = -(\mathbf{g}_\alpha, \nabla W), \tag{19}$$

$$W(\mathbf{r}) = \frac{g}{\mu} \frac{1}{\sqrt{4\pi}} \int U(\mathbf{r}') \, \frac{e^{-\mu|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}\, dV'; \tag{20}$$

$$\mathbf{g}_\alpha \equiv \{\psi_s^{v*}, \ \tau_\alpha \sigma \psi_s^v\} .$$

The coefficients of the expansion of the function $\tilde{\varphi}_\alpha^v(\mathbf{r})$ in a Fourier series of the form (9) have the form

$$p_{\alpha\varkappa}^v = (\mathbf{g}_\alpha, \boldsymbol{\varkappa}) \sqrt{\omega_\varkappa} W_{-\varkappa}; \tag{21}$$

$$W_\varkappa = \int W(\mathbf{r})\, \chi_\varkappa(\mathbf{r})\, dV .$$

Because of the spherical symmetry of the functions $U(\mathbf{r})$ and $W(\mathbf{r})$ the coefficients $W_\varkappa$, as is seen from Eqs. (21) and (6), are different from zero only in the half-space of $\varkappa$ in which $\varkappa_x < 0$. The coefficients $p_{\alpha\varkappa}^v$, and also, as is seen from Eq. (66) of Ref. 1, $\partial v_i / \partial p_{\alpha\varkappa}$ are different from zero only for $\varkappa_x > 0$.

Because of the small difference between the functions $\tilde{\varphi}_\alpha(\mathbf{r})$ and $\tilde{\varphi}_\alpha^v(\mathbf{r})$ in the neighborhood of the nucleon, in the second factor of the operator (18) we may replace $p_{\alpha,-\varkappa}$ approximately by $p_{\alpha\varkappa}^v$. It can be shown that this introduces into the result a fractional error of the order $(\mu a/g)^2$, which is small in the case of strong coupling, when the inequality (1) holds. After this replacement the second term of the operator (18) turns out equal to zero, since $p_{\alpha,-\varkappa}^v$ and $\partial v_i / \partial p_{\alpha\varkappa}$ are not both different from zero for any value of $\varkappa$. Thus in the further work application of the operator $\hat{\Omega}$ to functions of the form (17) will not involve differentiation with respect to the arguments v_i.

We now proceed to the calculation of the matrix elements of the perturbation. Since $\hat{\Omega}$ does not contain operators on the spin and charge degrees of freedom, and ψ_s^v does not contain the variables $p_{\alpha\varkappa}$ explicitly, in taking the matrix elements of the operator $\hat{\Omega}$, ψ_s^v is brought outside the sign of operation, forming a factor $\{\psi_{s'}^{v*}, \psi_s^v\} = \delta_{s's}$. Therefore, in the perturbation calculation we can ignore all states of the spin-charge motion that are different from the perturbed state s. In what follows it is always assumed that the spin-charge motion of the nucleon is not excited.

It is sufficient to carry out the calculation of the perturbation matrix elements with functions $\Phi(v, p)$ which, according to Ref. 2, have the form

$$\Phi = Q_\nu(\vartheta, \beta, \delta) \times \exp\left\{-\frac{1}{2}\sum_{\alpha\varkappa}\left[(p_{\alpha\varkappa} - p_{\alpha\varkappa}^v)^2 + \frac{1}{2}\ln\pi\right]\right\} \times \prod_l H_{n_l}(p_l - p_l^v), \tag{22}$$

where H_{n_l} are Chebyshev-Hermite polynomials of degree n_l, l are certain of the indices $\alpha\varkappa$ (indices of free mesons) and Q_ν is the wave function of the quasioscillator considered in Ref. 2. Here ν is a set consisting of the three quantum numbers of the above-mentioned oscillator, the half-integral positive number j [the square of the angular momentum of the rotator is equal to $j(j+1)$], the quantum number of the nucleon charge ϵ ($\epsilon = 0, 1$), and the integer quantum number of the doubled component of the nucleon spin along the axis OZ, σ. For the ground (nonisobaric) state of the rotator, i.e., for the state with $j = \frac{1}{2}$, the functions Q_ν are presented in Table I at the end of Ref. 2 ($Q_\nu = 2^{-\frac{1}{2}}\pi^{-1}C_i^\nu$).

We agree to denote mesonless states (free mesons are absent) by indices 0ν, one-meson states by indices $\alpha\varkappa\nu$, where $\alpha\varkappa$ is the index of the single free meson. States with two different free mesons will be denoted by indices $\alpha_1\varkappa_1\alpha_2\varkappa_2\nu$, where $\alpha_1\varkappa_1$ and $\alpha_2\varkappa_2$ are the indices of the free mesons. If in the function (22) $n_l = 2$ for one $l = \alpha\varkappa$, and the other $n_{l'}$ are all zero, we call such a state also a two-meson state, and give it the indices $2\alpha\varkappa\nu$.

If the initial state is mesonless, then the matrix elements $\hat{\Omega}$ are different from zero only for transitions into one-meson states. These matrix elements are equal to

$$(\alpha\varkappa\nu' \,|\, \hat{\Omega} \,|\, 0\nu) = 2^{-1/2}\, i\varkappa\sqrt{\omega_\varkappa}\, W_\varkappa(\varkappa, \mathbf{g}_\alpha^{\nu'\nu}); \tag{23}$$

$$\mathbf{g}_\alpha^{\nu'\nu} \equiv (\nu' \,|\, \hat{\mathbf{g}}_\alpha | \nu) \equiv (j'\varepsilon'\sigma' \,|\, \hat{\mathbf{g}}_\alpha \,|\, j\varepsilon\sigma) \tag{24}$$

$$\equiv \int Q_{\nu'}^{*} \mathbf{g}_\alpha Q_\nu d\omega; \; d\omega = \sin\delta\, d\vartheta d\beta d\delta\,.$$

The three components of the vector $\mathbf{g}_\alpha$ will be denoted, respectively, by $g_{\alpha 1}$, $g_{\alpha 2}$ and $g_{\alpha 3}$. In what follows only such matrix elements $g_{\alpha\beta}^{\nu'\nu}$ are needed in which the final state ν' as well as the initial state ν is nonisobaric ($j' = j = \frac{1}{2}$). The values of these matrix elements $g_{\alpha\beta}^{\epsilon'\sigma'|\epsilon\sigma}$ are given below:

$$g_{\alpha\beta}^{0-1|0-1} = g_{\alpha\beta}^{11|11} = \frac{1}{3}\delta_{\alpha 3}\delta_{\beta 3}; \tag{25}$$

$$g_{\alpha\beta}^{01|01} = g_{\alpha\beta}^{1-1|1-1} = -\frac{1}{3}\delta_{\alpha 3}\delta_{\beta 3};$$

$$g_{\alpha\beta}^{0-1|01} = -g_{\alpha\beta}^{1-1|11} = \frac{1}{3}(\delta_{\alpha 3}\delta_{\beta 1} + i\delta_{\alpha 3}\delta_{\beta 2});$$

$$g_{\alpha\beta}^{0-1|1-1} = -g_{\alpha\beta}^{01|11} = \frac{1}{3}(\delta_{\alpha 1}\delta_{\beta 3} + i\delta_{\alpha 2}\delta_{\beta 3});$$

$$g_{\alpha\beta}^{0-1|11} = \frac{1}{3}[\delta_{\alpha 1}\delta_{\beta 1} - \delta_{\alpha 2}\delta_{\beta 2} + i(\delta_{\alpha 1}\delta_{\beta 2} + \delta_{\alpha 2}\delta_{\beta 1})];$$

$$g_{\alpha\beta}^{01|1-1} = \frac{1}{3}[\delta_{\alpha 1}\delta_{\beta 1} + \delta_{\alpha 2}\delta_{\beta 2} + i(\delta_{\alpha 2}\delta_{\beta 1} - \delta_{\alpha 1}\delta_{\beta 2})].$$

The matrix $g_{\alpha\beta}^{\nu\nu'}$ is Hermitian: $g_{\alpha\beta}^{\nu'\nu} = (g_{\alpha\beta}^{\nu\nu'})^*$.

The nine quantities $g_{\alpha\beta}$, as is seen from Eq. (38) of Ref. 1, can be treated as a rotation matrix of a three-dimensional Cartesian coordinate system. In this connection the angles ϑ, β and δ appear as Eulerian angles. From this there follows the relations

$$\sum_{\alpha=1}^{3} g_{\alpha\beta}g_{\alpha\beta'} = \delta_{\beta\beta'}; \qquad \sum_{\beta=1}^{3} g_{\alpha\beta}g_{\alpha'\beta} = \delta_{\alpha\alpha'}, \tag{26}$$

which considerably simplify the further calculations.

Further on we shall also need matrix elements of $\hat{\Omega}$ for which the initial state (right-hand index) is a one-meson state. Here, besides the transitions into mesonless states, for which the matrix element is equal to the complex conjugate of (23), there are also the following nonvanishing matrix elements:

$$(\alpha, -\varkappa\nu'' \,|\, \hat{\Omega} \,|\, \alpha\varkappa\nu') = i\varkappa\delta_{\nu''\nu'}; \tag{27}$$

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'' \,|\, \hat{\Omega} \,|\, \alpha\varkappa\nu')$$

$$= 2^{-1/2}\, i\varkappa_1\sqrt{\omega_{\varkappa_1}}\, W_{\varkappa_1}(\varkappa_1, \mathbf{g}_\alpha^{\nu''\nu'})\,\delta_{\alpha\alpha_2}\delta_{\varkappa\varkappa_2}$$

$$+ 2^{-1/2}\, i\varkappa_2\sqrt{\omega_{\varkappa_2}}\, W_{\varkappa_2}(\varkappa_2, \mathbf{g}_\alpha^{\nu''\nu'})\,\delta_{\alpha\alpha_1}\delta_{\varkappa\varkappa_1};$$

$$(2\alpha\varkappa\nu'' \,|\, \hat{\Omega} \,|\, \alpha\varkappa\nu') = i\varkappa\sqrt{\omega_\varkappa}\, W_\varkappa(\varkappa, \mathbf{g}_\alpha^{\nu''\nu'}).$$

The matrix elements of the operator $\hat{\Omega}^2$ are easily obtained from the matrix elements of $\hat{\Omega}$ given above. The nonvanishing matrix elements of $\hat{\Omega}^2$ for which the initial state is mesonless (right-hand index 0ν) are of the forms

$$(0\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) = \frac{1}{2}\delta_{\nu'\nu}\sum_{\varkappa}\varkappa^4\omega_\varkappa W_\varkappa^2; \tag{28}$$

$$(\alpha\varkappa\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) = \sqrt{\omega_\varkappa/2}\,\varkappa^2 W_{-\varkappa}(\varkappa, \mathbf{g}_\alpha^{\nu'\nu}); \tag{29}$$

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) \tag{30}$$

$$= -\sqrt{\omega_{\varkappa_1}\omega_{\varkappa_2}}(\varkappa_1, \varkappa_2)\, W_{\varkappa_1}W_{\varkappa_2}[(\mathbf{g}_{\alpha_1}, \varkappa_1)(\mathbf{g}_{\alpha_2}, \varkappa_2)]^{\nu'\nu};$$

$$(2\alpha\varkappa\nu' \,|\, \hat{\Omega}^2 \,|\, 0\nu) = -2^{-1/2}\varkappa^2\omega_\varkappa[(g_\alpha, \varkappa)^2]^{\nu'\nu}. \tag{31}$$

As is well known, in order to make use of the ordinary perturbation theory it is further necessary to determine the differences of the energies of the system in the various states in zeroth approximation. According to Eqs. (12), (17) and (31) of Ref. 2, the energy differences needed in the calculation have the following values:

$$E_{\alpha\varkappa\nu'}-E_{0\nu}=\omega_{\varkappa}+H_{j'}-H_j; \tag{32}$$

$$E_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}-E_{0\nu}=\omega_{\varkappa_1}+\omega_{\varkappa_2}+H_{j'}-H_j;$$

$$E_{2\alpha\varkappa\nu'}-E_{0\nu}=2\omega_{\varkappa}+H_{j'}-H_j.$$

Here H_j is the energy of the quasioscillator, determined from Eq. (31) of Ref. 2.

On taking into account the form of the perturbation and Eq. (32), we can write the perturbed wave function of the system in first approximation in the form

$$\Psi=\psi_s^v\Big\{\Phi_{0\nu}-\frac{1}{2M_0}\sum_{\alpha\varkappa\nu'}\frac{(\alpha\varkappa\nu'\,|\,2\,(\mathbf{k},\hat{\Omega})+\hat{\Omega}^2\,|\,0\nu)}{\omega_{\varkappa}+\Delta_{\nu'}}\Phi_{\alpha\varkappa\nu'} \tag{33}$$

$$-\frac{1}{2M_0}\sum_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}{}'\frac{(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{\omega_{\varkappa_1}+\omega_{\varkappa_2}+\Delta_{\nu'}}\Phi_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}-\frac{1}{2M_0}\sum_{\alpha\varkappa\nu'}\frac{(2\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{2\omega_{\varkappa}+\Delta_{\nu'}}\Phi_{2\alpha\varkappa\nu'}\Big\},$$

where $\Delta_{\nu'}=H_{j'}-H_j$. The matrix elements of $\hat{\Omega}$ and $\hat{\Omega}^2$ determined from Eqs. (23) and (28)-(31) are to be substituted into the expression (33). The prime on the second summation of the expression (33) indicates that terms are to be omitted in which both of the indices $\alpha_1\varkappa_1$ coincide with the indices $\alpha_2\varkappa_2$. It is essential to note that the quantities involved in the matrix elements of $\hat{\Omega}$ and $\hat{\Omega}^2$ are different from zero only if $j'=j-1,\ j,\ j+1$. In particular, when the initial state ν is nonisobaric ($j=½$), transitions are allowed only to the states with $j'=1/2,\ 3/2$.

To prove the above-mentioned selection rules for $g_{\alpha\beta}^{\nu\nu'}$ we must start with the Eqs. (24) and take into account the fact that the functions $Q_{j\epsilon\sigma}$ are linear combinations of the functions $V_{sp}^j(\vartheta,\beta,\delta)$ with different s and p. The functions V_{sp}^j are defined in Ref. 2 by Eqs. (18), (20), (24) and (25). These functions are connected with the functions T_{sp}^j used in the work of Gel'fand and Shapiro[3] (Sec. 7) by the relations

$$V_{sp}^j(\vartheta,\beta,\delta) \tag{34}$$

$$=\frac{1}{2\pi}\sqrt{\frac{2j+1}{2}}(-i)^{2j-s-p}T_{sp}^{j*}(\vartheta,\pi-\delta,\beta).$$

Thus the functions $Q_{j\epsilon\sigma}(\vartheta,\beta,\delta)$ in the integrand in Eq. (24) are linear combinations of the functions $T_{sp}^{j*}(\vartheta,\pi-\delta,\beta)$ with the same index j. The selection rules mentioned above are easy to prove, if we make use of the well-known recurrence formulas for the functions $T_{sp}^j(\vartheta,\delta,\beta)$ (cf. Ref. 3, p. 88):

$$\cos\delta T_{sp}^j=c_{21}^pT_{sp}^{j+1}c_{21}^s+c_{22}^pT_{sp}^jc_{22}^s \tag{35}$$

$$+c_{23}^pT_{sp}^{j-1}c_{23}^s;$$

$$2^{-1/2}i\sin\delta e^{i\vartheta}T_{s+1,p}^j$$

$$=c_{21}^pT_{sp}^{j+1}c_{11}^s+c_{22}^pT_{sp}^jc_{12}^s+c_{23}^pT_{sp}^{j-1}c_{13}^s;$$

$$2^{-1/2}i\sin\delta e^{-i\vartheta}T_{s-1,p}^j$$

$$=c_{21}^pT_{sp}^{j+1}c_{31}^s+c_{22}^pT_{sp}^jc_{32}^s+c_{23}^pT_{sp}^{j-1}c_{33}^s;$$

$$2^{-1/2}i\sin\delta e^{i\beta}T_{s,p+1}^j$$

$$=c_{11}^pT_{sp}^{j+1}c_{21}^s+c_{12}^pT_{sp}^jc_{22}^s+c_{13}^pT_{sp}^{j-1}c_{23}^s;$$

$$2^{-1/2}i\sin\delta e^{-i\beta}T_{s,p-1}^j$$

$$=c_{31}^pT_{sp}^{j+1}c_{21}^s+c_{32}^pT_{sp}^jc_{22}^s+c_{33}^pT_{sp}^{j-1}c_{23}^s.$$

For the meaning of the coefficients c_{ik}^m see p. 52 of Ref. 3. If the quantity $g_{\alpha\beta}$ in the integrand of Eq. (24) contains $\cos\delta$, the first of the formulas (35) is to be used. If $g_{\alpha\beta}$ contains $\sin\delta$ multiplied by $\sin\vartheta$ or $\cos\vartheta$, the second and third formulas are to be used, and in the remaining cases the fourth and fifth.

The perturbations $-(\mathbf{k},\hat{\Omega})/M_0$ and $-\hat{\Omega}^2/2M_0$ do not "interfere", in the sense that they do not have nonvanishing matrix elements for one and the same transition. In particular, the matrix elements (23) and (29) are also different from zero in different half-spaces of $\varkappa$, so that the square of the absolute value of the matrix element of the total perturbation is here equal to the sum of the squares of the absolute values of the matrix elements of the separate terms. In such cases, as is well known, each of these perturbations can be introduced independently of the other, after which one can simply combine the corrections to the wave function and the energy caused by each of these perturbations separately.

Alongside the approximation method described above, we also considered ordinary adiabatic approximation: In the zeroth approximation it was assumed that the spin-charge motion of the nucleon and the oscillations of the meson field follow adiabatically the translational motion of the nucleon, and then the nonadiabatic terms were introduced as a small perturbation. Here the calculations turned out to be rather more complicated, mainly because of the fact that the matrix elements had to be calculated with more complicated wave functions, which also depended on the coordinate ξ of the translational motion. Besides this, it was also necessary in Eqs. (32) to add on the differences of the kinetic energies of the translational motions of the nucleon. The results naturally turned out to be equivalent to those obtained above, if expanded in powers of $1/M_0$.

3. ENERGY AND MESON-FIELD MASS OF THE NUCLEON

We first calculate the correction E_1 to the energy of the nucleon, caused by the perturbation $-(\mathbf{k}, \hat{\Omega})/M_0$. The first order correction is equal to zero, since $(0\nu|\hat{\Omega}|0\nu) = 0$. In the second approximation the energy correction is equal to

$$E_1^{(2)} = -\frac{1}{M_0^2}\sum_{\alpha\varkappa\nu'}\frac{|(\alpha\varkappa\nu'|(\mathbf{k},\hat{\Omega})|0\nu)|^2}{\omega_\varkappa + \Delta_{\nu'}} \tag{36}$$

$$= -\frac{1}{2M_0^2}\sum_{\alpha\varkappa\nu'}(\mathbf{k}, \boldsymbol{\varkappa})^2\, W_\varkappa^2\, |(\boldsymbol{\varkappa}, \mathbf{g}_\alpha^{\nu'\nu})|^2\, \frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{\nu'}}.$$

It must be noted that the perturbed state is fourfold degenerate (cf. Ref. 2). However, since $(0\nu'|(\mathbf{k}, \hat{\Omega})|0\nu) = 0$, the energy correction in second order is given by the same formula as in the nondegenerate case.

In Eq. (36) the summation over $\alpha\nu'$ can be carried out in the following way: first we replace $\Delta_{\nu'}$ in all terms by its value $\Delta_{3/2}$ for $j' = 3/2$. After this the sum over $\alpha\nu'$ becomes equal to

$$\sum_{\alpha\nu'}|(\boldsymbol{\varkappa}, \mathbf{g}_\alpha^{\nu'\nu})|^2\frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}} = \frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}} \tag{37}$$

$$\times\sum_{\alpha\nu'}(\boldsymbol{\varkappa}, \mathbf{g}_\alpha^{\nu\nu'})(\boldsymbol{\varkappa}, \mathbf{g}_\alpha^{\nu'\nu}) = \frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}}\sum_\alpha[(\boldsymbol{\varkappa}, \mathbf{g}_\alpha)^2]^{\nu\nu}.$$

As was already mentioned above, in the sum (36) only the terms $\mathbf{g}_\alpha^{\nu'\nu}$ with $j' = 1/2$, $3/2$ are different from zero, so that the replacement of $\Delta_{\nu'}$ by $\Delta_{3/2}$ changes only the terms with $j' = 1/2$ by the amount

$$-|(\boldsymbol{\varkappa}, \mathbf{g}_\alpha^{\nu'\nu})|^2\left\{1 - \frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}}\right\}.$$

Thus the exact expression of the sum (36) over α and ν' is

$$\frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}}\sum_\alpha[(\boldsymbol{\varkappa}, \mathbf{g}_\alpha)^2]^{\nu\nu} \tag{38}$$

$$+\sum_{\alpha\varepsilon'\sigma'\,(j'=1/2)}|(\boldsymbol{\varkappa}, \mathbf{g}_\alpha^{\nu'\nu})|^2\left\{1 - \frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}}\right\}.$$

Here the further summation over α is easy to perform, if we take account of Eqs. (25) and (26). The final result for the sum (38) is the expression

$$\sum_{\alpha\nu'}|(\boldsymbol{\varkappa}, \mathbf{g}_\alpha^{\nu'\nu})|^2\frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{\nu'}} \tag{39}$$

$$= \varkappa^2\left\{\frac{1}{3} + \frac{2}{3}\frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}}\right\}.$$

Substituting this in Eq. (36) and noting that the average of $(\mathbf{k}, \boldsymbol{\varkappa})^2$ over the angles is equal to $k^2\varkappa^2/3$, we find for the second-order correction to the energy the expression

$$E_1^{(2)} = -\frac{k^2}{6M_0^2}\sum_\varkappa \varkappa^4 W_\varkappa^2\left\{\frac{1}{3} + \frac{2}{3}\frac{\omega_\varkappa}{\omega_\varkappa + \Delta_{3/2}}\right\}. \tag{40}$$

If we introduce the operator $\hat{\omega} = \sqrt{\mu^2 - \Delta}$ (where Δ is the Laplace operator), the result (40) can be rewritten in the form

$$E_1^{(2)} = -\frac{k^2}{2M_0^2}S, \tag{41}$$

$$S = \frac{1}{3}\int \Delta W\left\{\frac{1}{3} + \frac{2}{3}\frac{\hat{\omega}}{\hat{\omega} + \Delta_{3/2}}\right\}\Delta W dV.$$

We now go on to the calculation of the correction to the energy due to the second perturbation term $-\Omega^2/2M_0$. By Eq. (28) this correction E_2 is in first order equal to

$$E_2^{(1)} = -\frac{1}{4M_0}\sum_\varkappa \varkappa^4\omega_\varkappa W_\varkappa^2 \tag{42}$$

$$= -\frac{1}{4M_0}\int \Delta W\hat{\omega}\Delta W dV.$$

From the formulas (28)-(31) the correction E_2 can without difficulty be found also in second approximation. But all corrections from the perturbation $-\Omega^2/2M_0$ give constant terms not dependent on $\mathbf{k}\varkappa$, which consequently have no effect on the value of the field mass of the nucleon, so that they have no particular interest for us here and will not be presented.

Just so it is pointless to calculate the corrections to the energy from the operator $-(\mathbf{k}\hat{\Omega})/M_0$ in orders higher than the second, since they contain the fourth and higher powers of **k**. The correction proportional to k^2 suffices for the calculation of the field mass of the nucleon. Consideration of higher powers of **k** does not make sense, since the theory is nonrelativistic.

According to Eqs. (16), (41) and (42), the total energy of the system, which appears in Eq. (16), can be written in the form

$$E = H + \frac{k^2}{2M_0} - \frac{k^2}{2M_0^2} S - \frac{1}{4M_0} \int \Delta W \hat{\omega} \Delta W dV. \tag{43}$$

Here H is the energy of the system in the approximation of the infinitely heavy (stationary) nucleon, given by Eqs. (17) and (31) of Ref. 2.

The term in Eq. (43) proportional to k^3 can be written in the form

$$k^2/2M, \quad \text{where} \quad 1/M = 1/M_0 - S/M_0^2. \tag{44}$$

Whereas M_0 is the mass of the "bare" nucleon (not interacting with the meson field), M represents the total mass of the real nucleon (interacting with the meson field). It is, namely, with M that the experimentally determined mass of the nucleon is to be identified. With a fractional error of the order $(S/M_0)^2$ Eq. (44) can be rewritten thus: $M = M_0 + S$. From this it is seen that S is the meson-field part of the nucleon mass.

The ratio $\Delta_{3/2}/\omega_\varkappa$ is in order of magnitude equal to $(\mu a/g)^2$. In the case of strong coupling, Eq. (1), it is considerably smaller than unity, so that in the formulas obtained above one can neglect the quantity $\Delta_{3/2}$ in comparison with $\omega_\varkappa$. As a result S, for example, takes the form

$$S = \frac{1}{3} \int (\Delta W)^2 dV. \tag{45}$$

We now indicate the criterion for applicability of the method of small perturbations which is used above. The term $-\Omega^2/2M_0$ can be regarded as a small perturbation if the energy change (42) that it causes is decidedly smaller than the energy differences (32). In order of magnitude the latter are equal (basically) to $\omega_\varkappa$, which can be arbitrarily large. But in determining the criterion of applicability of the perturbation method one must use in Eq. (32) some average $\bar{\omega}$, corresponding to the frequency of the oscillations of the meson field that interact most intensely with the nucleon. In Eq. (42) also one can give $\omega_\varkappa$ an average effective value $\bar{\omega}$ and take it in front of the sign of summation (or integration). Then, by Eqs. (42) and (45), $E_2^{(1)} \sim -3/4\, \bar{\omega}\, S/M_0$. The condition for applicability of the perturbation method can thus be written as : $S/M_0 \ll 1$.

The term $(\mathbf{k}, \hat{\Omega})/M_0$ can be regarded as a small perturbation if the energy correction (42) that it causes is much smaller than

$$k^2 S/2M_0^2 \ll \bar{\omega} \sim \sqrt{\mu^2 + a^{-2}}. \tag{46}$$

This inequality limits the nucleon momentum **k**.

4. SEMI-CLASSICAL CALCULATION OF THE MESON-FIELD MASS OF THE NUCLEON

The meson-field mass of the nucleon can be calculated without recourse to the perturbation method if the translational motion of the nucleon and the vibrations of the meson field are treated classically and the spin-charge motion of the nucleon quantum-mechanically. The Hamiltonian of the system has the following form

$$E = \frac{M_0}{2}\dot{\xi}^2 + \sum_{\alpha=1}^{3} \int \Big\{ \frac{1}{2} [\dot{\varphi}_\alpha^2 + (\nabla\varphi_\alpha)^2 + \mu^2\varphi_\alpha^2] - \frac{g}{\mu}\sqrt{4\pi}\, \tau_\alpha(\sigma, \nabla\varphi_\alpha)\, U(r) \Big\} dV. \tag{47}$$

In the state with the lowest energy of the system it is obvious that $\dot{\xi} = 0$, $\dot{\varphi}_\alpha = 0$. This corresponds to a stationary nucleon and a static meson field. The equilibrium form of the latter is determined from the condition of minimum potential energy of the system, expressed by Eq. (14) of Ref. 1. Such a minimization of the potential energy was carried out in Ref. 1 and the equilibrium static meson field so obtained turned out equal to $\tilde{\varphi}_\alpha^{(v)}(\mathbf{r})$ (cf. (19) and (20)]. By Eq. (47) the corresponding lowest energy of the system is

$$E_0 = \sum_{\alpha=1}^{3} \int \Big\{ \frac{1}{2} [(\nabla\tilde{\varphi}_\alpha^v)^2 + \mu^2\tilde{\varphi}_\alpha^{v2}] - \frac{g}{\mu}\sqrt{4\pi}\tau_\alpha(\sigma, \nabla\tilde{\varphi}_\alpha^v)\, U(r) \Big\} dV. \tag{48}$$

It was shown in Ref. 1 that the potential energy does not depend on the angles ϑ, β and δ. Moreover, because of the physical homogeneity of space it does not depend on ξ. Thus the system can perform a free motion in these four degrees of freedom. For such a freely "revolving" and translationally moving nucleon the meson field has the following form

$$\varphi_\alpha(\mathbf{r},t)=\tilde{\varphi}^v_\alpha(\mathbf{r}-\dot{\boldsymbol{\xi}}t,\ \vartheta,\beta,\delta). \tag{49}$$

Here ξ is constant, and ϑ, β and δ are functions of t.

Differentiating (49) with respect to the time, we get

$$\dot{\varphi}_\alpha=(\dot{\boldsymbol{\xi}},\nabla)\tilde{\varphi}^v_\alpha+\sum_{i=1}^{3}\frac{\partial\tilde{\varphi}^v_\alpha}{\partial v_i}\dot{v}_i,$$

where v_1, v_2 and v_3 are equal to ϑ, β and δ, respectively. Substituting this into (47) and noting that

$$\int\frac{\partial\varphi_\alpha}{\partial x_k}\frac{\partial\tilde{\varphi}^v_\alpha}{\partial v_i}dV \tag{50}$$

$$=\sum_{\beta\beta'}g_{\alpha\beta}\frac{\partial}{\partial v_l}g_{\alpha\beta'}\int\frac{\partial^2W}{\partial x_k\partial x_\beta}\frac{\partial W}{\partial x_{\beta'}}dV=0$$

because of the spherical symmetry of the function $W(r)$, we obtain

$$E=E_0+\frac{1}{2}(M_0+S)\dot{\xi}^2+T_r; \tag{51}$$

$$T_r=\frac{1}{2}\sum_{\alpha=1}^{3}\sum_{i=1}^{3}\dot{v}_i^2\int\left(\frac{\partial\tilde{\varphi}^v_\alpha}{\partial v_i}\right)^2dV \tag{52}$$

is the kinetic energy of "rotation" of the nucleon, unrelated to its translational motion. From Eq. (51) it is seen that the total inertial mass of the nucleon is equal to M_0+S. This result agrees with the approximate result of the quantum calculation.

5. INTRODUCTION OF THE MAGNETIC FIELD INTO THE HAMILTONIAN OF THE SYSTEM

For the introduction of the magnetic field into the Hamiltonian of a nucleon interacting with the meson field, it is convenient first to go over to new complex variables φ, π instead of the real variables φ_1, φ_2; π_1, π_2; $\varphi=2^{-1/2}(\varphi_1-i\varphi_2)$; $\pi=2^{-1/2}(\pi_1+i\pi_2)$. The result is that the Hamiltonian of the system, Eqs. (2)-(4), takes the form

$$\hat{H}=-\frac{1}{2M_0}\Delta_\xi+\int\Big[\pi^*\pi+\frac{1}{2}\pi_3^2+(\nabla\varphi^*,\nabla\varphi) \tag{53}$$
$$+\frac{1}{2}(\nabla\varphi_3)^2+\mu^2\Big(\varphi^*\varphi+\frac{1}{2}\varphi_3^2\Big)\Big]dV$$
$$-\frac{g}{\mu}\sqrt{4\pi}\int U(\mathbf{r}-\boldsymbol{\xi})(\boldsymbol{\sigma},\ \sqrt{2}\tau_+\nabla\varphi$$
$$+\sqrt{2}\tau_-\nabla\varphi^*+\tau_3\nabla\varphi_3)\,dV.$$

Here

$$\tau_+=2^{-1/2}(\tau_1+i\tau_2);\quad \tau_-=2^{-1/2}(\tau_1-i\tau_2).$$

The usual gauge-invariant introduction of the electromagnetic field leads to the replacement

$$\nabla\varphi\to\nabla\varphi-ie\mathbf{A}(\mathbf{r})\varphi,\quad \nabla\varphi^*\to\nabla\varphi^*+ie\mathbf{A}(\mathbf{r})\varphi^*,$$

$$\nabla_\xi\to\nabla_\xi-ie\,\frac{1+\tau_3}{2}\,\mathbf{A}(\boldsymbol{\xi})$$

in all terms of the Hamiltonian. Here **A** is the vector potential of the field. The magnetic field is assumed below to be static, homogeneous and parallel to the axis OZ. The corresponding vector potential can be chosen in the form $A_x=-\frac{1}{2}\mathfrak{H}y$, $A_y=\frac{1}{2}\mathfrak{H}x$, where $\mathfrak{H}$ is the intensity of the magnetic field.

Supposing the magnetic field sufficiently weak, we retain in the Hamiltonian only terms linear in $\mathfrak{H}$. The additional terms that appear in the Hamiltonian on the introduction of the magnetic field have the form

$$\hat{H}_{\mathrm{mag}}=\hat{H}_1+\hat{H}_2+\hat{H}_3+\hat{H}_4, \tag{54}$$

where

$$\hat{H}_1=\frac{ie}{4M_0}(1+\tau_3)\mathfrak{H}\left(\xi_1\frac{\partial}{\partial\xi_2}-\xi_2\frac{\partial}{\partial\xi_1}\right); \tag{55}$$

$$\hat{H}_2=-\frac{e}{4M_0}(1+\tau_3)\sigma_3\mathfrak{H}; \tag{56}$$

$$\hat{H}_3=e\mathfrak{H}\int\varphi_1\left(x\frac{\partial}{\partial y}-y\frac{\partial}{\partial x}\right)\varphi_2dV; \tag{57}$$

$$\hat{H}_4=\frac{eg\sqrt{\pi}}{\mu}\mathfrak{H} \tag{58}$$
$$\times\int U(\mathbf{r}-\boldsymbol{\xi})(x\sigma_y-y\sigma_x)(\tau_1\varphi_2-\tau_2\varphi_1)\,dV.$$

Here $\hat{H}_1$ is the operator for the interaction of the magnetic field with the orbital motion of the "bare" nucleon and $\hat{H}_2$ is the operator for the interaction of the magnetic field with the spin magnetic moment of the "bare" nucleon. This interaction is different from zero when the "bare" nucleon has charge

+ 1, and equal to zero when the charge of the "bare" nucleon is zero. Therefore, the operator factor $2^{-1/2}(1+\tau_3)$ appears in Eq. (56). $\hat{H}_3$ and $\hat{H}_4$ are the supplementary terms arising on the introduction of the magnetic field into the second and third terms of Eq. (53), respectively.

In Eqs. (57) and (58) we have again returned to the real fields φ_1 and φ_2. The further calculations will be conducted in the variables $\mathbf{R}$, $\tilde{\varphi}_\alpha(\mathbf{r})$, and $p_{\alpha\varkappa}$ introduced in accordance with Eqs. (8) and (9). In these variables $-i\nabla_\xi = -i\nabla_{\mathbf{R}} + \hat{\Omega}$, where the operator $\hat{\Omega}$ is defined by Eq. (13), and the operators (55), (57) and (58) have the forms

$$\hat{H}_1 = \frac{ie}{4M_0}(1+\tau_3)\mathfrak{H}\Big[R_1\frac{\partial}{\partial R_2} - R_2\frac{\partial}{\partial R_1} - i(R_2\hat{\Omega}_1 - R_1\hat{\Omega}_2)\Big]; \tag{59}$$

$$\hat{H}_3 = e\mathfrak{H} \times \int\Big[(x+R_1)\tilde{\varphi}_1\frac{\partial\tilde{\varphi}_2}{\partial y} - (y+R_2)\tilde{\varphi}_1\frac{\partial\tilde{\varphi}_2}{\partial x}\Big]dV; \tag{60}$$

$$\hat{H}_4 = \frac{ge\sqrt{\pi}}{\mu}\mathfrak{H}\int[(x+R_1)\sigma_2 - (y+R_2)\sigma_1](\tau_1\tilde{\varphi}_2 - \tau_2\tilde{\varphi}_1)U(r)\,dV. \tag{61}$$

6. CALCULATION OF THE MAGNETIC MOMENT OF THE NUCLEON

We now calculate the correction to the energy of the system caused by the magnetic field. The magnetic moment of the nucleon will then be defined as the coefficient of the magnetic field intensity in this correction. We shall start with the wave function of the system, Eqs. (15) and (33), which it is now convenient to rewrite in the form

$$\chi = L^{-3/2}e^{i\mathbf{k}\mathbf{R}}\Psi,\quad \Psi = \psi_s^v(\Phi_{0\nu} + \Phi_{0\nu}^{(1)}), \tag{62}$$

where $\Phi_{0\nu}^{(1)}$ is the correction to the wave function caused by the translational motion of the nucleon (and proportional to $1/M_0$):

$$\Phi_{0\nu}^{(1)} = -\frac{1}{2M_0}\Big\{\sum_{\alpha\varkappa\nu'}\frac{(\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{\omega_\varkappa + \Delta_{\nu'}}\Phi_{\alpha\varkappa\nu'} + \sum_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}{}' \frac{(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{\omega_{\varkappa_1} + \omega_{\varkappa_2} + \Delta_{\nu'}}\Phi_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'} + \sum_{\alpha\varkappa\nu'}\frac{(2\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)}{2\omega_\varkappa + \Delta_{\nu'}}\Phi_{2\alpha\varkappa\nu'}\Big\}. \tag{63}$$

Here we have already set $k = 0$, since the magnetic moment will be calculated below in the inertial system in which the total momentum of the nucleon is equal to zero.

The correction to the energy of the system caused by the weak magnetic field is given by

$$H_{\mathrm{mag}} = \int\{\chi^*\hat{H}_{\mathrm{mag}}\chi\}\,d\tau_R\,dp\,d\omega. \tag{64}$$

It must be kept in mind that Ψ does not depend on R; therefore, upon proper choice of the boundaries of the cyclic interval (in which χ is orthonormalized)

$$\int\mathbf{R}\{\chi^*, L\chi\}\,d\tau_R = 0,$$

where L is an arbitrary operator which does not depend on R. Thus in the integration of (64) all terms containing factors R_1 and R_2 drop out. In particular, the energy correction caused by the operator $\hat{H}_1$ is equal to zero.

Proceeding to the calculation of the correction H_2 from the operator $\hat{H}_2$, we must first of all note that according to Eqs. (7)-(9) and (29) of Ref. 1

$$\{\psi_s^{v*},\ \sigma_3\psi_s^v\} = |C_1|^2 - |C_2|^2 + |C_3|^2 - |C_4|^2 = 0.$$

Furthermore, by Eq. (25), we have

$$\int\Phi_{0\nu}^{v*}\tau_3\sigma_3\Phi_{0\nu}\,dp\,d\omega = g_{33}^{\nu\nu} = \begin{cases} 1/3 & \text{for}\quad \varepsilon = 1, \sigma = 1 \quad\text{or}\quad \varepsilon = 0, \sigma = -1, \\ -1/3 & \text{for}\quad \varepsilon = 1, \sigma = -1 \quad\text{or}\quad \varepsilon = 0, \sigma = 1. \end{cases} \tag{65}$$

Because of the orthogonality of $\Phi_{0\nu}$ and $\Phi_{0\nu}^{(1)}$ in the coordinates $p_{\alpha\varkappa}$

$$\int\Phi_{0\nu}^{(1)*}\tau_3\sigma_3\Phi_{0\nu}\,dp\,d\omega = 0.$$

Thus

$$H_2 = -(e/4M_0)\mathfrak{H}g_{33}^{\nu\nu}. \tag{66}$$

In the calculation of H_3 one must keep in mind Eqs. (9) and (19)-(22). Besides this, it is helpful to note the following property of the quantities $g_{\alpha\beta}(\vartheta, \beta, \delta)$: in the determinant $\|g_{\alpha\beta}\|$ each element is equal to its algebraic complement, taken with sign reversed (i.e., its minor). For example,

$$g_{33} = g_{12}g_{21} - g_{11}g_{22}. \tag{67}$$

As a result of the calculation one gets

$$(0\nu\,|\,\hat{H}_3\,|\,0\nu) = -\frac{1}{2}e\mathfrak{H}I^{(0)}g_{33}^{\nu\nu}, \tag{68}$$

where, as in Ref. 1 [cf. Eq. (46))],

$$I^{(0)} = \frac{2}{3}\int\left(\frac{\partial W}{\partial r}\right)^2 dV. \tag{69}$$

Furthermore, we readily obtain

$$(\alpha\varkappa\nu')\,|\,\hat{H}_3\,|\,0\nu) = \frac{e}{\sqrt{2\omega_\varkappa}}\mathfrak{H}W_{-\varkappa} \tag{70}$$

$$\times\,[(\delta_{\alpha 1}g_{22}^{\nu'\nu} - \delta_{\alpha 2}g_{12}^{\nu'\nu})\varkappa_x + (\delta_{\alpha 2}g_{11}^{\nu'\nu} - \delta_{\alpha 1}g_{21}^{\nu'\nu})\varkappa_y],$$

$$\sum_{\alpha\varkappa\nu'}\frac{1}{\omega_\varkappa}(\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)^*(\alpha\varkappa\nu'\,|\,\hat{H}_3\,|\,0\nu) \tag{71}$$

$$= -\frac{1}{3}e\mathfrak{H}g_{33}^{\nu\nu}I_1,$$

where

$$I_1 = \sum_\varkappa \varkappa^4 W_\varkappa^2/\omega_\varkappa.$$

The following expressions are calculated in an analogous way:

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{H}_3\,|\,0\nu) \tag{72}$$

$$= \frac{e}{2\sqrt{\omega_{\varkappa_1}\omega_{\varkappa_2}}}\mathfrak{H}\delta_{\nu'\nu}(\delta_{\alpha_1 1}\delta_{\alpha_2 2} - \delta_{\alpha_1 2}\delta_{\alpha_2 1})$$

$$\times\int\chi_{\varkappa_1}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)\chi_{\varkappa_2}\,dV,$$

$$\sum_{\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'}{}' \frac{1}{\omega_{\varkappa_1}+\omega_{\varkappa_2}}(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)^*(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu'\,|\,\hat{H}_3\,|\,0\nu) \tag{73}$$

$$= -e\mathfrak{H}g_{33}^{\nu\nu}\sum_{\varkappa_1\varkappa_2}\frac{(\varkappa_1,\ \varkappa_2)}{\omega_{\varkappa_1}+\omega_{\varkappa_2}}\varkappa_{1y}\varkappa_{2x}W_{\varkappa_1}W_{\varkappa_2}\int\chi_{\varkappa_1}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)\chi_{\varkappa_2}\,dV.$$

The expression (73) can be simplified by means of the following consideration. Since the operator Δ commutes with $A = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$, integrating by parts we obtain the following relation for an arbitrary function ($f(\varkappa_1^2, \varkappa_2^2)$ expansible in power series:

$$\int f(\varkappa_1^2,\ \varkappa_2^2)\chi_{\varkappa_1}\hat{A}\chi_{\varkappa_2}\,dV \tag{74}$$

$$= \int[f(-\Delta,\ \varkappa_2^2)\chi_{\varkappa_1}]\hat{A}\chi_{\varkappa_2}\,dV$$

$$= \int\chi_{\varkappa_1}\hat{A}f(-\Delta,\ \varkappa_2^2)\chi_{\varkappa_2}\,dV$$

$$= \int f(\varkappa_2^2,\ \varkappa_2^2)\chi_{\varkappa_1}\hat{A}\chi_{\varkappa_2}\,dV.$$

From this it is seen that the integrals (74) are different from zero only for $\varkappa_1^2 = \varkappa_2^2$. On this basis $(\omega_{\varkappa_1} + \omega_{\varkappa_2})$ can be replaced by $2\omega_{\varkappa_2}$ in (73). Then for the sum (73) one finds the value

$$\frac{1}{6}e\mathfrak{H}g_{33}^{\nu\nu}\int\Delta W\frac{1}{\hat{\omega}}\Delta W\,dV = \frac{1}{6}e\mathfrak{H}g_{33}^{\nu\nu}I_1. \tag{75}$$

The matrix element

$$(2\alpha\varkappa\nu'\,|\,\hat{H}_3\,|\,0\nu) = 0, \tag{76}$$

as one can readily convince oneself by performing first the integration over p and taking into account the orthogonality of the oscillator wave functions.

Proceeding to the calculation of the energy correction H_3, in the expression (63) we drop out the $\Delta_{\nu'}$ occurring together with $\omega_\varkappa$, as was already done in the derivation of Eq. (45). Then, using Eqs. (63), (68), (71), (75) and (76), we obtain

$$H_3 = (0\nu\,|\hat{H}_3\,|\,0\nu) \tag{77}$$

$$+\int[\Phi_{0\nu}^{(1)*}\hat{H}_3\Phi_{0\nu} + \Phi_{0\nu}^*\hat{H}_3\Phi_{0\nu}^{(1)}]\,dp\,d\omega$$

$$= -{}^1\!/_2\,e\mathfrak{H}g_{33}^{\nu\nu}[I^{(0)} - (1/3M_0)I_1].$$

In the calculation of H_4 one again makes use of Eqs. (9) and (19)-(22), and also of (67). One finds

$$(0\nu\,|\,\hat{H}_4\,|\,0\nu) = -e\mathfrak{H}g_{33}^{\nu\nu}(g/3\mu)\sqrt{4\pi}\,K; \tag{78}$$

$$K = \int rU\frac{\partial W}{\partial r}\,dV. \tag{79}$$

Furthermore,

$$(\alpha\varkappa\nu'\,|\,\hat{H}_4\,|\,0\nu) = \frac{ge\sqrt{\pi}}{\mu\sqrt{2\omega_\varkappa}}\mathfrak{H} \tag{80}$$

$$\times\int[(xg_{12}^{\nu'\nu} - yg_{11}^{\nu'\nu})\delta_{\alpha 2} - (xg_{22}^{\nu'\nu} - yg_{21}^{\nu'\nu})\delta_{\alpha 1}]\,U\chi_\varkappa\,dV;$$

$$\sum_{\alpha\varkappa\nu'}\frac{1}{\omega_\varkappa}(\alpha\varkappa\nu'\,|\,\hat{\Omega}^2\,|\,0\nu)^*(\alpha\varkappa\nu'\,|\,\hat{H}_4\,|\,0\nu) \tag{81}$$

$$= \frac{ge\sqrt{\pi}}{3\mu}g_{33}^{\nu\nu}\mathfrak{H}K_1;$$

$$K_1 = \int rU\frac{\partial}{\partial r}\frac{\Delta}{\hat{\omega}}W\,dV.$$

On carrying out the integration over p one easily shows that

$$(\alpha_1\varkappa_1\alpha_2\varkappa_2\nu' \mid \hat{H}_4 \mid 0\nu) = 0, \qquad (82)$$

$$(2\alpha\varkappa\nu' \mid \hat{H}_4 \mid 0\nu) = 0.$$

In the calculation of H_4 we shall again drop $\Delta_{\varkappa'}$ in comparison with $\omega_{\varkappa}$. Then, using Eqs. (63), (78) and (81)-(82), we obtain

$$H_4 = -e\mathfrak{H}g_{33}^{\nu\nu}(g/3\mu)\sqrt{4\pi}\,[K + K_1/2M_0]. \qquad (83)$$

Summing, finally, the expressions (66), (77) and (83), we obtain the resultant energy correction caused by the magnetic field,

$$H_{\mathrm{mag}} = -e\mathfrak{H}g_{33}^{\nu\nu}\Big[\frac{1}{2}I^{(0)} + \frac{g}{3\mu}\sqrt{4\pi}K \qquad (84)$$

$$+ \frac{1}{6M_0}\Big(\frac{3}{2} - I_1 + \frac{g}{\mu}\sqrt{4\pi}K_1\Big)\Big].$$

The magnetic moment of the nucleon, defined as the coefficient of $\mathfrak{H}$ (with sign reversed) in the expression (84), is

$$m = eg_{33}^{\nu\nu}\Big[\frac{1}{2}I^{(0)} + \frac{g}{3\mu}\sqrt{4\pi}K \qquad (85)$$

$$+ \frac{1}{6M_0}\Big(\frac{3}{2} - I_1 + \frac{g}{\mu}\sqrt{4\pi}K_1\Big)\Big].$$

From this it is seen that according to (65) the magnetic moments of the proton and neutron are equal in magnitude and opposite in sign.

The experimentally observed difference of the absolute magnitudes of the magnetic moments of the proton and neutron can in general not be obtained in the approximation limit of strong coupling, in which the wave function of the system has the form

$$\Psi = \psi_s\Phi(\xi, q). \qquad (86)$$

To prove this in general, we assume that (86) represents the proton state of the system. In our notation the operator for the charge of the system[4] can be written in the form

$$\hat{\varepsilon} = \hat{\varepsilon}' + {}^1/_2(1 + \tau_3);$$
$$\hat{\varepsilon}' = -i\sum_{\varkappa}(q_{1\varkappa}\partial/\partial q_{2\varkappa} - q_{2\varkappa}\partial/\partial q_{1\varkappa}). \qquad (87)$$

The function (86) is an eigenfunction of this operator and corresponds to the eigenvalue + 1. From this it follows, since $\{\psi_s^*, \tau_3\psi_s\} = 0$, that

$$\overline{\varepsilon'} = \int \Phi^*\hat{\varepsilon}'\Phi d\xi\, dq = \frac{1}{2}. \qquad (88)$$

Since the operator for the vibrational energy of the meson field (cf. Eq. (1) of Ref. 2] is real, the function

$$\Psi_1 = \psi_s\Phi^*(\xi, q) \qquad (89)$$

is also an eigenfunction of the energy operator and corresponds to the same energy as does the function (86). It is easy to see that (89) is a neutron state of the system. Indeed, in this state

$$\overline{\varepsilon'} = \int \Phi\hat{\varepsilon}'\Phi^* d\xi\, dq \qquad (90)$$

$$= -\int \Phi[\hat{\varepsilon}'\Phi]^* d\xi\, dq = -\frac{1}{2},$$

and consequently the eigenvalue of the charge operator (87) is equal to zero.

In an entirely analogous way, by considering the operator of the total angular momentum of the system, one can show that the states (86) and (89) correspond to opposite orientations of the spin of the nucleon.

If we now compare the corrections to the energy of the system, Eqs. (56)-(58), caused by the magnetic field, for the states (86) and (89), they turn out precisely identical. Consequently, the magnitudes of the magnetic moments of the proton and neutron states of the system are found to be identical.

Thus it can be possible to obtain a difference of the magnitudes of the magnetic moments of proton and neutron only by renouncing the multiplicative form of the wave function, Eq. (86), with the factor ψ_s, which was determined in Ref. 1. This is not to be achieved by a more precise consideration of the transitional motion of the nucleon, but by departing from the limit of strong coupling and introducing the corresponding corrections.

7. DISCUSSION OF RESULTS

If in Eq. (85) we drop the last two terms (containing I_1 and K_1), which appeared because of our taking into account the translational motion of the nucleon, the value of the magnetic moment $\overline{m}$ turns out to be identical with that found in the work of Pauli and Dancoff[4]. In the comparison it must be noted that our parameter g is equal to the parameter $2^{-1/2}g$ of Ref. 4. The agreement of our value of m, obtained in the symmetrical theory, with the value of m obtained in Ref. 4 in the charged meson theory assures us that the interaction of the nucleon with neutral mesons does not influence the size of the magnetic moment (at least in the case of the infinitely heavy nucleon).

It must be noted that in Ref. 4 the magnetic moment was determined from the density of the meson current, which led to the difficulties in connection with the impossibility of satisfying the equation of continuity of the current inside the extended nucleon. In our work this difficulty is not encountered, since the magnetic moment of the nucleon is determined from the expressoin for the energy of the nucleon in the magnetic field.

Investigation shows that in the case $\mu a \gg 1$ [a is the effective radius of the form factor $U(r)$] the results of the theory depend essentially on the shape of the form factor $U(r)$. Since in the present state of science there is no sound basis for choosing this shape, an impermissable arbitrariness appears in the theory in the case $\mu a \gg 1$. Therefore, we confine ourselves below to the consideration of the case $\mu a \ll 1$, in which the results are insensitive to the choice of the form factor. In this case Eq. (20) can be approximately rewritten as follows:

$$W(r) \approx \frac{g}{\mu\sqrt{4\pi}} \int \frac{U(r')}{|\mathbf{r}-\mathbf{r}'|} dV'. \qquad (91)$$

Expanding the functions U and W in the integrands in Eqs. (75) and (81) in series of the form

$$U(r) = (2\pi R)^{-1/2} \sum_k U_k \frac{\sin kr}{r},$$

where R is the radius of the fundamental region in which the basis functions are orthonormal and $k = n\pi / R$ ($n = 1, 2, 3, \ldots$), and carrying out the integration over the volume term by term, we obtain

$$I_1 = \frac{4\pi g^2}{\mu^2} \sum_k \frac{U_k^2}{k}, \quad K_1 = \frac{g\sqrt{4\pi}}{\mu} \sum_k \frac{U_k^2}{k}. \qquad (92)$$

From this it is seen that the sum of the two last terms in Eq. (85) is zero. Thus by Eqs. (69), (79) and (91), one gets

$$m = eg_{33}^{vv}\left[\frac{1}{2} I^{(0)} + \frac{g\sqrt{4\pi}}{3\mu} K + \frac{1}{4M_0}\right] \qquad (93)$$
$$= \frac{1}{6} eg_{33}^{vv}\left[\left(\frac{g}{\mu}\right)^2 \iint \frac{U(r)U(r')}{|\mathbf{r}-\mathbf{r}'|} dV\, dV' + \frac{3}{2M_0}\right].$$

Consequently, in the case $\mu a \ll 1$ the inclusion of the translational motion of the nucleon to first approximation does not change the value of the magnetic moment of the nucleon.

By Eqs. (45) and (91) one gets for the meson-field mass of the nucleon

$$S = \frac{4\pi}{3}\left(\frac{g}{\mu}\right)^2 \int U^2 dV. \qquad (94)$$

The quantities (93) and (94) can be evaluated readily if we replace the form factor by the function

$$U(r) = \begin{cases} 3/4\pi a^3 & \text{for } r < a \\ 0 & \text{for } r > a, \end{cases} \qquad (95)$$

taking advantage of the above-mentioned small sensitivity of the results to the shape of the form factor (in the case $\mu a \ll 1$). Then we get

$$m = eg_{33}^{vv}[g^2/5\mu^2 a + 1/4M_0], \qquad (96)$$

$$S = g^2/\mu^2 a^3. \qquad (97)$$

To estimate the parameters of the theory we replace in Eq. (96) the mass M_0 of the "bare" nucleon by the mass M of the physical nucleon, and equate the quantity (96) to the experimental value of the magnetic moment of the nucleon. For the latter we take the arithmetical average of the magnetic moments of the proton and neutron, equal to 2.4 nuclear magnetons. Then for $M/\mu = 6.7$ we get

$$(g/\mu a)^2 \mu a \approx 2.5. \qquad (98)$$

From this it follows that for $\mu a \ll 1$ the criterion (1) for strong coupling is satisfied.

We now pass to the consideration of the criterion for the heavy nucleon approximation ($M_0 \gg S$). By Eq. (97), it can be written

$$(g/\mu a)^2/\mu a \ll M/\mu = 6.7. \qquad (99)$$

This inequality limits the strength of the coupling, and in the case $\mu a \ll 1$ can be fulfilled only for weak or intermediate coupling. Multiplying Eqs. (98) and (99), we get $(g/\mu a)^2 \ll 4$.

CONCLUSIONS

1. In the limiting case of strong coupling the magnetic moments of proton and neutron are found to be identical (in magnitude) even with inclusion of the translational motion of the nucleon.

2. In the case $\mu a \ll 1$ comparison with experiment of the nucleon magnetic moment calculated for the strong-coupling limit and the heavy nucleon confirms the criterion of strong coupling.

3. The heavy-nucleon approximation is not applicable in the limiting case of strong coupling.

4. In the case of strong coupling the meson-field mass of the nucleon is not small in comparison with M_0, and possibly even exceeds M_0. In the latter case M_0 turns out to be of the order of or smaller than the mass of the mesons playing an important part in the interaction with the nucleon, so that it is of interest to consider the relativistic motion of a nucleon in the meson field, including the possibility of the formation of nucleon pairs.

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On the Effect of Radio Waves on the Properties of Plasma (Ionosphere)

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A successive method is developed for determining the electron-velocity distribution function in plasma in an alternating electric field and a constant magnetic field, assuming the collisions between the electrons and molecules or ions to be elastic. Expressions are derived and analyzed for the mean electron energies, for the conductivity and the dielectric constant of the plasma, and for the effective number of electron collisions.

1. INTRODUCTION

IF an electromagnetic wave is propagated in the ionosphere, the intense alternating electric field of the wave may change substantially the energy of the electrons, and consequently the conductivity and the dielectric constant of the plasma. To evaluate these changes, one must first determine the electron-distribution function.

Considering only the elastic collisions between the electrons and the heavy particles (molecules or ions) of the plasma we know that with each collision the electron loses only a small portion of its energy [on the order of $\delta = 2m/M$, where m is the electron mass and M the molecule (or ion) mass]. It is therefore possible to obtain the distribution function in a simpler manner by representing the collision integral in the Boltzmann equation in differential form. Furthermore, since δ is small, the mean square velocity of the electrons is many times greater than the mean directed velocity even in strong electric fields. The symmetrical portion (which depends only on the modulus of the velocity) of the distribution function is correspondingly much greater than its asymmetric portion. Taking these into account, Davydov[2] simplified the kinetic equation considerably and solved it for the case of stationary electric and magnetic fields.

In nonstationary fields the electron energy is changed directly only by the electric field. Thus, depending on the ratio between the time t_E required to change substantially the electric field (for example, if $E = E_0 \cos \omega t$, $t_E \sim 1/\omega$), and the relaxation time of the electron energy ($t_r \approx 1/\delta\nu$, where ν is the collision frequency of the electron), it is possible to distinguish among cases in which the fields vary slowly ($t_E \gg t_r$) or rapidly ($t_E \ll t_r$). In the former (quasi-stationary) case the solution obtained in Ref. 1 for a time-dependent electric field intensity E is correct. In a rapidly-varying field, when $t_r \gg t_E$ the mean electron energy cannot change as fast as the electric field; the symmetrical portion of the distribution function should therefore be stationary, to a zero-order approximation. Let us also note that since the current relaxation time t_j is much smaller than the electron relaxation time t_r, because $t_j \sim 1/\nu \ll \delta\nu \sim t_r$, the ratio between t_E and t_j affects the nature of the distribution in rapidly-varying fields ($t_E \ll t_r$).

The case where the electric field varies periodically with time is discussed in Refs. 2 and 3 (in the absence of a magnetic field) and in Ref. 4 (assuming a constant magnetic field). However, owing to incorrect solution methods, some of the results obtained of References 3 and 4 are in error (cf. below).* An analogous problem was also solved in an analysis of the cross-modulation effect of radio waves in the ionosphere, where corrections were derived for the Maxwell distribution function for the case of a relatively weak alternating electric field, when the energy delivered by the field to the electrons is much less than their thermal energy (Refs. 6 and 7 and Sec. 64 of Ref. 8).

The purpose of this article is to solve the kinetic equations, assuming an alternating electric field and a constant magnetic field (without the above-mentioned restrictions on the field intensity), and to analyze the expressions for the conductivity and the dielectric constant of the plasma. Section 2 discusses the method for calculating the distribution function, really an expansion into a series in powers of the parameter t_E / t_r (for rapidly varying fields) or t_r / t_E (for slowly-varying fields). A zero-order approximation is obtained for the distribution function. In Sec. 3 we obtain and analyze expressions for the average electron energy and for the conductivity and dielectric constant of the plasma. In Sec. 4 we investigate the possibility of using in our calculations the values of ϵ and σ obtained from elementary-theory equations.

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*The expression for the distribution function, derived in Ref. 4, was corrected by its authors in a later work[5] (see footnote 3).

We show that in the case of collisions with molecules the use of these equations leads to but a slight discrepancy with the results of the kinetic theory. Generally speaking, this discrepancy is more significant in the case of collisions with ions.

Estimates show that in many cases the incident wave may change substantially the average electron energy and the properties of the plasma.

2. CALCULATION OF DISTRIBUTION FUNCTION

Consider a spatially-homogeneous plasma located in an electric and magnetic field. We assume the collisions between the electrons and the molecules or ions of the gas to be elastic. The electron distribution function

$$f(\mathbf{v}, t) = f_0(v, t) + \frac{\mathbf{v}}{v}\mathbf{f}_1(v, t) + \chi(\mathbf{v}, t)$$

obeys in this case the following approximate system of equations, derived in Ref. 1:

$$\frac{\partial f_0}{\partial t} + \frac{e}{3mv^2}\frac{\partial}{\partial v}(v^2\mathbf{E}\mathbf{f}_1) - \frac{1}{v^2}\left\{\nu\,\frac{v^2 kT}{M}\frac{\partial f_0}{\partial v} + \nu\frac{v^3 m}{M}f_0\right\} = 0, \tag{1}$$

$$\frac{\partial \mathbf{f}_1}{\partial t} + \frac{e\mathbf{E}}{m}\frac{\partial f_0}{\partial v} + \frac{e}{mc}[\mathbf{H}\mathbf{f}_1] + \nu\mathbf{f}_1 = 0, \tag{1'}$$

wherein terms on the order of χ were neglected in the derivation of Eq. (1). (As shown in Ref. 1, $\chi \sim \sqrt{8}\,|f_1| \sim \delta f_0$). The normalization condition for the distribution function is

$$4\pi\int_0^\infty f_0(v, t)\,v^2\,dv = 1. \tag{2}$$

In Eq. (1) e is the electron charge, k the Boltzmann constant, T the plasma temperature, E and H the electric and magnetic field intensities, and $\nu(v)$ the frequency of collisions between the electron and the molecules (or ions). In the case of collisions with molecules, we have

$$\nu(v) = \pi a^2 N_M v, \tag{3}$$

where a is the "radius" of the molecule and N_M the molecule concentration; in the case of collisions with ions, on the other hand, we have (see Ref. 8)

$$\nu(v) = \frac{2\pi N_i e^4}{m^2 v^3}\ln\left(1 + \frac{\rho_m^2 m^2}{e^4}v^4\right), \tag{3'}$$

where N_i is the ion concentration, ρ_m is the maximum collision parameter, which can be assumed to equal the Debye radius[8] (at frequencies that are not too high).

Let us now assume that the electric field varies rapidly, i.e., $t_E << t_r$. In this case the solution of Eq. (1) can be solved by successive approximation:

$$f_0(v, t) = f_{00} + f_{01} + \ldots, \tag{4}$$

$$\mathbf{f}_1(v, t) = \mathbf{f}_{10} + \mathbf{f}_{11} + \ldots$$

Neglecting in the zero-order approximation the variation of the distribution function due to the collisions (on the order of f/t_r) as compared with the first term of the equation ($\partial f/\partial t \sim f/t_E$), we have $\partial f_{00}/\partial t = 0$ or

$$f_{00} = f_{00}(v), \tag{5}$$

i.e., in the zero-order approximation the symmetrical portion of the distribution function is independent of time (see Introduction). On the other hand, the dependence of f_{00} on v is determined, as it should be, not by the equation for the zero-order approximation (4), but by the requirement that the boundary conditions for the next (first-order) approximation be satisfied (see below).

Inserting now (5) into (1′), we obtain f_{10} (the electric field is assumed for simplicity periodic, $E = E_0\cos\omega t$, and the magnetic field H is assumed constant):

$$\mathbf{f}_{10} = -\frac{eE_0}{2m}\frac{df_{00}}{dv}\left\{\frac{(i\omega+\nu)^2(\mathbf{E}_0/E_0) + \omega_H^2\cos\beta\,(\mathbf{H}/H) + \omega_H(i\omega+\nu)\,[\mathbf{H}\mathbf{E}_0]/HE_0}{(i\omega+\nu)\,[\omega_H^2 + (i\omega+\nu)^2]}e^{\omega t} + \frac{(-i\omega+\nu)^2(\mathbf{E}_0/E_0) + \omega_H^2\cos\beta\,(\mathbf{H}/H) + \omega_H(-i\omega+\nu)\,[\mathbf{H}\mathbf{E}_0]/HE_0}{(-i\omega+\nu)\,[\omega_H^2 + (-i\omega+\nu)^2]}e^{-i\omega t}\right\}, \tag{6}$$

where $\omega_H = eH/mc$ is the gyromagnetic frequency, and β the angle between H and E_0.

Next, inserting f_{00} and f_{10} into the equation for the first approximation f_{01} and integrating it with respect to time, we obtain in expression for f_{01} the following term

$$\Delta f_{01} = \frac{\delta}{2v^2}\frac{\partial}{\partial v}\left\{\nu v^2\left(\frac{e^2E_0^2}{3m^2\delta}\varphi(v)\frac{df_{00}}{dv} + \frac{kT}{m}\frac{df_{00}}{dv} + vf_{00}\right)\right\}t,$$

which increases without limit as $t \to \infty$. Stipulating the existence of the distribution function, i.e., the boundedness of f_{01} at $t \to \infty$, we set the expression in the braces equal to zero, and obtain

$$f_{00} = C\exp\left\{-\int_0^v \frac{mv\,dv}{kT + (e^2E_0^2/3m\delta)\,\varphi(v)}\right\}. \tag{7}$$

Here

$$\varphi(v) = \frac{\cos^2\beta}{\omega^2+\nu^2} + \frac{\sin^2\beta}{2}\left(\frac{1}{(\omega+\omega_H)^2+\nu^2} + \frac{1}{(\omega-\omega_H)^2+\nu^2}\right), \tag{7'}$$

and the constant C is defined by the normalization condition (2). Thus a unique solution to the system of equations (1) is obtained only by satisfying the supplementary condition that the solution be bounded at $t \to \infty$.

Integrating the equations for f_{01} and f_{11} with respect to time, we can obtain the following approximation (with accuracy to within a time-independent function, which is determined by specifying that f_{02} be bounded at $t \to \infty$), etc. Comparing the successive terms of expansion (4) we find that in case of collisions with molecules the resultant expressions are correct provided the following condition is satisfied

$$\frac{1{,}5\,\delta\nu_0}{\omega}\left(1+\frac{eE_0}{\nu_0\sqrt{6kTm\delta}}\mu\right)^{1/2} \approx \frac{1{,}2\cdot10^{-14}N_M\sqrt{T}}{\omega}\left(1+1{,}4\cdot10^{16}\frac{E_0}{N_MT}\mu\right)^{1/2} \ll 1, \tag{8}$$

where $\nu_0 = \nu$ and $\mu \lesssim 1$, namely:

$$\mu = \begin{cases} \cos\beta + \sin^2\beta\,\dfrac{\sqrt{2}\,\nu_0 eE_0}{\omega_H^2\sqrt{3kTm\delta}} & \text{if}\quad \omega_H^2 \gg \nu_0^2 + \dfrac{\sqrt{2}\,eE_0\nu_0}{\sqrt{3kTm\delta}}, \\ 1 & \text{if}\quad \omega_H^2 \ll \nu_0^2 + \dfrac{\sqrt{2}\,eE_0\nu_0}{\sqrt{3kTm\delta}}. \end{cases}$$

In the numerical coefficient we assumed $\delta = 3.4 \times 10^{-5}$ and $\pi a^2 = 4.3 \times 10^{-16}$ (see Ref. 8); E_0 is in millivolts per meter.

In case of slowly-varying fields, reversing the inequality sign in (8) we obtain for f_{00} and f_{10} the expressions similar to (6) or (7) but with E_0 being replaced by $\sqrt{2E_0}$ cos ωt in f_{00}. Thus we have

$$f_{00} = C(t)\exp\left\{-\int_0^v \frac{mvdv}{kT + (2e^2E_0{}^2/3m\delta)\,\varphi(v)\cos^2\omega t}\right\}, \tag{9}$$

where the constant $C(t)$ is determined by the normalization condition (2). For $\omega t \to 0$, i.e., for $E \to E_0$, the distribution function (9) agrees with that obtained in Ref. 1 for the stationary case. At $H \to 0$ the distribution function (7) agrees with that obtained in Refs. 2 and 3. However, (9) does not agree with the function obtained under the same conditions in Ref. 3 (Section 7). In connection with this let us note that Refs. 2 and 3 attempted to solve Eqs. (1) by expanding them into a Fourier series, and although this led to separation of the variables, it also resulted in an infinite system of interrelated differential equations. Since general analysis of such a system is exceedingly difficult (the termination method was used in Reference 3, but the authors of this article did not investigate fully the conditions under which this method is applicable,* and therefore the results they obtained for the case of low-frequency fields (Ref. 3, Sec. 7) are not quite correct, even though the distribution obtained in Refs. 2 and 3 for rapidly varying fields does not agree with (7). Let us note that Ref. 3 investigated not the approximate system of Eqs. (1), but a complete infinite system of equations, equivalent to the Boltzmann equation. The method developed in this article can also be used to solve that problem. The distribution function (7) does not agree with that obtained in Ref. 4.**

In the case when the principal role is played by collisions between electrons and ions, expression (7) is valid for the distribution function if the following condition is satisfied:

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*The attempt to approximate c, made in Sec. 5 of Ref. 3 (from Sec. 3 of the same reference) is not sufficiently consistent: instead of function f_{00}^c which should be obtained in this approximation, function f_{00}^b was used (approximation c, Sec. 3), which is incorrect within the framework of the method employed.

**Recently the authors of that reference derived a correct expression for the distribution function,[5] agreeing with (7). The same expression was derived for the distribution function by Fain.[9] The kinetic equations were solved in these investigations as in Ref. 3. Fain also investigated the applicability of this method and arrived at a condition that is equivalent to (8) (see also Ref. 10).

$$\left(\sqrt{2/3}\,eE_0/\omega\sqrt{kTM}\right)+\delta\nu_{0i}/\omega\ll 1. \qquad (10)$$

Neither the question of obtaining the distribution function in case of collisions with ions when conditions (10) are not satisfied (stationary and quati-stationary fields), nor gyro-resonance are considered in this article.*

The solution above is for a plane-polarized electric field ($\mathbf{E}=\mathbf{E}_0\cos\omega t$). However, in anisotropic plasma ($H\neq 0$), the plane electromagnetic wave breaks up into two elliptically-polarized waves. It is therefore interesting to determine the electron distribution function for the case of an elliptically-polarized electric field **E**. Resolving **E** along the three principal polarization axes we have

$$\mathbf{E}=\mathbf{E}_{\parallel 0}\cos\omega t+\mathbf{E}_{\perp 0}^{+}e^{i\omega t}+\mathbf{E}_{\perp 0}^{-}e^{-i\omega t}$$

($\mathbf{E}_{1\cdot 1}$ is the plane-polarized field with $\mathbf{E}_{11}\parallel\mathbf{H}$; $\mathbf{E}_{\perp}^{+}$ and $\mathbf{E}_{\perp}^{-}$ are the circularly-polarized fields in a plane perpendicular to H and rotating in the same (minus) or opposite (plus) direction as the electron in the magnetic field), it is easy to show that the same expression (7) is valid for the distribution function, provided that $E_0^2\ \varphi(v)$ is replaced by

$$E_0^2\varphi(v)\to\frac{E_{\parallel 0}^2}{\omega^2+\nu^2}+\frac{2E_{\perp 0}^{+2}}{(\omega+\omega_H)^2+\nu^2}+\frac{2E_{\perp}^{-2}}{(\omega-\omega_H)^2+\nu^2}. \qquad (11)$$

In particular, if the field is circularly polarized in a plane perpendicular to H, it can be seen from (11) that the effect of the magnetic field is actually equivalent to an increase (or decrease) in the frequency of the electric field, a physically understandable effect.

3. MEAN ELECTRON ENERGY, PLASMA CONDUCTIVITY AND DIELECTRIC CONSTANT

Using the expression obtained above for the distribution function, let us compute the mean electron energy $\overline{\mathcal{E}}$, and the plasma conductivity σ and dielectric constant ϵ. Integrating the expressions for the mean energy and for the electric-current density with respect to the angular variables and employing the orthogonality of the functions f_0, f_1, and χ we obtain[1]

$$\overline{\mathcal{E}}=2\pi m\int_0^\infty v^4 f_0\,dv; \qquad (12)$$

$$\mathbf{j}=\left(\sigma+i\omega\frac{\varepsilon-1}{4\pi}\right)\mathbf{E}=\frac{4\pi eN}{3}\int_0^\infty v^3\mathbf{f}_1\,dv.$$

Using the Hermite properties of the tensors ϵ and σ and aligning the z axis with the magnetic field, we obtain from (12) and (7) the following expression for the components of the tensors ϵ and σ:

$$\sigma_{zz}=\frac{8}{3\sqrt{\pi}}\frac{e^2N}{m}\int_0^\infty F\frac{\nu}{\omega^2+\nu^2}\,du;$$

$$\sigma_{xx}=\sigma_{yy}=\frac{8}{3\sqrt{\pi}}\frac{e^2N}{m}\int_0^\infty F\nu\,du\,\frac{1}{2}\left\{\frac{1}{(\omega-\omega_H)^2+\nu^2}+\frac{1}{(\omega+\omega_H)^2+\nu^2}\right\};$$

$$\sigma_{xy}=-\sigma_{yx}=i\frac{8}{3\sqrt{\pi}}\frac{e^2N}{m}\int_0^\infty F\nu\,du\,\frac{1}{2}\left\{\frac{1}{(\omega+\omega_H)^2+\nu^2}-\frac{1}{(\omega-\omega_H)^2+\nu^2}\right\};$$

$$\frac{\varepsilon_{zz}-1}{4\pi}=-\frac{8}{3\sqrt{\pi}}\frac{e^2N}{m}\int_0^\infty F\,du\,\frac{1}{\omega^2+\nu^2}; \qquad (12')$$

$$\frac{\varepsilon_{yy}-1}{4\pi}=\frac{\varepsilon_{xx}-1}{4\pi}=-\frac{8}{3\sqrt{\pi}}\frac{e^2N}{m}\int_0^\infty F\,du\,\frac{1}{2\omega}\left\{\frac{\omega-\omega_H}{(\omega-\omega_H)^2+\nu^2}+\frac{\omega+\omega_H}{(\omega+\omega_H)^2+\nu^2}\right\};$$

$$\frac{\varepsilon_{xy}}{4\pi}=-\frac{\varepsilon_{yx}}{4\pi}=-i\frac{8}{3\sqrt{\pi}}\frac{e^2N}{m}\int_0^\infty F\,du\,\frac{1}{2\omega}\left\{\frac{\omega-\omega_H}{(\omega-\omega_H)^2+\nu^2}-\frac{\omega+\omega_H}{(\omega+\omega_H)^2+\nu^2}\right\};$$

$$\sigma_{xz}=\sigma_{zx}=\sigma_{yz}=\sigma_{zy}=0;$$

$$\varepsilon_{xz}=\varepsilon_{zx}=\varepsilon_{yz}=\varepsilon_{yz}=0,$$

where a dimensionless variable $u=v\sqrt{2kT/m}$ was introduced;

$$u=v\sqrt{2kT/m};\ F=f_{00}\frac{u^4}{1+(e^2E_0^2/3kTm\delta)\,\varphi};$$ f_{00} and φ

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*The many difficulties arising in the analysis of the distribution function in stationary fields in case of collisions with ions were pointed out to the author by V. L. Ginzburg. I take advantage of this opportunity to thank him for attentive examination of the results of this investigation.

are given by expressions (7) and (7′). In the absence of a magnetic field, it can clearly be seen from (12′) that $\sigma_{xy} = \sigma_{yx} = 0$ and $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma$, with analogous relationships for ϵ.

a) Magnetic field $H = 0$. In the absence of a magnetic field (for collisions with molecules) we obtain from (12):

$$\overline{\mathscr{E}} = \frac{3}{2} kT (\gamma + q^2)^{1/2} \frac{W_{\gamma/2-5/4;\, \gamma/2+3/4}(\gamma + q^2)}{W_{\gamma/2-1/4;\, \gamma/2+3/4}(\gamma + q^2)};$$
$$\sigma = \frac{8}{3\sqrt{\pi}} \frac{e^2 N}{m\nu_0} (\gamma + q^2)^{1/2} \frac{W_{\gamma/2-3/4;\, \gamma/2+1}(\gamma + q^2)}{W_{\gamma/2-1/4;\, \gamma/2+3/4}(\gamma + q^2)};$$
$$\frac{\varepsilon - 1}{4\pi} = -\frac{e^2 N}{m\nu_0^2} \frac{W_{\gamma/2-5/4;\, \gamma/2+3/4}(\gamma + q^2)}{W_{\gamma/2-1/4;\, \gamma/2+3/4}(\gamma + q^2)}. \tag{13}$$

Here $W_{\mu,\lambda}(z)$ is the Whittaker function of order μ, λ; to simplify the notation we again introduce the dimensionless parameters

$$\gamma = \frac{e^2 E_0^2}{3kTm\delta\nu_0^2} \approx \frac{3{,}4 \cdot 10^{32} E_0^2}{N_M^2 T^2}; \tag{13′}$$
$$q = \frac{\omega}{\nu_0}; \quad \nu_0 = \sqrt{\frac{2kT}{m}} \pi a^2 N_M \approx 2{,}36 \cdot 10^{-10} N_M \sqrt{T},$$

where γ is a quantity on the order of the square of the ratio of the energy delivered to the electron by the constant field E_0 to its thermal energy; ν_0 is a frequency on the order of the number of electron collisions in the absence of an electric field. In the numerical coefficients we assumed $\delta = 3.4 \times 10^{-5}$ and $\pi a^2 = 4.3 \times 10^{-16}$ cm^2; E_0 is in millivolts per meter. If $\gamma \gg 1$ we have from (12):

$$\overline{\mathscr{E}} = 3kTx (1 + \gamma / q^2) I_{3/2}(x) / I_{1/2}(x),$$
$$\sigma = \frac{16}{\sqrt{3\pi}} \frac{eN}{E_0} \sqrt{\frac{kT\delta x}{m}} (1 + \gamma / q^2)^{1/2} \frac{I_2(x)}{I_{1/2}(x)} \left(1 - \frac{6}{\sqrt{2\gamma}} \frac{I_3(x)}{I_2(x)}\right); \tag{14}$$
$$\frac{\varepsilon - 1}{4\pi} = -\sqrt{6} \frac{eN}{E_0\nu_0} \sqrt{\frac{kT\delta}{m}} \frac{I_{3/2}(x)}{I_{1/2}(x)} \left(1 - \frac{5}{\sqrt{2\gamma}} \frac{I_{5/2}(x)}{I_{3/2}(x)}\right),$$

where

$$I_p(x) = \frac{1}{\Gamma(p+1)} \int_0^\infty e^{-t^2-2tx} t^p \, dt \tag{14′}$$
$$= \frac{e^{x^2/2}}{2^{(p+1)/2}} D_{-p-1}(\sqrt{2x}),$$

if $x \gg 1$

$$I_p(x) = (2x)^{-p-1} \{1 - (p+1)(p+2)/(2x)^2\}$$

(here $D_h(z)$ are the parabolic-cylinder functions) and

$$x = \sqrt{3}\, \omega^2 m \sqrt{\delta} / 2\pi a^2 N_M e E_0 \approx 660\, \omega^2 / E_0 N_M$$

(x is on the order of $(\omega/\nu)^2$, where ν is the electron collision frequency in a strong electric field). The functions $I_p(x)$ are plotted in Fig. 1. According to Eqs. (14) and (14′), $\overline{\mathscr{E}}$ increases monotonically with E_0 and T and decreases monotonically with ω and N_M. If $x \gg 1$ (high frequencies) we obtain from (14) and (14′):

$$\overline{\mathscr{E}} = \frac{3}{2} kT (1 + \gamma / q^2),$$
$$\sigma = \frac{8}{3\sqrt{\pi}} \frac{e^2 N \nu_0}{m\omega^2} (1 + \gamma / q^2)^{1/2},$$
$$\frac{\varepsilon - 1}{4\pi} = -\frac{e^2 N}{m\omega^2}.$$

In this case the distribution function is Maxwellian with an effective electron temperature

$$T_{эф} = T (1 + \gamma / q^2) = T (1 + e^2 E_0^2 / 3kTm\delta\omega^2).$$

If $x \ll 1$ (low frequency):

$$\overline{\mathscr{E}} = \frac{\Gamma(5/4)}{\sqrt{3}\,\Gamma(3/4)} \frac{eE_0}{\sqrt{\delta}\, \pi a^2 N_M}, \tag{15}$$
$$\sigma = \frac{\sqrt{\pi}}{2^{1/4} 3^{3/4} \Gamma(3/4)} \frac{N e^{3/2} \delta^{1/4}}{\sqrt{\pi a^2 N_M m E_0}} \approx 1{,}5 \cdot 10^{10} \frac{N}{\sqrt{N_M E_0}},$$
$$\frac{\varepsilon - 1}{4\pi} = -\frac{2\Gamma(5/4)}{\sqrt{3}\,\Gamma(3/4)} \frac{eN\delta^{1/2}}{\pi a^2 N_M E_0}$$
$$\approx -1{,}67 \cdot 10^{11} \frac{N}{N_M E_0}.$$

In this case the distribution function is of the Druyvesteyn type with an effective field $E_{ef.} = E_0 / \sqrt{2}$, as it should be, for if $x \ll 1$, i.e., if $\omega \ll \nu$, the alternating electric field acts on the average as a constant field $E = E_{ef} = E_0 / \sqrt{2}$. Let us also introduce corrections for the expressions for $\overline{\mathscr{E}}$, σ, and ϵ in a weak field ($\gamma \ll 1$). If $\omega^2 \gg \nu_0^2$ we have:

$$\overline{\mathscr{E}} = \frac{3}{2} kT (1 + \gamma / q^2);$$
$$\sigma = \frac{8}{3\sqrt{\pi}} \frac{e^2 N \nu_0}{m\omega^2} (1 + \gamma / q^2)^{1/2};$$
$$(\varepsilon - 1) / 4\pi = -e^2 N / m\omega^2.$$

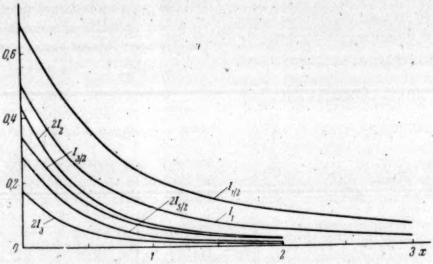

FIG. 1

If $\omega^2 << \nu_0^2$

$$\overline{\mathscr{E}} = \frac{3}{2}kT(1 + {}^2/_3\gamma);$$

$$\sigma = \frac{4}{3\sqrt{\pi}}\frac{e^2N}{m\nu_0}[1 - 2\gamma(1 - \ln 2)];$$

$$I_p(x) = (2x)^{-p-1}\{1 - (p+1)(p+2)/(2x)^2\}$$

Figures 2, 3 and 4 show plots of $\overline{\mathcal{E}}$, σ, and $(\epsilon - 1)/4\pi$) (related to their values at $E \to 0$) vs. $q = \omega/\nu_0$ for various values of γ. It is clear from the figures that at $\gamma >> 1$ the mean electron energy and plasma conductivity and dielectric constant depend strongly on the electric field intensity. It is interesting to note that at low frequencies ($x << 1$), σ decreases with increasing E_0 and the mean electron energy increases, and with it the energy delivered the wave to the plasma molecules (see Eq. 15). If we assume here that the absorption of radio waves is proportional to the conductivity, an apparent contradiction arises: the energy delivered by the wave to the plasma molecules increases with the field in a strong low-frequency field, and at the same time the absorption of the wave decreases. This, however, does not contradict the law of conservation of energy, for the energy of the wave increases as the square of E_0, while the mean energy of the plasma electrons is proportional to E_0 [(from (15)]. Consequently, even though the total energy delivered by the wave to the plasma molecules increases with E_0, its relative value decreases; therefore, the absorption also decreases, and exhibits a relative decrease in the amplitude of E on the given section of the path.

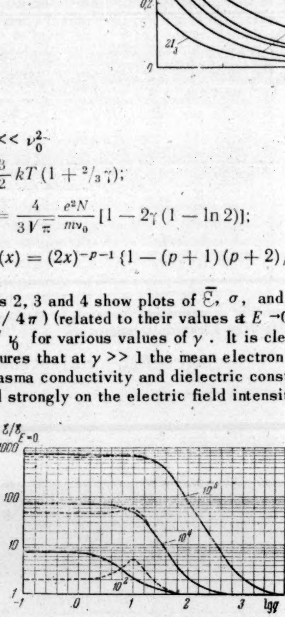

FIG. 2. Dependence of $\mathcal{E}/\mathcal{E}_{E=0}$ on $q = \omega/\nu_0 = 4.24 \times 10^9\,\omega/N_M\sqrt{T}$ for the values of γ indicated on the curves, and for $\omega_H/\nu_0 = 10$. Solid lines are for $\beta = 0$, dotted for $\beta = \pi/2$.

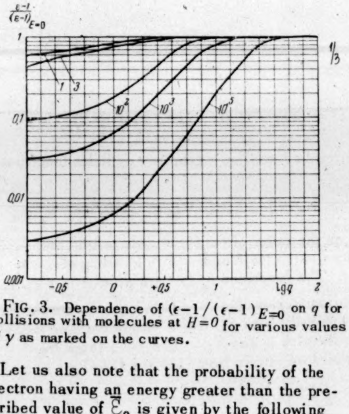

FIG. 3. Dependence of $(\epsilon-1/(\epsilon-1)_{E=0}$ on q for collisions with molecules at $H=0$ for various values of γ as marked on the curves.

Let us also note that the probability of the electron having an energy greater than the prescribed value of $\mathcal{E}_0$ is given by the following equation

$$P(\mathscr{E} \geqslant \mathscr{E}_0) = \int_{v_0}^{\infty} v^2 f_{00}\,dv,$$

where $v_0 = \sqrt{2\overline{\mathcal{E}}/m}$. It is now easy to obtain an expression for $P\ (\mathcal{E} \geq \mathcal{E}_0)$ for various limiting cases. In particular, for $\gamma >> 1$ we have

$$P(\mathcal{E} \geqslant \mathcal{E}_0) = I_{1/2}(t_0, x) / I_{1/2}(x), \qquad (16)$$

$$I_{1/2}(t_0, x) = \frac{2}{\sqrt{\pi}} \int_{t_0}^{\infty} t^{1/2} \exp\{-t^2 - 2tx\}\, dt; \qquad (16')$$

$$t_0 = \frac{\mathcal{E}_0}{2kTx\,(1 + \gamma/q^2)}.$$

It is clear from (16) and (16′) that in a strong field the electron energy is less likely to deviate substantially from its mean value $\overline{\mathcal{E}}$ at low frequencies than at high frequencies.

b) Magnetic field $H \neq 0$. If the effect of the constant magnetic field is taken into account, the computation of $\overline{\mathcal{E}}$, σ, and ϵ is in general much more complicated; however, in some particular instances it is possible to convert the expressions for $\overline{\mathcal{E}}$, σ, and ϵ to those discussed above by introducing effective parameters E_0', ω', which take the magnetic field into account.

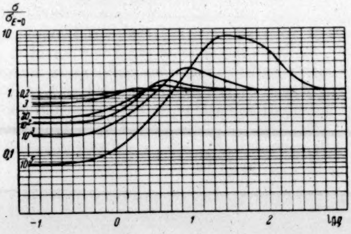

FIG. 4. Dependence of $\sigma/\sigma_{E\to 0}$ on q for collisions with molecules (at $H = 0$) for the various values of γ marked on the curves.

It is evident that if $\beta = 0$ (β is the angle between E and H) the magnetic field does not affect the symmetrical portion of the distribution function. Consequently, the average electron energy, and also the components of the conductivity and dielectric constant tensors (σ_{zz}, ϵ_{zz}), which are parallel to H are given by the expressions analyzed previously ($E_0' = E_0$; $\omega' = \omega$). For the remaining components of tensors σ and ϵ we obtain the somewhat more complicated expressions (12). In several limiting cases they can be simplified. For example, if $\gamma >> 1$, $x >> 1$, and $x\,(\omega - \omega_H)^2/\omega^2 >> 1$ the expressions for σ_{xx} etc. are the same as obtained in the weak-field theory [with the temperature replaced by its effective value T_{ef}, as given in (20)], namely;

$$\sigma_{xx} = \sigma_{yy} = \frac{8}{3\sqrt{\pi}} \frac{e^2 N \nu_0}{m} \frac{1}{2} \qquad (17)$$

$$\times \left(\frac{1}{(\omega - \omega_H)^2} + \frac{1}{(\omega + \omega_H)^2}\right)(1 + \gamma/q^2)^{1/2};$$

$$\sigma_{xy} = -\sigma_{yx} = -i\,\frac{8}{3\sqrt{\pi}} \frac{e^2 N \nu_0}{m} \frac{1}{2}$$

$$\times \left(\frac{1}{(\omega - \omega_H)^2} - \frac{1}{(\omega + \omega_H)^2}\right)(1 + \gamma)/q^2)^{1/2};$$

$$\frac{\varepsilon_{xx} - 1}{4\pi} = \frac{\varepsilon_{yy} - 1}{4\pi} = -\frac{e^2 N}{m\,(\omega^2 - \omega_H)^2};$$

$$\frac{\varepsilon_{xy}}{4\pi} = -\frac{\varepsilon_{yx}}{4\pi} = -i\,\frac{e^2 N}{m\,(\omega^2 - \omega_H^2)} \frac{\omega_H}{\omega}.$$

The magnetic field affects the electron distribution most when $\beta = \pi/2$. In this case we have for $\gamma >> 1$ and $x\,(\omega + \omega_H)^2/\omega^2 >> 1$

$$\overline{\mathcal{E}} = \overline{\mathcal{E}}_{H=0}(E_0', \omega'); \qquad (18)$$

$$\sigma_{xx} = \sigma_{yy} = \frac{1}{2}\left(1 + \left(\frac{\omega - \omega_H}{\omega + \omega_H}\right)^2\right)\sigma_{H=0}(E_0', \omega'),$$

$$\sigma_{xy} = -\sigma_{yx} = -i\,\frac{2\omega\omega_H}{(\omega + \omega_H)^2}\,\sigma_{H=0}(E_0', \omega');$$

$$\frac{\varepsilon_{xx} - 1}{4\pi} = \frac{\varepsilon_{yy} - 1}{4\pi}$$

$$= -\sqrt{6}\,\frac{eN}{\nu_0 E_0'}\sqrt{\frac{kT\delta}{m}}\,\frac{I_{3/2}(x')}{I_{1/2}(x')}\left\{\frac{\omega^2 - \omega_H^2}{(\omega + \omega_H)^2}\right.$$

$$\left. + \frac{(\omega - \omega_H)^2}{(\omega + \omega_H)^2}\,\frac{5}{2x'}\,\frac{I_{5/2}(x')}{I_{3/2}(x')}\right\},$$

$$\frac{\varepsilon_{xy}}{4\pi} = -\frac{\varepsilon_{yx}}{4\pi}$$

$$= -i\sqrt{6}\,\frac{eN}{\nu_0 E_0'}\sqrt{\frac{kT\delta}{m}}\,\frac{\omega_H}{\omega}\,\frac{I_{3/2}(x')}{I_{1/2}(x')}\left\{\frac{\omega_H^2 - \omega^2}{(\omega + \omega_H)^2}\right.$$

$$\left. + \frac{(\omega - \omega_H)^2}{(\omega + \omega_H)^2}\,\frac{5}{2x'}\,\frac{I_{5/2}(x')}{I_{3/2}(x')}\right\},$$

$$E_0' = \frac{E_0}{\sqrt{2}}\sqrt{1 + \left(\frac{\omega - \omega_H}{\omega + \omega_H}\right)^2}; \qquad (18')$$

$$\omega' = |\omega - \omega_H|; \quad x' = \frac{\sqrt{3}}{2}\,\frac{\omega'^2 m \sqrt{\delta}}{\pi a^2 N_M e E_0}$$

If, on the contrary, $x(\omega_1 + \omega_H)^2/\omega^2 \ll 1$, the effect of the magnetic field is insignificant; for the mean energy of the electrons and for the diagonal components of tensors σ and ϵ ($\sigma_{xx} = \sigma_{yy} = \sigma_{zz}, \ldots$) we have $E_0' = E_0$; $\omega' = \omega$. The remaining components have the following form:

$$\sigma_{xy} = \sigma_{yx} = -i\,\frac{2\sqrt{2\pi}}{3^{1/4}\Gamma(3/4)}\,\frac{\omega\omega_H m^{1/2}\delta^{3/4}e^{1/2}N}{(E_0\pi a^2 N_M)^{3/2}}; \quad (19)$$

$$\frac{\varepsilon_{xy}}{4\pi} = -\frac{\varepsilon_{yx}}{4\pi} = i\,\frac{2\Gamma(5/4)}{\sqrt{3}\,\Gamma(3/4)}\,\frac{\omega_H}{\omega}\,\frac{eN\sqrt{\delta}}{E_0\pi a^2 N_M}.$$

Figures 2 and 5 show plots of the functions ϵ and σ_{xx} (at $\beta = \pi/2$) vs. $q = \omega/\nu_0$ for $\omega_H/\nu_0 = 10$ and for various values of γ . The mean electron energy $\mathcal{E}/\mathcal{E}_{E=0}$, is seen clearly from (18) and (18′) to increase resonantly in the vicinity of the gyro-frequency [$\omega \sim \omega_H$, compare (13)] ; this resonance is observed when the following conditions $\gamma \gg 1$; $x(\omega_H/\omega)^2 \gg 1$, are satisfied, as can also be seen from Fig. 2. Under these conditions $\sigma/\sigma_{E=0}$ decreases resonantly (see Fig. 5), while at $\omega = \omega_H$ increases, and $\mathcal{E}$ decreases with increasing E_0, the same as at low frequencies in the absence of a magnetic field (see Eq. 15).

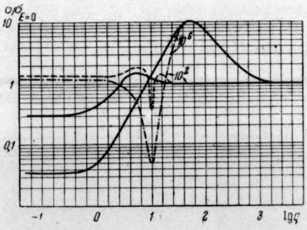

FIG. 5. Dependence of $\sigma/\sigma_{E=0}$ on q (Collisions with molecules for $\omega/\nu_0 = 10$ and for various values of γ marked on the curve). Solid curves are for $\beta = 0$, dotted for $\beta = \pi/2$.

Let us also note that the case $\beta = \pi/2$ also occurs for longitudinal propagation of the electromagnetic wave. The wave breaks up here into two waves – ordinary and extraordinary – circularly polarized in a plane perpendicular to H . For each of these waves, as can be clearly seen from (11), $E_0' = E_0\sqrt{2}$; ; $\omega' = |\,\omega \pm \omega_H\,|$. Comparing (19) with (18) and (18′) we see that in first approximation both circularly-polarized waves, which make up a plane polarized wave, exert an equal effect of the electrons in all cases except gyro-resonance ($\omega \sim \omega_H$, $x(\omega_H/\omega)^2 >> 1$). In the case of gyro-resonance, however, the increase in electron energy is due only to the extraordinary wave.

If angle β is arbitrary and the following conditions are satisfied

$$x/\cos^2\beta \gg 1; \quad (2x/\sin^2\beta)(\omega - \omega_H)^2/\omega^2 \gg 1,$$

the distribution function is Maxwellian with an effective temperature

$$T_{\text{эф}} = T\left\{1 + \frac{e^2E_0^2}{3kTm\delta}\left[\frac{\cos^2\beta}{\omega^2} + \frac{\sin^2\beta\,(\omega^2 + \omega_H^2)}{(\omega^2 - \omega_H^2)^2}\right]\right\}, \quad (20)$$

i.e., the same expressions hold for $\overline{\mathcal{E}}$, σ, and ϵ as for weak fields, except that T is replaced by T_{ef} (see Eq. 17). Let us note that the electron gas also has this effective temperature (20) in case of collisions with ions, provided the following conditions are satisfied.

$$\omega \gg \nu_{0i}; \quad |\,\omega - \omega_H\,| \gg \nu_{0i}. \quad (21)$$

Here ν_{0i} is on the order of the frequency of collisions between electrons and ions in the absence of an electric field. If $x(\omega + \omega_H)^2 \ll 1$, and $\gamma \gg 1$, the distribution function (in case of collisions with molecules is of the Druyvesteyn type with an effective field $E_{ef} = E_0/\sqrt{2}$; expressions (15), (19) are valid for $\overline{\mathcal{E}}$, ϵ, and σ .

4. EFFECTIVE FREQUENCY OF ELECTRON COLLISIONS

The expressions for σ and ϵ obtained above with the aid of the kinetic theory are in general quite complicated. It becomes therefore advantageous to investigate the possibility of calculating σ and ϵ from the elementary theory equations:

$$\sigma_{(\text{el})} = \frac{e^2N}{m}\,\frac{\nu_{\text{эф}}}{\omega^2 + \nu_{\text{эф}}^2}; \quad (22)$$

$$\varepsilon_{(\text{el})} = 1 - \frac{4\pi e^2N}{m}\,\frac{1}{\omega^2 + \nu_{\text{эф}}^2},$$

For this purpose it is necessary to determine the effective electron-collision frequency (ν_{ef}) and to compare the values of the conductivity and dielectric constant (σ_{el}, ϵ_{el}) from Eqs. (22), with those (σ_k, ϵ_k), obtained from kinetic theory.

Let us first consider the case of a weak electric field ($e^2E_0^2/3kTm\delta(\omega^2+\nu_0^2 \ll 1$), when the effect of the wave of the plasma can be neglected.* Here, as is well known, it is possible to determine the effective electron-collision frequency ν_{ef0} at high frequencies ($\omega \gg \nu_0$) in such a way that the conductivity and the dielectric constant, calculated from the kinetic theory and from Eq. (22), are in agreement (see Refs. 11-13):

$$\nu_{\text{эф}\,0} = \frac{8}{3\sqrt{\pi}}\nu_0 \tag{22'}$$

$$= \frac{8\sqrt{2}}{3\sqrt{\pi}}\sqrt{\frac{kT}{m}}\pi a^2 N_M \text{ (for collisions with molecules)}$$

$$\nu_{\text{эф}\,0i} = \frac{4}{3\sqrt{\pi}}\nu_{0i}$$

$$= \frac{2\sqrt{2\pi}}{3}\frac{e^4N_i}{m^{1/2}(kT)^{3/2}}\ln\left\{\frac{(kT)^3}{2\pi N_i e^6}\right\} \text{ (for collisionswith ions).}$$

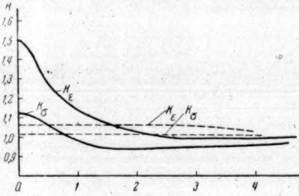

FIG. 6. Dependence of K_σ and K_ϵ on q for collisions with molecules. The dotted lines show curves for strongelectric fields (at $\gamma = 10^5$).

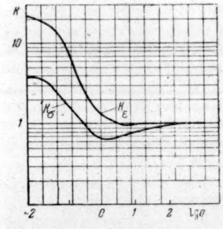

FIG. 7. Dependence of K_σ and K_ϵ on $\log q_i$ for collisions with ions.

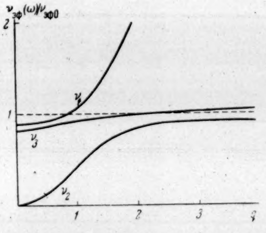

FIG. 8. Dependence of $\nu_{ef}(\omega)/\nu_{ef0}$ on q for collisions with molecules.

If the condition $\omega \gg \nu_0$ is not satisfied then σ_{el} and ϵ_{el} no longer equal σ_k and ϵ_k. It is therefore necessary to introduce correction coefficients K_σ and K_ϵ, which are determined in the following manner:

$$\sigma_{\text{к}} = K_\sigma \sigma_{\text{эл}}; \quad \varepsilon_{\text{к}} - 1 = K_\varepsilon(\varepsilon_{\text{эл}} - 1).$$

The coefficients K_σ and K_ϵ plotted in Figs. 6 and 7 are functions of the frequency. In the case of collisions with molecules (Fig. 6) the correction coefficients K_σ and K_ϵ are nearly equal to unity; on the other hand, in the case of collisions with ions (Fig. 7) they may differ considerably from unity (at $\omega \lesssim 10\,\nu_{0i}$),* and this circumstance must be taken into account in the calculation of the conductivity and of the dielectric constant.

In a strong electric field, K_{ef} can be obtained from effective electron gas temperature T_{ef}. In the case of collisions with molecules we have here $\nu_{ef} = \nu_{ef0}\sqrt{T_{ef}/T}$, where T_{ef} is determined by the following relationship:

---

*Let us note that the weak-field condition specified here is valid in the case of collisions with ions only if $\omega \gg \nu_{0i}$.

*It must be noted that in general the parameter ν_{ef} can be defined in such a way as to make σ_{el} and ϵ_{el} equal to σ_k and ϵ_k; for this purpose it is necessary to examine $\nu_{ef}(\omega)$, as was done earlier (see Ref. 8, Sec. 61). However, analysis shows that such determination of the effective collision frequency leads to three expressions for $\nu_{ef}(\omega)$, as shown in Figs. 8 (collisions with molecules) and 9 (collisions with ions). As can be seen from the figures, the indicated three expressions for $\nu_{ef}(\sigma)$ differ considerably even at the same value of ω and vary substantially with ω. Therefore ν_{ef} loses in this case its physical significance of an electron collision frequency ($\nu_{ef2} \to 0$ at $\omega \to 0$). In addition, the requirement that ϵ_{el} and x_{el} be equal to ϵ_k and σ_k calls for the use of at least two substantially-varying functions $\nu_{ef}(\omega)$. There is no sense in doing this and it is more convenient to employ the coefficient K_σ and K_ϵ. In strong fields the use of ν_{ef} leads to even more serious difficulties.

$$\frac{T_{(ef)}}{T}=1+\frac{e^2E_0^2}{3kTm\delta}\left\{\frac{\cos^2\beta}{\omega^2+\nu^2_{(ef)0}T_{(ef)}/T}+\frac{\sin^2\beta}{2}\left[\frac{1}{(\omega-\omega_H)^2+\nu^2_{(ef)0}T_{(ef)}/T}+\frac{1}{(\omega+\omega_H)^2+\nu^2_{(ef)0}T_{(ef)}/T}\right]\right\}. \quad (23)$$

Comparing now the values of the conductivity and dielectric constant calculated with the simple equations (22), (22′), (23) and those obtained with the kinetic-theory equations (Sec. 3), we see that in strong fields (as well as weak ones) the coefficients K_σ and K_ϵ are nearly equal to unity as before: at $x \ll 1$ (low frequencies), $K_\sigma \approx 1.03$ and $K_\epsilon \approx 1.07$ (see, for example, the dotted curve on figures 6), and at $x \gg 1$ (high frequencies) $K_\sigma \to 1$; $K_\epsilon \to 1$.

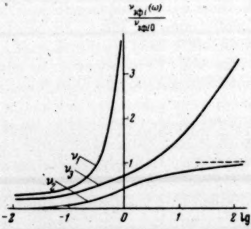

FIG. 9. Dependence of $\nu_{ef}(\omega)/\nu_{ef0}$ on $q_i=\omega/\nu_{0i}$ for collisions with ions.

In the case of collisions with ions, if we restrict ourselves to high frequencies ($\omega \gtrsim 10\,\nu_{0i}$), Eqs. (22) and (22′), used in conjunction with the effective electron temperature T_{ef} as defined by equation (20), result in strongfield values of σ and ϵ that are even in closer agreement with the results of the kinetic theory than the weak-field values.**

In conclusion the author thanks Ia. L. Al' pert for guidance in this investigation and for many advises and comments.

** The latter circumstance follows from the fact that the effective frequency of collisions between electrons and ions (22) decreases with increasing T_{ef}, i.e., the condition $\omega \gg \nu_{efi}$ is better satisfied in a strong field than in a weak one. Let us also recall that if the electron temperature T_{ef} differs from the ion temperature T, a somewhat different expression is used to calculate ν_{ef0i} than (22) (see Ref. 8, Sec. 61).

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SOVIET PHYSICS JETP VOLUME 3, NUMBER 6 JANAUARY, 1957

New Nonpiezoelectric Dielectrics with Very High Dielectric Permeability and Small Conductivity

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It is shown that a combination favoring polarization of the internal field by means of relaxation polarization of ions leads to a very high dielectric permeability in a wide range of frequencies (up to very high) with freedom from spontaneous polarization and other piezoelectric effects. We obtained nonpiezoelectric dielectrics with $\epsilon \approx 1000$ and with small conductivity.

ONE of the authors in cooperation with A. I. Demeshina has earlier discovered relaxation polarization, giving a very high dielectric permeability at low frequencies (50-20,000 cycles per second). This was found in polycrystalline dielectrics made of rutile (TiO_2) containing small impurities of oxides of the alkali earth metals[1].

In the present work the problem was to bring about experimental conditions in a solid dielectric such that the internal field associated with the polarization[2] would be associated with an ionic relaxation displacement which would have a sufficiently small time of relaxation. At the same time, the dielectric permeability was to be very high even at high frequencies. The basic difficulty in this program consists in bringing about the possibility of comparatively large ionic displacements while still preserving the perovskite structure, which is favorable for polarization.

If one adds ions of bismuth, which have the same radius (1.2 A) as ions of strontium, to strontium titanate, then the ions of bismuth are able partly to displace ions of strontium without changing the crystal structure. Also, owing to the differences of valence (ions of strontium are divalent, ions of bismuth are trivalent) the electrical neutrality of the lattice can be maintained only if holes are formed with the specified fractional substitution, as is the case in solid solutions of the substitutional type. Ia. M. Ksendzov showed that the presence of holes is very probable. Breaking up the lattice of the perovskite type can lead to the appearance of ionic relaxation polarization.

Experimental work in this direction has showed that dielectrics of strontium titanate in the presence of a small addition of bismuth trioxide actually possess very high relaxation polarization, and moreover this is quite clearly expressed. Their dielectric permeability remains very high up to very high frequencies*. In Figs. 1 and 2 are curves of the dependence of the dielectric permeability ϵ and of the tangent of the angle of dielectric losses, tan δ, on temperature with various frequencies for one of the new dielectrics. The shape of these curves is characteristic of relaxation polarization: an increase of frequency displaces the maximum of tan δ and ϵ toward the direction of high temperatures. The presence of clearly expressed temperature maxima in ϵ and tan δ is an indication that the relaxation polarization is not affected by other processes, for example, by an increased conductivity. The falling part of the curve $\epsilon = f(T)$ corresponds to a dielectric permeability with a zero frequency ϵ_0 which decreases with rising temperatures. With sufficiently low temperature the dielectric permeability is reduced in value and approaches the elastically dependent polarization ϵ_∞ ($\sim$200-250).

The series of experiments showed that the new dielectrics, in spite of the temperature maximum of ϵ, cannot be put into the class of piezoelectrics, because of the following characteristics: 1) the temperature maxima of ϵ and tan δ shift with increasing frequency toward the direction of high temperature; 2) nonlinearity of polarization is absent and the dielectric permeability is independent of field strength; 3) dielectric hysteresis is absent. The presence of the temperature maximum of ϵ corresponds to "classical relaxation polarization." This was observed earlier in such an evident form only in polar liquids and some polar polymers, but it was not brought out in such high values of ϵ.

The circumstance that the conductivity of the strontium-bismuth titanates (SBT) is relatively very small makes it possible to estimate from the experimental data in Figs. 1 and 2 several of the dielectric parameters, for example, the coefficients of the internal field, the activation energy of ionic relaxation, and so on.

If one does not take into account the differences between the fields acting on various ions, then one can introduce and average field E, which is equal to

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* We discovered similar, but more weakly expressed, properties in the dielectric systems calcium titanate-bismuth titanate and barium titanate-bismuth titanate.

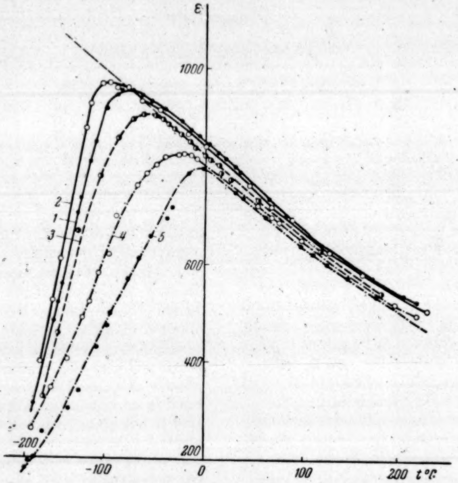

FIG. 1. The dependence of ϵ on the temperature, using various frequencies for the dielectric SVT.
1 -- $f = 5000$ cycles, 2 -- $f = 50$ cycles, 3 - $f = 50$ kc, 4 -- $f = 1$ mc, 5 -- $f = 10$ mc.

$$E = E_{\mathrm{ave}} + \beta_1 I' + \beta_2 I_0, \qquad (1)$$

where E_{ave} is the average macroscopic field, I' is the electrical moment of the unit of volume, dependent on the relaxation polarization and I_0 is the same for the elastic (electronic and ionic) polarization. Then the condition for a maximum in tan δ will have the following form:

$$(\omega\tau)' = \frac{\varepsilon_\infty - 1 + (4\pi/\beta_1)}{\varepsilon_0 - 1 + (4\pi/\beta_1)} \sqrt{\frac{\varepsilon_0}{\varepsilon_\infty}}, \qquad (2)$$

where $\tau = (2\nu)^{-1} \exp(U/kT)$, U is the potential barrier which the relaxing particles surmount and ν is their frequency of oscillation in position. It is essential that the condition (2) does not include the coefficient of the internal field β_2, which was created by the elastic polarization.

It is possible to show that an approximate condition for a temperature maximum in ϵ has the following form:

$$(\omega\tau)'' \approx \frac{\varepsilon_{\max} - \varepsilon_\infty}{2U\varepsilon_{\max}\,\mathrm{tg}\,\delta''} kT_1, \qquad (3)$$

where tan δ'' relates to the temperature T_1 of maximum ϵ and to the frequency ω, and k is Boltzman's constant.

Expressions (2) and (3) correspond to the assumption that all the weakly bound ions take the same relaxation time. It is well known from experimental data that ϵ_0 and ϵ_∞ can be expressed through the effective "elastic" polarizability of the unit volume α_{eff} and the relaxation polarizability of the unit volume $\alpha' = n'q^2\varkappa^2/12\,kT$ (q is the charge of the relaxing ion, $\varkappa$ is its displacement and n' is the number of weakly bound ions in 1 cm^3)

$$\frac{\varepsilon_0 - 1}{\varepsilon_0 + 2} = \frac{4\pi(\alpha_{\mathrm{eff}} + \alpha')}{1 - (\beta_2\alpha_{\mathrm{eff}} + \beta_1\alpha')}, \qquad (4)$$

$$\frac{\varepsilon_\infty - 1}{\varepsilon_\infty + 2} = \frac{4\pi\alpha_{\mathrm{eff}}}{1 - \beta_2\alpha_{\mathrm{eff}}}. \qquad (5)$$

The combination of Eqs. (2) and (5), and also an estimate of the size of α_{eff} from additional considerations, makes it possible to estimate U, β_1,

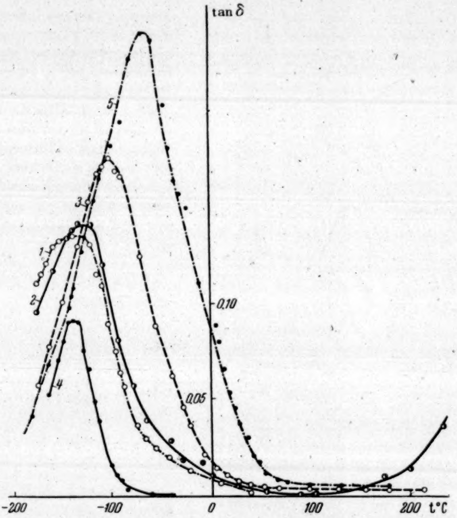

FIG. 2. The dependence of tan δ on temperature, using various frequencies for the dielectric SVT.
1 -- $f = 5000$ cycles, 2 -- $f = 50$ kc, 3 -- $f = 1.5$ mc, 4 -- $f = 50$ cycles, 5 -- $f = 10$ mc.

β_2 and α', using experimental data. For the low temperatures (and low frequencies) $U \sim 1$ ev, while with high temperatures (and high frequencies) $U \sim 0.2$-0.3 ev. The coefficient $\beta_1 \sim 0.01$, $\beta_2 \sim +5$. The small value of β_1 confirms the assumption that the concentration of lattice defects, which governs the relaxation polarization, is small. The high value of dielectric permeability is determined by the simultaneous effect of the large internal field created by elastic polarization (β_2 is large and positive) and relaxation polarization. A more detailed analysis of the experimental data says that the new dielectrics have at least two collections of relaxation times: a diffuse collection of longer relaxation times and a concentrated collection of short relaxation times. Calculation using the two specified collections of relaxation times allows one to explain the intrease of tan δ_{max} with frequency and also several other facts.

As has been pointed out above, the electrical conductivity of the new dielectrics is extremely small (Fig. 3).

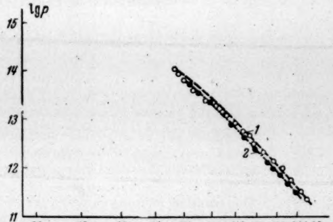

FIG. 3. The dependence of the resistivity ρ on temperature for the dielectric SVT. 1 -- forward direction. 2 -- reverse direction.

At low temperatures a large drop of current, dependent on the accumulation of space charge, occurs with time. In addition to this, if the specimen remains at room temperature with the voltage applied until the current is practically constant,

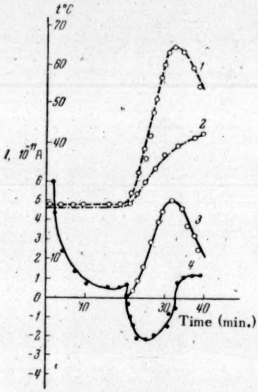

FIG. 4. The dependence of current and various electrode temperatures on time for the dielectric SVT. *1*-- temperature on the "hot" electrode, *2*-- temperature of the "cold" electrode, *3*-- the difference in temperature of the electrodes, *4*-- current through the sample.

and then the specimen is heated, the current changes its sign. Specially prepared samples showed that the reverse current can be determined by the rapidity of change of the temperature difference of the electrodes. The corresponding curves are in Fig. 4. One can think that with a change of temperature gradient in the sample with time, a reverse thermal diffusion current arises, determined by the recombination of space charge. In view of the small direct current conductivity, this reverse current can exceed the direct current, and the total current changes sign.

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The Shape of the Dispersion Signal in Nuclear Magnetic Resonance with Strong High-Frequency Magnetic Fields

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Approximate solutions are given for the system of equations which describe the behavior of the components of the magnetization vector in a strong high-frequency magnetic field for intermediate relaxation times; both symmetrical and asymmetrical sinusoidal modulation are considered. The approximation is investigated and the limits of its applicability indicated. The solution is used to derive expressions for the longitudinal and transverse relaxation times. The theoretical conclusions are found to be in agreement with the results of experimental studies.

INTRODUCTION

THE shapes of the dispersion and absorption signals in nuclear magnetic resonance depend on the relaxation times (longitudinal and transverse), which are characteristics of the internal fields, and on the external conditions (the strength of the high-frequency magnetic field, the amplitude, frequency and wave-shape of the modulating magnetic field, etc.). Although numerous analyses of the signal shape have been given in the literature[1-10] there are still many experimentally encountered cases which have not been considered.

The present paper is concerned with the signal shape when a "strong" high-frequency magnetic field is applied; sinusoidal modulation of the longitudinal magnetic field is assumed. We shall be interested in the intermediate relaxation-time case, i.e., the case in which the period of the modulating field T_m is comparable to the relaxation times (the longitudinal relaxation time is designated by T_1 and the transverse relaxation time by T_2). By "strong" high-frequency magnetic field, as is customary[1,5,10,11], is meant a field such that the vector **M**, which describes the nuclear magnetization, is highly perturbed or turned over at resonance. The case of intermediate relaxation times is of interest since it is frequently encountered in practice.

In preparation for the analysis of the signal shape, which follows, we introduce the dimensionless parameters:

$$\nu = \frac{1}{\omega_m T_1}, \quad \mu = \frac{1}{\omega_m T_2}, \quad k = \frac{H_m}{H_1}, \tag{1}$$

$$\varkappa = \frac{H_c}{H_m}, \quad \lambda = \frac{\omega_m H_m}{|\gamma| H_1^2}$$

and the dimensionless time $x = \omega_m t$ (t is the time; ω_m is the angular frequency of the modulating field; H_m is the amplitude of the modulating magnetic field; H_1 is the amplitude of the high-frequency magnetic field and H_c is a fixed quantity which is superimposed on the resonance value of the dc magnetic field H_0.

A general expression for the dispersion signal u in a strong high-frequency field ($\lambda < 1$) has been given in Refs. 1 and 10. The solution is given as a series expansion in terms of the parameter λ. When the longitudinal magnetic field is sinusoidally modulated, the zeroth approximation has the form

$$u^0(x) = u^0(x_0)\frac{G(x_0)}{G(x)} \tag{2}$$

$$\times \exp\{\nu(x_0 - x) + (\mu - \nu)[H(x_0) - H(x)]\}$$

$$+ M_0 \int\limits_{x_0}^{x} \frac{\nu k(\varkappa + \sin\eta)}{G(x)G(\eta)}$$

$$\times \exp\{\nu(\eta - x) + (\mu - \nu)[H(\eta) - H(x)]\}\, d\eta,$$

where x_0 is the initial time and M_0 is the constant component of the magnetization vector

$$G(x) = [1 + k^2(\varkappa + \sin x)^2]^{1/2},$$

$$H(\eta) - H(x) = \int\limits_{x}^{\eta} [G(\xi)]^{-2}\, d\xi.$$

In the symmetrical modulation case ($\varkappa = 0$), the shape of the dispersion signal is given by the expression[10]

$$u^{(0)}(x) = u^{(0)}(x_0)\frac{g(x_0)}{g(x)} \exp\{f(x_0) - f(x)\} \tag{3}$$

$$+ M_0 \int\limits_{x_0}^{x} \frac{\nu k \sin\eta}{g(x)\,g(\eta)} \exp\{f(\eta) - f(x)\}\, d\eta,$$

where

$$f(x) = \nu x + \frac{\mu - \nu}{\sqrt{1 + k^2}} \operatorname{arc\,tg}(\sqrt{1 + k^2}\,\operatorname{tg} x),$$

$$g(x) = [1 + k^2 \sin^2 x]^{1/2}.$$

Examination of Eqs. (2) and (3) shows that in the steady-state ($x_0 \to -\infty$) the form of the dispersion signal is determined by the second term in Eqs. (2) and (3). It has been shown in Ref. 10 that in the first approximation the steady-state value of $u^{(1)}$ is zero. Hence, if the parameter λ is small, the shape of the dispersion signal u is given by the zeroth approximation, Eqs. (2) and (3) (to second order terms in λ). In the following the zero symbol will be omitted in the quantity $u^{(0)}(x)$.

1. SYMMETRICAL MODULATION

We shall consider the form of the steady-state dispersion signal $u_{st}(x)$ for symmetrical modulation ($\varkappa = 0$), i.e., the case in which the longitudinal magnetic field H_z is of the form $H_z = H_0 + H_m \sin x$. From Eq. (3) we have

$$u_{st}(x) = \frac{M_0}{g(x)} e^{-f(x)} \int\limits_{-\infty}^{x} \frac{\nu k \sin\eta}{g(\eta)} e^{f(\eta)} d\eta. \tag{4}$$

The analysis of this signal is difficult because the integral which appears in Eq. (4) cannot be expressed in terms of tabulated functions.

Under certain conditions, Eq. (4) can be simplified considerably. We introduce the notation

$$C_\tau = \int\limits_{-\infty}^{x_\tau} \frac{\nu k \sin\eta}{g(\eta)} e^{f(\eta)} d\eta, \tag{5}$$

$$J(x, x_\tau) = \int\limits_{x}^{x_\tau} \frac{\nu k \sin\eta}{g(\eta)} e^{f(\eta)} d\eta.$$

Then

$$u_{st}(x) = [M_0/g(x)]\, e^{-f(x)} [C_\tau + J(x, x_\tau)]; \tag{6}$$

$$C_\tau = \frac{u_{st}(x_\tau)}{M_0} g(x_\tau) e^{f(x_\tau)}. \tag{7}$$

The quantity x_T is the time corresponding to the onset of resonance, i.e., $u(x_T) = 0.5\, u(x_m)$ where x_m is the time at which the signal reaches its maximum value. It has been shown in Ref. 10 that $x_T < 0$.

We now ascertain the values of ν, μ and k in Eq. (6) for which $J(x, x_T)$ can be neglected as compared with the quantity C_T. Analysis of Eq. (5) indicates that in the region from $-|x_T|$ to $|x_T|$, the quantity $J(x, x_T)$ assumes its maximum values at $x = 0$ and $x = |x_T|$. We are concerned with the intermediate relaxation time case, i.e, the case in which $T_m T_1$ varies within the limits 0.1 to 10 (as has been pointed out in Refs. 3 and 12, $T_2 < T_1$ and the quantity T_m/T_2 can vary over wider limits). To estimate the relative magnitudes of the quantities C_T and $J(x, x_T)$, the values of $J(0, -|x_T|)$, $J(|x_T|, -|x_T|)$ and C_T were computed by numerical integration, using the following values for the parameters: ν from 0.001 to 5, μ from $\mu = \nu$ to 20 for k equal to 10 and 20 and x_T equal to 0.2 and 0.1, respectively. The results are presented graphically in Fig. 1 which shows the different regions of the dependence of μ on ν. The curves which bound the regions $k = 10$ and $k = 20$ are plotted using the conditions:

$$J(0, -|x_\tau|) = 0.1 C_\tau,$$

$$J(|x_\tau|, -|x_\tau|) = 0.1 C_\tau.$$

The following condition is satisfied within these regions:

$$-J(x, -|x_\tau|) < 0.1 C_\tau.$$

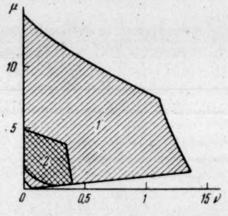

FIG. 1. Limits of applicability of the approximate solutions for two values of the quantity k. Regions 1 and 2 correspond to $k = 20$ and $k = 10$, respectively.

Thus for a specified value of k, if for a given value of the qantity ν, the value of μ lies within the region indicated, then the quantity $J(x, x_T)$ in Eq. (16) can be neglected (for a signal width of $2|x_T|$ and 10 percent accuracy). Then we obtain the following expression for u_{st} from Eqs. (6) and (7):

$$u_{st}(x) = u_{st}(x_\tau) \frac{g(x_\tau)}{g(x)} \exp\{f(x_\tau) - f(x)\}. \quad (8)$$

this expression is valid in the resonance region.

Comparing Eqs. (8) and (3), we see that the steady state and transient solutions have the same form; hence, for rapid passage through resonance, the form of the dispersion signal remains fixed, although its magnitude varies with time.

The rate at which the signal builds up can be used to determine the longitudinal relaxation time T_1 in those cases for which the build-up time is large compared with the modulation period. From Eqs. (3) and (8) it follows that

$$u(x) = \frac{C_0}{g(x)} e^{-f(x)} + u_{st}(x), \quad (9)$$

$$C_0 = u(x_0)\, g(x_0) e^{f(x_0)}.$$

After a time 2m (i.e., after n modulation periods) the magnitude of the signal will be

$$u(x + 2\pi n) \quad (10)$$

$$= \frac{C_0}{g(x)} \exp\{-f(x) - 2\pi n\nu\} + u_{st}(x).$$

From Eqs. (9) and (10) we have

$$\nu = \frac{1}{2\pi n} \ln\left[\frac{u(x) - u_{st}(x)}{u(x + 2\pi n) - u_{st}(x)}\right]. \quad (11)$$

If $\nu \to 0$ then from Eqs. (5) and (6) $u_{st}(x) \to 0$ and

$$\nu = \frac{1}{2\pi n} \ln\left[\frac{u(x)}{u(x + 2\pi n)}\right]. \quad (12)$$

To make practical use of this scheme, the screen of the oscilloscope can be photographed with exposure times equal to 3-5 modulation periods. Using Eqs. (11) and (12) the value of T_1 can be determined from the ratio of the amplitudes of several pulses which appear in sequence on the photograph.

In Ref. 10 a detailed analysis of the shape of the dispersion signal in the transient state was given for symmetrical modulation with $\nu \to 0$. It has been indicated above that the conditions for which Eq. (8) is valid are such as to make the transient and steady state solutions identical. Under these conditions an analysis of Eq. (8), similar to that which was carried out in Ref. 10, indicates that the dispersion signal is bell-shaped and asymmetrical. We now introduce the following notation: x_m is the time which corresponds to $u(x_m) = u_{\max}$; x_a and x_b indicate the instants at which $u(x) = 0.5\, u_{\max}$ ($x_a < x_m$, $x_b > x_m$). For $\nu < \mu$ we have

$$\frac{1}{k} = 0,144\Delta\left[1+\sqrt{1-3,74\,\frac{\varepsilon^2}{\Delta^2}}\right]; \qquad (13)$$

$$\frac{1}{k} = 0,289\Delta\left[1-0,935\,\frac{\varepsilon^2}{\Delta^2}\right] \quad \text{for } \frac{\varepsilon}{\Delta} < 0.2; \qquad (14)$$

$$x_m^2 = 0,461\,\frac{\Delta}{k}\left[1-\frac{3,464}{\Delta k}\right], \qquad (15)$$

$$|x_m| = 0,353\varepsilon\left[1-\frac{3,115}{k^2}\right];$$

$$\mu = -{}^1/_2 k^2 \sin 2x_m, \qquad (16)$$

where $\Delta = x_a + x_b$ is the half-width of the signal and $\epsilon = (|x_a| - |x_m|) - (x_b + |x_m|)$ is a measure of the signal asymmetry.

As in Ref. 10, using Eqs. (13)-(16), it is possible to determine the transverse relaxation time T_2 from a knowledge of the half-width and asymmetry of the signal; also, if the strength of the modulating field H_m is known, the amplitude of the high-frequency magnetic field H_1 can be found. We may note that this method can be used only when the high-frequency magnetic field H_1 is sufficiently homogeneous over the region of the sample; if there are any homogeneities, Δ and ϵ become average values determined by the different environmental conditions in the various parts of the sample.

2. ASYMMETRICAL MODULATION

Experimentally it is necessary to vary the time interval between the signal pulses which occur as the magnetic field is modulated[5,11]. Under these conditions the magnitude of the dc magnetic field differs from the resonance value, i.e., H_z is of the form

$$H_z = H_0 + H_m(\varkappa + \sin x), \qquad (17)$$

in which $\varkappa$ can have either sign depending on the sign of H_c.

For asymmetrical modulation the shape of the dispersion signal is given by Eq. (2). We are interested only in the steady-state solution. In order to find this solution it is necessary to compute the integral $I = H(\eta) - H(x)$ [cf. Eq. (2)].

Using certain approximations it is possible to find I. It follows from Eq. (17) that the magnitude of the displacement term H_c must be smaller than the modulation amplitude H_m. Since it is always true that $\varkappa \leq 1$ we may make the substitution $\varkappa = \sin\psi$. It also follows from Eq. (17) that the resonance conditions obtain when $x_{\text{res}} \cong -\psi$. We divide I into two integrals $I = I_1 + I_2$, where

$$I_1 = \int_x^{x-\Delta} [G(\xi)]^{-2}\, d\xi; \quad I_2 = \int_{x-\Delta}^{\eta} [G(\xi)]^{-2} d\xi. \qquad (18)$$

If the half-width of the signal Δ is small, then I_1 can be computed by expanding $\sin x$ in a series about the resonance value.

$$I_1 = \frac{1}{p}\,\operatorname{arctg}\,[p(\xi+\psi)]\,\Big|_x^{x-\Delta}, \qquad (19)$$

where $p = k\cos\psi$.

Substituting I_1 and I_2 in Eq. (2), dividing the integral $\int_{-\infty}^{x}$ into two parts as in the symmetrical case and keeping only the part which is of importance in the resonance region, we get

$$u_{\text{st}}(x) = u_{\text{st}}(x_\tau)\,\frac{G(x_\tau)}{G(x)}\,\exp\{F(x_\tau) - F(x)\}, \qquad (20)$$

where

$$F(x) = \nu x + \frac{\mu-\nu}{p}\,\operatorname{arctg}\,[p(x+\psi)].$$

Comparing Eqs. (8) and (20), we see that the more exact symmetrical-modulation solution (8) will have the same form as (20) when $\tan x \cong x$, $\sqrt{1+k^2} \cong k$, i.e., for sufficiently large values of k and small values of x. In the asymmetrical-modulation case the role of the parameter k is played by the quantity p.

Using the analysis developed in Ref. 10 we can now examine Eq. (20); when $\nu < \mu$ (the notation is the same as that used in the symmetric-modulation case) we have:

$$1/p = 0,144\Delta\,[1+\sqrt{1-3,74(\varepsilon^2/\Delta^2)}]; \qquad (21)$$

$$1/p = 0.289\Delta\,[1-0.935\varepsilon^2/\Delta^2] \qquad (22)$$

$$\text{for } \varepsilon/\Delta < 0,2;$$

$$(x_m+\psi)^2 = 0,461\,\frac{\Delta}{p}\,[1-3,464/\Delta p], \qquad (23)$$

$$|x_m+\psi| = 0,353\varepsilon;$$

$$\mu = -p^2(x_m+\psi). \qquad (24)$$

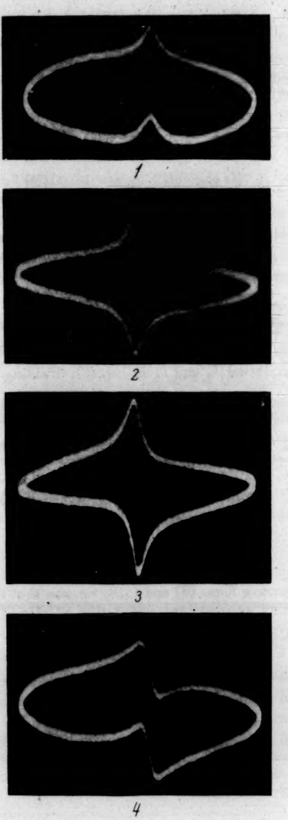

FIG. 2. The change in the shape of the dispersion signal as the quantities ν and μ are varied. These pictures were obtained with water solutions as follows: *1* – 0.001 molar $CuSO_4$, *2* -- 0.02 molar $Fe(NO_3)_3$, *3* – molar $CuSO_4$, *4* – 2 molar $Fe(NO_3)_3$; f_m = 50 cps, H_m = 19 gauss, H_1 = 1 gauss. The values of ν and μ are, respectively: *1* -- 0.0035 and 6.9; *2* -- 0.57 and 10; *3* -- 3.7 and 17; *4* -- 59 and 78.

Equations (21)-(24) can be used to determine the transverse relaxation time T_2 and the amplitude of the high-frequency magnetic field H_1 when the signal pulse does not fall at the center of the oscilloscope pattern. Comparing Eqs. (21)-(24) with Eqs. (13)-(16), we see that the expressions are identical up to terms of the order of k^{-2}.

An estimate of the limits of applicability of Eqs. (21)-(24) shows that these expressions are accurate to within 10 per cent for values up to $\varkappa = 0.65$ ($k = 20$) and up to $\varkappa = 0.5$ ($k = 10$).

It is still necessary to verify that the expressions given above for symmetric modulation Eqs. (13)-(16) and asymmetrical modulations Eqs. (21)-(24) are being used in cases for which the experimental values of μ and ν lie within the limits of the regions shown in Fig. 11. We may note that in asymmetrical modulation the parameter p should be used instead of k in Fig. 1 and the regions $\mu(\nu)$ should be compressed as $\varkappa$ is increased.

EXPERIMENTAL RESULTS

The theoretical results obtained in Secs. 1 and 2 were checked with an electrically compensated crossed-coil setup. Proton resonances were studied in all cases. The experiments were performed in distilled water, aqueous solutions of varying concentrations of $CuSO_4$ and of $Fe(NO_3)_3$, paraffin, glycerin and various types of synthetic rubber.

It has been shown above that the form and magnitude of the dispersion signal for a strong high-frequency magnetic field are determined by the values of the quantities T_1, T_2, H_1, H_m, ω_m and H_c. If for a given value of k, the values of ν and μ lie within the regions shown in Fig. 1, then the signal is bell-shaped and is given by Eqs. (8) and (20). On the other hand, if the value of ν corresponding to this value of k is outside the region, then the effect of the quantity $J(x, x_\tau)$ becomes important and the curve is distorted.

The oscilloscope photographs shown in Fig. 2 illustrate the change which takes place in the dispersion signal when ν and μ are increased. These photographs were made with aqueous $CuSO_4$ solutions and $Fe(NO_3)_3$ solutions under the following conditions: $f_m = 50$ cps, $H_m = 19$ gauss and $H_1 = 1$ gauss. The corresponding values of ν and μ are given for each photograph. The theoretical predictions regarding the dependence of the signal-shape on ν and μ are borne out in these photographs.

To verify the functional dependence of the half-width Δ on k, given in Eqs. (14) and (22), we found the experimental dependence of this quantity

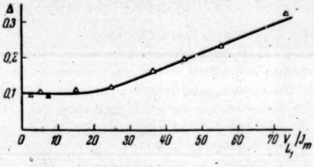

FIG. 3. The signal half-width Δ as a function of the ratio of the high-frequency voltage in the driving coil to the modulation current; this ratio is proportional to $1/k$.

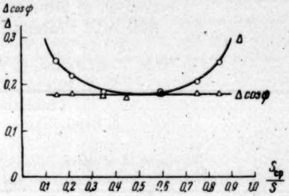

FIG. 4. The signal half-width Δ as a function of position on the oscilloscope pattern.

$$[S_{\mathrm{rel}}/S = 1/2(1+\varkappa)].$$

on the ratio V_{L1}/J_m, which is proportional to $1/k$ (V_{L1} is the amplitude of the voltage in the driving coil and is proportional to H_1. J_m is the current in the modulating coil and is proportional to H_m [13]). This dependence is shown in Fig. 3 which was obtained using a 0.064 molar solution of $CuSO_4$ with $\nu = 0.26$, $\mu = 7.1$ and $\epsilon/\Delta < 0.2$.

The strong-field case ($\lambda < 1$), which is the subject of this paper, is realized experimentally when $V_{L1}/J_m > 30$. It is apparent from Fig. 3 that for $V_{L1}/J_m < 25$ the half-width Δ increases in proportion to $V_{L1}J_m$; this is in agreement with the theoretical prediction. The region $V_{L1}/J_m < 10$, in which the quantity Δ remains constant, corresponds to the weak-field case ($\lambda > 1$); here, the width of the signal, as is well known[3,4,14], is determined by the transverse relaxation time T_2 and the modulation rate is independent of the strength of the high-frequency magnetic field H_1.

An experimental investigation was also undertaken to determine the dependence of the half-width Δ on the position of the signal on the oscilloscope pattern in the asymmetrical-modulation case.

A typical curve is shown in Fig. 4 which was obtained using a 0.064 molar solution of $CuSO_4$ with $f_m = 50$ cps, $H_m = 19$ gauss and $H_1 = 1$ gauss. It is evident from this figure that the quantity $\Delta \cos \psi$ remains constant to within 5 percent; thus, within experimental errors, the changes in the signal-width owing to variation in position on the oscilloscope pattern are given correctly by Eq. (22).

In conclusion, the author wishes to thank Professor S. D. Gvozdover for valuable advice in the course of this work and also E. G. Pozniakii, A.V. Luk'ianov and T. M. Cherkasov for undertaking the numerical integrations.

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Production of Slow π^+-Mesons in Photographic Emulsion Nuclei by 460 MEV Protons and Neutrons of 400 MEV Effective Energy

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The method of an emulsion chamber was used to investigate the production of charged mesons by 460 mev protons and neutrons of 400 mev effective energy.

INTRODUCTION

THE purpose of this research was the investigation of the production characteristics of slow π-mesons in photographic emulsion nuclei by 460 mev protons and neutrons of 400 mev effective energy, the analysis of stars accompanying the production of mesons and their comparison with stars not containing π-mesons. The use of the emulsion chamber technique developed by one of the authors[1] enabled one to follow the complete path of produced π-mesons from the place of their ionization terminal to the place of production, and thus gave the opportunity to observe a large number of stars with the production of charged π-mesons and to plot angular and energy distributions of π-mesons in the region of low energies.

There are many papers in the literature devoted to the study of emulsion stars produced by high energy nucleons. From the results of these papers there were obtained the fundamental characteristics of stars in agreement with the theory of nucleonic cascade, which is generated when a fast nucleon is captured by a nucleus. The investigation of stars with the production of mesons of opposite sign can give additional information about the nucleonic cascade.

1. PROCEDURE

The methodology of the emulsion chamber, previously described in detail[1], was used in the investigation. Each of the two utilized emulsion chambers consisted of 20 layers of photographic emulsion of 6 mm total thickness and 42 mm in diameter, placed in a specially constructed thin-walled cassette (plate-holder). The chambers were exposed to beams of fast neutrons and protons brought out of the synchrocyclotron of the USSR Academy of Sciences' Institute for Nuclear Problems; one chamber in a beam of neutrons with a 400 mev effective energy for the production of mesons--the other in a 460 mev proton beam.

In the investigation, emulsions prepared by D.M. Samoilovich were used, permitting the effective recording of proton tracks with energies up to 300 mev. The relationship of range-energy for these emulsions was calculated from the known percentage composition of their constituents (85% AgBr) and verified by measuring the average path of the μ-mesons, produced during the $\pi \rightarrow \mu$ decay ($\overline{R}_\mu = 590\,\mu$). The emulsion shrinkage coefficient was determined by comparing the thickness of the emulsion chamber before and after development, and also by comparing the average track lengths of μ-mesons of the $\pi \rightarrow \mu$ decay located in the plane of the emulsion and making with it an angle of nearly 90°.

The scanning of the emulsion layers was done with MBI-2 microscopes under a total magnification of 225×; the measurements under a magnification of 2025×. While scanning, the ends of π-mesons were recorded and "starless" π-mesons were identified by the grain density gradient and by the characteristic "willowiness" of the track determined by multiple scattering. Special attention was paid to the observer's accuracy in reporting the relationship between the numbers of π^+- and π^--mesons in the emulsion. As shown by special check, the effectiveness of recording the π-meson ends was nearly 100%.

The π^+ and π^--mesons found by this method were then followed to the place of their production or to the place where they emerged from the emulsion chamber. In order to facilitate the tracing of the tracks of charged particles while passing from one layer to the next, a roentgen grid coordinate system with 30-35 μ lines, 3 mm apart was superimposed on the emulsion layers.

The chambers used in the investigation allowed the study of production characteristics of π-mesons with energies up to 40 mev.

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* Deceased.

** This communication is based on the results of research carried out in 1952-1953.

2. THE ENERGY AND ANGULAR DISTRIBUTIONS OF PRODUCED π-MESONS

As a result of scanning the emulsion layers and following the tracks of π^+- and π^--mesons in neighboring layers, there were found 193 cases of π^+- and π^-- meson production by neutrons (185 π^-- and 8 π^+-mesons) and 121 cases of π^+- and π^-- meson production by protons (63 π^+- and 58 π^--mesons).

The correct notion of energy and angular distribution of produced mesons requires edge effect corrections determined by the finite dimensions of the emulsion chamber. To calculate the edge effect each recorded case of a π-meson production was assigned to coefficient $\varkappa$ expressing the statistical weight of a meson with the given production characteristics. The value of the coefficient $\varkappa$ ($\varkappa = 1/p$, where p is the probability of recording a given meson in the emulsion chamber) was determined from the path length of the meson and its orientation in the emulsion chamber and was calculated for each case of π-meson production by means of specially constructed nomograms (see Appendix).

The energy distribution of π^+- and π^--mesons produced by protons and π^--mesons produced by neutrons is shown in Fig. 1. (The energy distribution of π^+-mesons produced by neutrons is not given because the number of cases found was small.) It is seen from the Figure that the spectra of π^+-mesons produced by protons are comparable within experimental error to the spectra of π^+-mesons produced by neutrons, as is to be expected from the principle of charge symmetry. The displacement of the spectra (ΔE = 15 mev) can be explained by the action of the nuclear Coulomb field displacing the spectra 7.5 mev in opposite directions. It is worth noting that the 7.5 mev value of the shift agrees with the value of the Coulomb barrier height for photoemulsion nuclei. The spectrum of π^--mesons produced by protons is characterized by a weaker dependence on energy in comparison with the spectra of π^+-mesons produced by protons and π^--mesons produced by neutrons.

The angular distributions of π^+- and π^--mesons produced by protons and π^--mesons produced by neutrons are shown in Fig. 2. It can be seen that in accordance with the principle of charge symmetry, the angular distributions of π^+-mesons produced by protons are similar to those of π^--mesons produced by neutrons. It is difficult to give a more detailed interpretation of the results obtained for the angular distribution of produced π-mesons because a substantial part of the energy spectrum in the region of small energies may be determined by mesons which have lost their energy at the expense of scattering by nucleons within the nucleus.

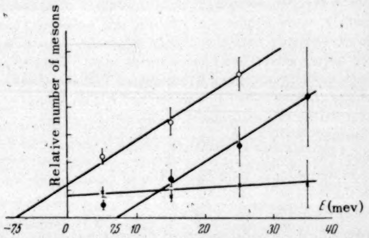

FIG. 1. The energy distribution of π-mesons produced by 460 mev protons and neutrons of 400 mev effective energy in photoemulsion nuclei. ○–π^--mesons, produced by neutrons; ●–π^+-mesons produced by protons; ×–π^--mesons produced by protons.

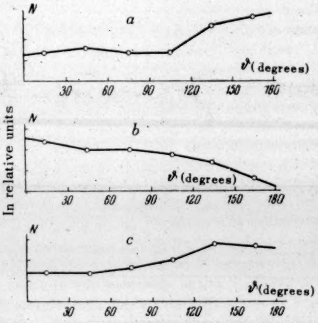

FIG. 2. The angular distributions of π-mesons produced by 460 mev protons and neutrons of 400 mev effective energy. a–π^--mesons produced by neutrons; b–π^--mesons produced by protons; c–π^+-mesons produced by protons.

3. THE RELATIONSHIP BETWEEN THE NUMBERS OF PRODUCED π^+- AND π^--MESONS. PRODUCTION CROSS SECTION.

The calculation of the number of small energy π^+-

and π^--mesons with corrections for the edge effect gives for the π^-/π^+ ratio in the case of meson production by neutrons a value of $(\pi^-/\pi^+)_n = 17 \pm 8$ and for the ratio π^+/π^- in the case of meson production by protons a value of $(\pi^+/\pi^-)_p = 2.2 \pm 0.7$. Since the photoemulsion nuclei consist of approximately equal numbers of protons and neutrons, then in accordance with the charge symmetry principle, the above ratios must be in agreement. The experimentally observed discrepancy in the values of the ratios $(\pi^-/\pi^+)_n$ and $(\pi^+/\pi^-)_p$ can be explained by the difference in the action of the Coulomb field on the energy spectra of π^+- and π^--mesons.

In accordance with the value of Coulomb shift in the energy spectra of π-mesons, $E = 15$ mev, established in Sec. 2, π^+-mesons in the energy interval of 15 to 30 mev and π^--mesons in the energy interval 0 to 15 mev were used to calculate the ratios $(\pi^-/\pi^+)_n$ and $(\pi^+/\pi^-)_p$, corrected for the Coulomb shift. With this there were obtained values of the $(\pi^-/\pi^+) = 3^{+3}_{-1.5}$ and $(\pi^+/\pi^-)_p = 2.5 \pm 0.5$ ratios which agreed within error limits; the large error in the ratio $(\pi^-/\pi^+)_n$ is associated with the small numbers of recorded cases.

The production cross section of π^+- and π^--mesons of energies under consideration ($E_\pi < 40$ mev) produced by 460 mev protons in photoemulsion nuclei were obtained by comparing the number of produced π-mesons in a definite emulsion volume with the proton beam density. The density of the proton beam was determined visually by counting with a microscope the number of tracks. With the emulsion used it was possible to make these measurements quite reliable for the horizontal tracks of the 460 mev proton beam.

The number of mesons N_0^π produced in 1 cm^3 of emulsion is determined by the expression $N_\sigma^\pi = \bar{\varkappa} N^\pi/V$, where N^π is the number of recorded π-mesons ends, V is the volume of the scanned emulsion, $\bar{\varkappa}$ is the multiplier, which takes into account the edge effect and is determined by the conditions $\bar{\varkappa} = \Sigma \varkappa/N^\pi$. The production cross section for π^--mesons is equal to:

$$\sigma = N_0^\pi / c\ I_p, \tag{1}$$

where c is the number of atoms in 1 cm^3 of emulsion, I_p is the proton current through 1 cm^2 of the photoemulsion.

Using Eq. (1), the mean cross sections of π-meson production in photoemulsion nuclei in the energy interval $0 < E_\pi < 40$ mev were found to be equal to:

$$\sigma_{\pi^+} = (2.9 \pm 0.9)\ 10^{-27}\ \text{cm}^2;$$

$$\sigma_{\pi^-} = (1.3 \pm 0.5)\ 10^{-27}\ \text{cm}^2;$$

$$\sigma_{\pi^+ + \pi^-} = (4.2 \pm 1.4)\ 10^{-27}\ \text{cm}^2.$$

Another evaluation of π-meson production cross section was made by comparing the number of produced π-mesons with the number of stars produced by protons in the same emulsion volume, on the assumption that the star production cross section is equal to $\sigma_{\text{star}} = 0.58\,\sigma_{\text{geom}}$ and $\sigma_{\text{geom}} = 0.6 \times 10^{-24}$ cm^2 (the calculation made for the emulsion used). In this case the cross section had a value

$$\sigma(\pi^+ + \pi^-) = (4.9 \pm 1{,}5)\ 10^{-27}\ \text{cm}^2$$

agreeing with the first result within error limits.

The determination of the π^--meson production cross section by neutrons was made by comparing the number of produced mesons with the number of stars produced by neutrons, with the same assumptions concerning σ_{star} and σ_{geom}. Moreover, the taking into account of the nature of the neutron spectrum and the dependence of π-mesons production cross section on neutron energy, leads one to the conclusion that in practice monochromatic neutrons with an effective energy $E_n = (400 \pm 40)$ mev are involved in meson production. The value of the cross section of π^--mesons by neutrons in photoemulsion nuclei was found to be equal to:

$$(\sigma_{\pi^-})_n = (12 \pm 8)\ 10^{-27}\ \text{cm}^2$$

The large error is associated with the uncertainty in the neutron spectrum in the region of small energies.

From a comparison of $(\sigma_{\pi^-})_n$ with $(\sigma_{\pi^+})_p$ it is seen that $(\sigma_{\pi^-})_n > (\sigma_{\pi^+})_p$, although $E_p > E_n$. This is associated with the action of the nuclear Coulomb field which displaces differently the spectra of π^+- and π^--mesons.

4. ANALYSIS OF STARS ACCOMPANYING THE PRODUCTUON OF π-MESONS

In analyzing stars accompanying the production of π-mesons, the energy of the particles was determined with the aid of calibration graphs $E = f(R)$ (R--stops in flight of particles) for the paths terminating within one emulsion layer, and from the grain density $E = f(dN/dR)$ for the tracks of particles leaving a given emulsion layer. Such a treatment of the rays permitted the plotting of a rough curve for particles ejected from stars accompanying π-meson production.

The curve $E = f(dN/dR)$ was obtained experimentally for π-mesons and recalculated for protons by the formula

$$\frac{dN}{dR}(E_p) = \frac{dN}{dR}\left(\frac{E_\pi M_p}{M_\pi}\right)$$

In the current investigation, the particles ejected from stars were not identified and their energy was calculated from the curves of $E(R)$ or $E(dN/dR)$ for protons. The errors arising from such consideration were taken into account during the analysis of experimental data.

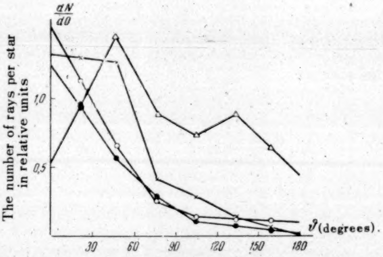

FIG. 3. The angular distribution of slow particles ($E < 30$ mev), ejected from stars under the action of 460 mev protons and neutrons of 400 mev effective energy. O – stars with the production of π^--mesons by neutrons; ● – stars with the production of π^+-mesons by protons; × – stars with the production of π^--mesons by protons; Δ – stars without production of π-mesons by protons.

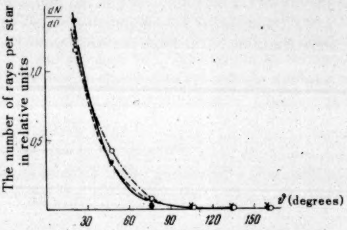

FIG. 4. The angular distribution of fast particles ($E > 30$ mev) ejected from stars with the production of mesons by 460 mev protons and neutrons of 400 mev effective energy. × – stars with π^--mesons produced by neutrons; ● – stars with π^+-mesons produced by protons; O – stars with π^--mesons produced by protons.

The angular distributions and the magnitude of the anisotropy $[N(0\text{-}90°) - N(90\text{-}180°)]/[N(0\text{-}90°) + N(90\text{-}180°)]$ of slow and fast particles ejected from photoemulsion nuclei by the action of fast nucleons are shown in Figs. 3 and 4 and in Table I.

As can be seen from Fig. 3 and Table I, the angular distribution of particles with energies $E < 30$ mev is peaked forward, which indicates the presence of a large number of slow cascade particles, their number being especially large in stars accompanying the production of mesons[2].

The value obtained for the anisotropy of "mesonless" stars is in good agreement with the measurements of other authors. Bernardini *et al.*[2] obtained a value of 0.25 for the anisotropy in the angular distribution of "black rays", in the case of stars produced by 400 mev protons; Blau *et al.*[3], for stars produced by 300 mev neutrons, 0.33.

TABLE I.

The value of anisotropy of slow ($E \leq 30$ mev) and fast ($E > 30$ mev) particles.

Type of Star	(p, π^+) $E_p = 460$ mev	(n, π^-) $\bar{E}_n = 400$ mev	(p, π^-) $E_p = 460$ mev	Stars without π-meson
Anisotropy Value:				
Slow Particles	0.61 ± 0.09	0.46 ± 0.06	0.47 ± 0.08	0.25 ± 0.06
Fast Particles	0.96 ± 0.04	0.80 ± 0.05	0.96 ± 0.04	—

The angular distribution of particles with energies $E > 30$ mev in stars accompanying the production of π-mesons is shown in Fig. 4. As was to be expected, these particles are ejected mostly in the forward direction, since they are mainly cascade particles, Unfortunately, the insufficient sensitivity of the emulsion used in the investigation (300 mev for protons) did not permit a reliable analysis of stars which are not accompanied by the production of π-mesons. The mean values for the numbers of rays per star which accompany the production of π^+- and π^--mesons are given in Table II; therein are also the results of Bernardini *et al.*[2] for stars not accompanying the production of mesons.

TABLE II. The mean number of rays per star.

Star Type	Mean number of rays per star	Mean number of fast particles ($E > 30$ mev) per star	Mean number of slow particles ($E < 30$ mev) per star
(n, π^-) . .	2.9 ± 0.1	0.9 ± 0.07	2.0 ± 0.1
(p, π^-) . .	3.9 ± 0.3	0.9 ± 0.1	3.0 ± 0.2
(p, π^+) . .	2.4 ± 0.2	0.8 ± 0.1	1.6 ± 0.2
$(p$, without $\pi)$	3.4 ± 0.2[2]	0.85 ± 0.07[2]	2.5 ± 0.2 [2]

It can be seen from Table II that the number of fast ($E > 30$ mev) charged particles in stars is the same for the various methods of meson production. If one assumes that the production of mesons takes place through nulceon-nucleon interaction, then the obtained results indicate the large overcharge of fast nucleons. The number of slow particles for the considered cases of π-meson production varies and is thus characterized by the fact that the slow particles compensate for the change in nuclear charge brought in by the primary protons and by the produced π^+- or π^--meson.

The general nature of the energy spectra of particles ejected from stars accompanying the meson production is illustrated by Fig. 5, where it is seen that the curves coincide in the regions of high energy and diverge regularly in the region of small energies. The calculation of the mean energy carried out by charged particles of the star gives a value of approximately 100 mev, independent of the meson producing reaction type. About 75 percent of the total energy is taken out by fast ($E > 30$ mev) particles.

If one assumes that for the stars under consideration the energy removed by the neutrons is equal to the total energy of the charged particles, then the full energy appearing in the star E_Σ is approximately equal to the energy of the nucleon which produced the star $E_\Sigma = 2(E_{\text{mean}} + QN_{\text{mean}}) + \epsilon_\pi = 2(104 + 9.4) + 160 = 440$ mev. Here E_{mean} is the mean energy of the charged particles in the star, Q is the binding energy of the nucleon in the nucleus, N_{mean} is the mean number of charged particles in the star and ϵ_π is the total energy of the meson.

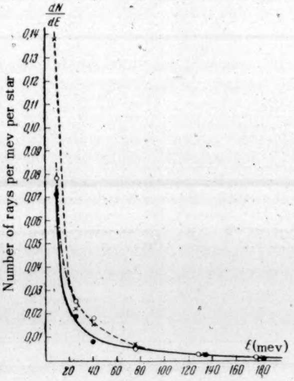

FIG. 5. Energy distributions of particles ejected from stars accompanying the production of π-mesons by 460 mev protons and neutrons of 400 mev effective energy. × -- stars with π^--mesons produced by protons; ● -- stars with π^+-mesons produced by protons; O -- stars with π^--mesons produced by neutrons.

In this investigation there were also measured a number of fast protons ejected from stars accompanying the production of π^+- and π^--mesons for the cases when the produced proton is ejected forward. Obviously, on the assumption of the nucleon-nucleon mechanism of π-meson production, the production of π^+- and π^--mesons by protons and neutrons must be accompanied by a different number of protons in the final state. If it is assumed that the mesons are produced on the nucleus surface, then the above-mentioned difference in the number of fast protons accompanying the production of π-mesons must be especially noticeable in the case where the produced π-meson is ejected forward. However, as follows from Table III, the number of fast protons ejected from stars is the same for the different methods of meson production.

TABLE III. Mean number of fast protons for stars accompanying the production of π-mesons ejected forward.

Production Reaction	Mean number of protons with $E > 30$ mev
(p, π^+)	0.76 ± 0.17
(p, π^-)	0.74 ± 0.11
(n, π^-)	0.86 ± 0.11

This result can be explained by the relatively large probability of π-meson production with a consequent overcharge of the nucleons participating in the reaction. The obtained results cannot be explained by assuming a strong overcharge of cascading nucleons in the nucleus up to the instant of π-meson production. Actually, with this assumption, the spectra of π^+- and π^--mesons should be identical, which is contradicted by experiment.

In conclusion, the authors express sincere thanks to Prof. M. G. Meshcheriakov and Prof. Dzhelepov for the opportunity to carry out these experiments with the synchrocyclotron of the Institute for Nuclear Problems of the Academy of Sciences, USSR, and to D. M. Samoilovich for the preparation of photoemulsions, and also to V. V. Matveev and B. B. Liubimov for assistance with the investigation.

APPENDIX

EDGE EFFECT CORRECTIONS

In order to get the correct idea of the energy and angular distribution of the produced π-mesons, corrections had to be introduced for the edge effect determined by the terminal dimensions of the emulsion chamber. We shall consider separately the edge effect associated with the finite thickness of the emulsion chamber and the edge effect determined by the finite thickness and diameter of the emulsion chamber.

a) *The edge effect associated with the finite thickness of the emulsion chamber.* Consider the mesons produced at some point O and having a given path l and given angle ϑ (angle relative to the direction of the proton beam). Obviously, all such mesons lie in the surface of a cone with a vertex angle θ and generatrix l (Fig. 6 *a*). Moreover, if one considers angle φ (in the plane of cone generation), then it is clear from physical considerations that the mesons produced with a given l and θ must be uniformly distributed relative to this angle. However, the finite emulsion chamber thickness h leads to the conclusion that of all these mesons only those will be recorded whose ends lie on the arcs of a circle with radius $R = l \sin\theta$. The correction coefficient k is equal to the ratio of the circumference to the above-mentioned arcs.

$$k = \frac{\pi}{\varphi_1 + \varphi_2} = \frac{2k_1k_2}{k_1 + k_2}, \tag{2}$$

where $k_1 = \pi/2\varphi_1$ and $k_2 = \pi/2\varphi_2$ are the correction coefficients for mesons ejected into the upper and lower hemisphere, respectively. In practice, the corrections for the edge effect connected with the finite emulsion chamber thickness were made for each meson (with a given l, ϑ and φ) individually with the aid of the nomogram shown in the Fig. 6*b* and enabling one to find the coefficients k_1 and k_2.

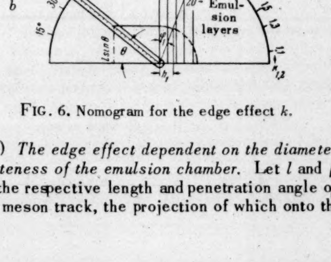

FIG. 6. Nomogram for the edge effect k.

b) *The edge effect dependent on the diameter finiteness of the emulsion chamber.* Let l and β be the respective length and penetration angle of the meson track, the projection of which onto the emulsion plane has a given direction **n** (Fig. 7*a*). Then it is evident that of the mesons with the given direction **n** only those will be recorded which are produced within the limits of the cross-hatched area S_1. Since the above consideration does not depend on the choice of the direction **n**, then for all the tracks of length l and angle β the edge effect correction dependent on the finiteness of the chamber diameter will be equal to the ratio of the circle area S to the cross-hatched area S_1.

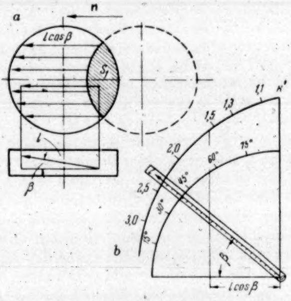

FIG. 7. Nomogram for the edge effect k'.

$$k' = S / S_1.$$

The determination of the value of k' was made for each track individually with the aid of the nomogram shown in Fig. 7*b*.

In this manner the true number of produced mesons can be calculated if to each located π-meson one assigns a statistical weight $\varkappa$ equal to the product of the coefficients k and k':

$$\varkappa = kk'.$$

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The Theory of a Fermi Liquid

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A theory of the Fermi liquid is constructed, based on the representation of the perturbation theory as a functional of the distribution function. The effective mass of the excitation is found, along with the compressibility and the magnetic susceptibility of the Fermi liquid. Expressions are obtained for the momentum and energy flow.

AS is well known, the model of a Fermi gas has been employed in a whole series of cases for the consideration of a system of Fermi particles, in spite of the fact that the interaction among such particles is not weak. Electrons in a metal serve as a classic example, Such a state of the theory is unsatisfactory, since it leaves unclear what properties of the gas model correspond to reality and what are intrinsic to such a gas.

For this purpose we must keep in mind that the problem is concerned with definite properties of the energy spectrum ("Fermi type spectrum"), for whose existence it is necessary, but not sufficient that the particles which compose the system obey Fermi statistics, i.e., that they possess half-integer spin. For example, the atoms of deuterium interact in such a manner that they form molecules. As a result, liquid deuterium possesses an energy spectrum of the Bose type. Thus the presence of a Fermi energy spectrum is connected not only with the properties of the particles, but also with the properties of their interaction.

A liquid of the Bose type was first considered by the author of the present article in application to the properties of He II. It follows from the character of the spectrum of such a liquid that a viscous liquid of Bose particles necessarily possesses superfluid properties. The converse theorem that a liquid consisting of Fermi particles cannot be superfluid, in accord with the above, is in general form not true.

1. THE ENERGY AS A FUNCTIONAL OF THE DISTRIBUTION ENERGY

If we consider a Fermi gas at temperatures which are low in comparison with the temperature of degeneration, and introduce some weak interaction between the atoms of this gas, then, as is known, the collision probability for a given atom, which is found in the diffuse Fermi zone, is proportional not only to the intensity of the interaction, but also to the square of the temperature. This shows that for a given intensity of interaction, the "indeterminacy of the momenta", associated with the finite path length, is also small for low temperatures, not only in comparison with the size of the momentum itself, but also in comparison with the width of the Fermi zone, proportional to the first power of the temperature.

As a basis for the construction of the type of spectrum under consideration, is the assumption that, as we gradually "turn on" the interaction between the atoms, i.e., in the transition from the gas to the liquid, classification of the levels remains invariant. The role of the gas particles in this classification is assumed by the "elementary excitations" (quasi-particles), each of which possesses a definite momentum. They obey Fermi statistics, and their number always coincides with the number of particles in a liquid. The quasi-particle can, in a well-known sense, be considered as a particle in a self-consistent field of surrounding particles. In the presence of a self-consistent field, the energy of the particle depends on the state of the surrounding particles, but the energy of the whole system is no longer equal to the sum of the energies of the individual particles, and is a functional of the distribution function.

We consider an infinitely small change in the distribution function of quasi-particles n Then we can write down the change in the energy density of the system in the form

$$\delta E = \int \varepsilon \delta n d\tau, \tag{1}$$

where $d\tau = dp_x dp_y dp_z / (2\pi\hbar)^3$. The quantity $\epsilon(p)$ is a function of the derivative of the energy with respect to the distribution function. It corresponds to a change in the energy of the system upon the addition of a single quasi-particle with momentum p, and it can be regarded as the Hamiltonian function of the added quasi-particle with given momentum in the self-consistent field.

However, we have not taken it into account in Eq. (1) that the particles possess spin. Since the spin is a quantum mechanical quantity, it cannot be considered by classical means. We must therefore consider the distribution function of the statistical matrices in regard to spin, and replace Eq. (1) by the following:

$$\delta E = \mathrm{Sp}_\sigma \int \varepsilon \delta n d\tau, \tag{2}$$

where Sp_σ is the spur over the spin states. The quantity ϵ in the general case is also an operator which depends on the spin operators. If we have an equilibrium liquid, which is not in an external magnetic field, then, because of isotropy, the energy cannot depend on the spin operators. We limit ourselves to the consideration of particles with $s = \frac{1}{2}$.

We can show that just this energy ϵ enters into the formula for the Fermi distribution of the quasi-particles. Actually, it is reasonable to determine the entropy of the liquid by the following way:

$$S = -\mathrm{Sp}_\sigma \int \{n \ln n + (1-n)\ln(1-n)\}\, d\tau. \tag{3}$$

By means of a variation, subject to the additional conditions

$$\delta N = \mathrm{Sp}_\sigma \int \delta n d\tau = 0, \quad \delta E = \mathrm{Sp}_\sigma \int \varepsilon \delta n\, d\tau = 0,$$

we can obtain the Fermi distribution

$$n(\varepsilon) = [e^{(\varepsilon-\mu)/\Theta} + 1]^{-1}. \tag{4}$$

from this equation. We note that ϵ, being a functional of n, naturally depends on the temperature also.

In correspondence with (4) the heat capacity of a Fermi liquid at low temperatures will be proportional to the temperature. It is determined by the same formula as for the Fermi gas, with one exception, that in place of the real mass m of the particles therein, we place the effective mass of the quasi-particles

$$m^* = \frac{p}{\partial\varepsilon\, \partial p}\Big|_{p=p_0}, \tag{5}$$

where p_0 is the limiting momentum of the Fermi distribution of quasi-particles at absolute zero.

Not only $\epsilon(p)$ for a given distribution, but also the change in e produced by a change in n, is of essential importance for the theory of the Fermi liquid:

$$\delta\varepsilon(p) = \mathrm{Sp}_{\sigma'} \int f(\mathbf{p}, \mathbf{p}')\, \delta n'\, d\tau'. \tag{6}$$

Being a second variational derivative, the function f is a symmetric relative to p and p$'$; moreover, it depends on the spins.

If the principal distribution n is isotropy, then the function f in the general case contains terms of the form $\varphi_{ik}(\mathbf{p}, \mathbf{p}')\sigma_i\sigma_k'$, where σ_i is the spin operator, and if the interaction is exchange, only terms of the form

$$\varphi(\mathbf{p}, \mathbf{p}')(\sigma\sigma').$$

will appear.

We can consider the function f from the following point of view. The number of acts of scattering of quasi-particles per unit volume per unit time can be written in the form

$$dW = \frac{2\pi}{\hbar}\,|F(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_1', \mathbf{p}_2')|^2\,\delta(\varepsilon_1 + \varepsilon_2 - \varepsilon_1' - \varepsilon_2')\,n_1 n_2 (1 - n_1')(1 - n_2')\,d\tau_1\,d\tau_2\,d\tau_1', \tag{7}$$

where conservation of momentum is assumed: $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_1' + \mathbf{p}_2'$. The quantity f is nothing else but $-F(\mathbf{p}_1, \mathbf{p}_2; \mathbf{p}_1, \mathbf{p}_2)$, i.e., the amplitude of the scattering on 0^0 (with opposite sign). Generally speaking, this amplitude is complex, its imaginary part being determined by the total effective scattering cross section. Inasmuch as we assume that the real acts of scattering are highly improbable, we can neglect the imaginary part.

2. RELATIONS WHICH FOLLOW FROM THE PRINCIPLE OF GALILEAN RELATIVITY

If we deal with a liquid which is not in an external field, then it follows from the principle of Galilean relativity that the momentum arriving at a unit volume must be equal to the density of mass flow*. Inasmuch as the velocity of the quasi-particle is $\partial\epsilon/\partial\mathbf{p}$, and the number of quasi-particles coincides with the number of real particles, we have

$$\mathrm{Sp}_\sigma \int \mathbf{p} n\,d\tau = \mathrm{Sp}_\sigma \int m\,\frac{\partial\varepsilon}{\partial\mathbf{p}}\,n\,d\tau. \tag{8}$$

Therefore, the variational derivatives with respect to n ought to be the same on both sides of this equation. Then

$$\frac{1}{m}\,\mathrm{Sp}_\sigma \int \mathbf{p}\,\delta n\,d\tau = \mathrm{Sp}_\sigma \int \frac{\partial\varepsilon}{\partial\mathbf{p}}\,\delta n\,d\tau + \mathrm{Sp}_\sigma \mathrm{Sp}_{\sigma'} \int \frac{\partial}{\partial\mathbf{p}} f(\mathbf{p}, \mathbf{p}')\,\delta n' n\,d\tau\,d\tau'.$$

Since the quantity δn is arbitrary, we obtain

$$\frac{\mathbf{p}}{m} = \frac{\partial\varepsilon}{\partial\mathbf{p}} + \mathrm{Sp}_{\sigma'} \int \frac{\partial f}{\partial\mathbf{p}'}\,n'\,d\tau' = \frac{\partial\varepsilon}{\partial\mathbf{p}} - \mathrm{Sp}_{\sigma'} \int f\,\frac{\partial n'}{\sigma\mathbf{p}'}\,d\tau' \tag{9}$$

(the left side is understood as the unit matrix in the spins).

If we deal with the isotropic case, then it is sufficient that the Eq. (9) hold for the spurs, i.e.,

$$\frac{\mathbf{p}}{m} = \frac{\partial\varepsilon}{\partial\mathbf{p}} - \frac{1}{2}\,\mathrm{Sp}_\sigma \mathrm{Sp}_{\sigma'} \int f\,\frac{\partial n'}{\partial\mathbf{p}'}\,d\tau'. \tag{10}$$

We note that this formula determines the function ϵ through the quantity f with accuracy to within a constant.

Let us consider Eq. (10) for momenta close to the boundary of the Fermi distribution. For low temperatures, the function $\partial n/\partial p$ will differ slightly from the δ-function. For this reason, we can carry out the integration in Eq. (10) over the absolute value of the momentum, leaving only the integration over the angle. This gives the following relation between the real and the effective masses:

$$\frac{1}{m} = \frac{1}{m^*} + \frac{p_0}{2(2\pi h)^3}\,\mathrm{Sp}_\sigma \mathrm{Sp}_{\sigma'} \int f \cos\theta\,d\Omega. \tag{11}$$

Inasmuch as, in this formula, both of the vector arguments in f correspond to the Fermi surface, the function f depends only on the angle between them.

3. COMPRESSIBILITY OF THE FERMI LIQUID

Let us express the compressibility (at absolute zero) by the more appropriate quantity for us, $\partial\mu/\partial N$. For this purpose, we note that as a consequence of homogeneity, the chemical potential μ depends only on the ratio N/V. Consequently, we have

$$\frac{\partial\mu}{\partial N} = -\frac{V\partial\mu\,\partial V}{N} = -\frac{V^2}{N}\frac{\partial p}{\partial V}. \tag{12}$$

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* This conclusion does not apply in particular to electrons in a metal. For them, p is not the momentum, but the quasi-momentum.

For the square of the velocity of sound, we have

$$c^2 = \frac{\partial p}{\partial (mN/V)} = \frac{1}{m}\left(N\frac{\partial\mu}{\partial N}\right). \qquad (13)$$

Thus the problem reduces to the calculation of the derivative $\partial\mu/\partial N$. Inasmuch as $\mu = \epsilon(p_0) = \epsilon_0$, the change in the chemical potential $\delta\mu$ which is brought about as a result of the change in the total number of particles δN, will be equal to[*]

$$\delta\mu = \frac{1}{2}\,\mathrm{Sp}_\sigma\mathrm{Sp}_{\sigma'}\int f\delta n'\,d\tau' + \frac{\partial\epsilon_0}{\partial p_0}\delta p_0. \qquad (14)$$

The second term is connected with the fact that for a change δN the limiting momentum p_0 changes by an amount δp_0.

For the case of spin ½, δN and δp_0 are connected by the relation

$$\delta N = 8\pi p_0^2\delta p_0 V/(2\pi\hbar)^3. \qquad (15)$$

The value of the function under the integral in Eq. (14) is appreciable only for values of momentum close to p_0. Therefore, we can carry out integration over the absolute value of p, obtaining

$$\mathrm{Sp}_\sigma\mathrm{Sp}_{\sigma'}\int f\delta n'd\tau' = \frac{1}{8\pi V}\mathrm{Sp}_\sigma\mathrm{Sp}_{\sigma'}\int f do\,\delta N. \qquad (16)$$

We get from Eq. (14), with the help of Eqs. (15) and (16):

$$\partial\mu/\partial N = \mathrm{Sp}_\sigma\mathrm{Sp}_{\sigma'}\int f do/16\pi V \qquad (17)$$
$$+ (2\pi\hbar)^3/8\pi p_0 m^* V.$$

Now let us make use of Eq. (11) and express the effective mass m^* in the expression that has been obtained by the mass of the particles, m. We have

$$\frac{\partial\mu}{\partial N} = \frac{1}{16\pi V}\int\mathrm{Sp}_\sigma\mathrm{Sp}_{\sigma'}f(1-\cos\theta)\,do + \frac{(2\pi\hbar)^3}{8\pi p_0 m V}.$$

Furthermore, multiplying the resultant equation by $N/m = (1/m)8\pi p_0^3 V/3(2\pi\hbar)^3$, we find an expression for the square of the sound velocity:

$$c^2 = \frac{p_0^2}{3m^2} + \frac{1}{6m}\left(\frac{p_0}{2\pi\hbar}\right)^3\int\mathrm{Sp}_{\sigma,\sigma'}f(1-\cos\theta)\,do. \qquad (18)$$

4. MAGNETIC SUSCEPTIBILITY

We calculate the magnetic susceptibility of a Fermi liquid. If the system is located in a magnetic field H, then the additional energy of a free particle in this field is equal to $\beta\sigma\mathrm{H}$. Moreover, we must also consider the fact that the form of the distribution function also changes in the presence of a magnetic field. Consequently, in calculating the magnetic susceptibility, we must keep in mind that

$$\delta\varepsilon = -\beta(\sigma\mathbf{H}) + \mathrm{Sp}_{\sigma'}\int f\delta n'\,d\tau', \qquad (19)$$

i.e., it is impossible to neglect the effect of the term containing f. We write f in the form

$$f = \varphi + \psi(\sigma\sigma'), \qquad (20)$$

where the second term takes into account the exchange interaction between the particle s. Furthermore, in calculating δn, which depends on the field, the change in the chemical potential $\delta\mu$ does not have to be considered. This change appears as a quantity of second order of smallness relative to the field H, while $\delta\epsilon$ is of first order with respect to the field. Therefore, we can substitute $\delta n = (\partial n/\partial\epsilon)\,\delta\epsilon$ in Eq. (19). We then have

$$\delta\varepsilon = -\beta(\sigma\mathbf{H}) + \mathrm{Sp}_{\sigma'}\int f\frac{\partial n'}{\partial\varepsilon'}\delta\varepsilon'\,d\tau'. \qquad (21)$$

We shall look for $\delta\epsilon$ in the form

$$\delta\varepsilon = -\gamma(\sigma\mathbf{H}). \qquad (22)$$

The quantity γ is defined by Eq. (21)[*]

$$\gamma = \beta + \frac{1}{2}\int\psi\frac{\partial n}{\partial\varepsilon'}\gamma' d\tau'. \qquad (23)$$

Remembering the δ-character of $\partial n/\partial\epsilon$, we than obtain

$$\gamma = \beta - {}^1\!/_2\overline{\psi}_0\gamma\,(\partial\tau/\partial\varepsilon)_0. \qquad (24)$$

Here the index zero indicates that the values of all functions are taken at $p = p_0$; the bar over the symbol indicates averaging over the angles. On the other hand, the susceptibility is defined by the relation

$$\chi\mathbf{H} = \beta\,\mathrm{Sp}\int n\sigma d\tau$$

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[*] Equation (14) is obtained as a result of taking the spur of the analogous expression which contains the spin operators.

[*] Here we make use of the relation $\mathrm{Sp}_{\sigma'}(\sigma\sigma')\sigma' = 1/3\,\sigma\,\mathrm{Sp}_{\sigma'}(\sigma'\sigma') = 1/2\,\sigma$.

or

$$\chi \mathbf{H} = -\beta \operatorname{Sp} \int \frac{\partial n}{\partial \varepsilon} \gamma (\mathbf{H} \sigma) \sigma \, d\tau = \frac{\mathbf{H}}{2} \beta \gamma \left(\frac{d\tau}{d\varepsilon}\right)_0 \qquad (25)$$

Hence, we get finally,

$$\frac{1}{\chi} = \frac{2}{\beta \gamma_0 (\partial\tau/\partial\varepsilon)_0} \qquad (26)$$

$$= \frac{2}{\beta^2 (d\tau/d\varepsilon)_0} \left(1 + \frac{1}{2} \overline{\psi}_0 \left(\frac{d\tau}{d\varepsilon}\right)_0\right).$$

Further, we can replace $(d\tau/d\epsilon)_0$ by the coefficient α in the linear heat capacity law. Then

$$1/\chi = \beta^{-2} \{2\pi^2 k^2/3\alpha + \overline{\psi}_0\}. \qquad (27)$$

It is then evident that there does not exist in the liquid the relation between the heat capacity and the susceptibility that exists in gases. The term with $\overline{\psi}_0$ takes the exchange interaction into account and is large for liquids. Thus, for He^3, analysis of the experimental data[1] shows that $\overline{\psi}_0$ is negative and amounts to about 2/3 of the first term.

5. THE KINETIC EQUATION

In the absence of a magnetic field and for a neglect of the magnetic spin-orbit interaction, ϵ does not depend on the operator σ and the kinetic equation in the quasi-classical approximation takes the form

$$\frac{\partial n}{\partial t} + \frac{\partial n}{\partial \mathbf{r}} \frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}} \frac{\partial \varepsilon}{\partial \mathbf{r}} = I(n). \qquad (28)$$

The necessity of calculation of dervatives of the energy ϵ with respect to the coordinates in the absence of an external field is connected with the fact that ϵ is a functional of n, and the distribution function n depends on the coordinates.

We find the expression for the momentum flux. For This purpose, we multiply the left and right sides of the equation above by the momentum p_i and integrate over all phase space. We have

$$\frac{\partial}{\partial t} \operatorname{Sp} \int p_i n \, d\tau + \operatorname{Sp} \int p_i \left(\frac{\partial n}{\partial x_k} \frac{\partial \varepsilon}{\partial p_k} - \frac{\partial n}{\partial p_k} \frac{\partial \varepsilon}{\partial x_k}\right) d\tau \qquad (29)$$

$$= \operatorname{Sp} \int p_i I(n) \, d\tau.$$

As a consequence of the conservation of momentum for collisions, the right side of the equation is zero, while the left side yields, after simple transformations,

$$\frac{\partial}{\partial t} \int p_i n \, d\tau + \frac{\partial}{\partial x_k} \int p_i \frac{\partial \varepsilon}{\partial p_k} n \, d\tau$$

$$- \int p_i \frac{\partial}{\partial p_k} \left(n \frac{\partial \varepsilon}{\partial x_k}\right) d\tau = 0. \qquad (30)$$

Finally, integrating the three integrals by parts, we get

$$\frac{\partial}{\partial t} \int p_i n \, d\tau + \frac{\partial}{\partial x_k} \int p_i \frac{\partial \varepsilon}{\partial p_k} n \, d\tau \qquad (31)$$

$$+ \int n \frac{\partial \varepsilon}{\partial x_i} d\tau = 0.$$

The integral $\operatorname{Sp} \int n (\partial\epsilon/\partial x_i) d\tau$ can be represented in the form [see (2)]

$$\operatorname{Sp} \int n \frac{\partial \varepsilon}{\partial x_i} d\tau = \operatorname{Sp} \frac{\partial}{\partial x_i} \int n \varepsilon \, d\tau - \operatorname{Sp} \int \varepsilon \frac{\partial n}{\partial x_i} d\tau$$

$$= \frac{\partial}{\partial x_i} [\operatorname{Sp} \int n \varepsilon \, d\tau - E].$$

Thus we finally obtain the law of conservation of momentum:

$$\frac{\partial}{\partial t} \operatorname{Sp} \int p_i n \, d\tau + \frac{\partial \Pi_{ik}}{\partial x_k} = 0, \qquad (32)$$

where the tensor of momentum flux is

$$\Pi_{ik} = \operatorname{Sp} \int p_i \frac{\partial \varepsilon}{\partial p_k} n \, d\tau + \delta_{ik} \left[\operatorname{Sp} \int n \varepsilon \, d\tau - E\right]. \qquad (33)$$

In a similar way we obtain the expression for the energy flow. We multiply the left and right sides of the kinetic equation (28) by ϵ and integrate over all phase space. We have

$$\operatorname{Sp} \int \varepsilon \frac{\partial n}{\partial t} d\tau + \operatorname{Sp} \int \varepsilon \left(\frac{\partial n}{\partial \mathbf{r}} \frac{\partial \varepsilon}{\partial \mathbf{p}} - \frac{\partial n}{\partial \mathbf{p}} \frac{\partial \varepsilon}{\partial \mathbf{r}}\right) d\tau$$

$$= \operatorname{Sp} \int \varepsilon I(n) \, d\tau.$$

As a consequence of the conservation of energy under collisions, the right side is zero while the left side reduces without difficulty to the form

$$\int \varepsilon \frac{\partial n}{\partial t} d\tau + \frac{\partial}{\partial \mathbf{r}} \int n \varepsilon \frac{\partial \varepsilon}{\partial \mathbf{p}} d\tau = 0.$$

Taking Eq. (2) into account, we have finally,

$$\frac{\partial E}{\partial t} + \operatorname{div} \mathbf{Q} = 0, \qquad (34)$$

where the energy flow is

$$Q = \mathrm{Sp}\int n\varepsilon \frac{\partial \varepsilon}{\partial \mathbf{p}} d\tau. \tag{35}$$

In the solution of concrete kinetic problems it is necessary to keep in mind the following circumstances. For such a solution we usually write down the function n in the form of a sum of equilibrium functions n_0 and correction δn. In this case the departure of the tensor of momentum flow Π_{ik} and the vector of energy flow Q from their equilibrium values will result as a consequence of the direct change of the function n by the quantity δn, as well as from the change in ϵ which comes about as a result of the functional dependence of ϵ on n [Eq. (2)].

In conclusion, I express my gratitude to I. M. Khalatnikov and A. A. Abrikosov for fruitful discussions.

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The Scattering of Photons by Nucleons and Nuclear Isobars

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The scattering cross section for photons against nucleons is computed, taking into account absorption and the excited states of the nucleons (isobars). The excited states of the nucleons are described by a relativistic equation for particles of spin and isotopic spin 3/2. The coupling constants are taken from Refs. 1 and 2 where they were obtained by comparison of experiments on the scattering and photoproduction of mesons on protons. Comparison of the results of this calculation with experimental data on the scattering of photons by protons would provide and additional check on the admissibility of the isobaric representation and on the correctness of the values chosen for the constants.

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1. INTRODUCTION

A SUFFICIENTLY strong interaction between nucleons and mesons produces around the nucleons a charged cloud of virtual mesons. The presence of such a charged cloud lends to the nucleons an anomalous magnetic moment $\gamma e\hbar/2mc$ instead of the normal magnetic moment $e\hbar/2mc$ predicted by the Dirac theory for spin 1/2 particles. Therefore, the scattering of photons by nucleons will not be described by the Klein-Nishina formula. The anomalous magnetic moment can be described phenomenologically by adding to the nucleonic Lagrangian the expression

$$\tfrac{1}{2}(\gamma-1)(e\hbar/2mc)\,\bar{u}\gamma_\mu\gamma_\nu F_{\mu\nu}u, \qquad (1.1)$$

proposed by Pauli[3]. Computations for the scattering of photons by protons carried out by Batdorf and Thomas and by Powell[4] making use of Pauli's expression, yield a scattering cross section 25 percent greater than that obtained from the Klein-Nishina formula for 100 mev γ-rays. It is clear, however, that the virtual meson cloud is deformed in proportion to the energy of the photon, with the largest deformation expected when the frequency of the photon is $\mu c^2/\hbar$. The value of the anomalous magnetic moment therefore depends on the energy of the photon. It should also be noted that an analysis of meson photoproduction, while taking account of the anomalous magnetic moment as suggested by Pauli[5], does not bring about agreement with experiment. All this makes it appear quite doubtful that an analysis of photon-proton scattering according to Eq. (1.1) would give the correct result.

Sachs and Foldy[6] have analyzed more recently the scattering of photons by nucleons by taking into account the presence of virtual mesons and their interactions with photons. Carrying out the computations in the weak interaction approximation and neglecting nucleonic recoil, they obtain the cross section for scattered photons, for the pseudoscalar meson theory, in the form of a resonance curve with maximum at $E_\gamma = \mu c^2$. However, an analogous though more exact calculation of the anomalous magnetic moment of a nucleon[7], including nucleonic recoil, is well known to yield only the correct sign and not the correct magnitudes of the proton and neutron magnetic moments. Thus Sachs' and Foldy's computations will apparently not agree with experiment.

In the present article, the scattering of light by nucleons will be considered in the spirit of a semiphenomenological theory[1] according to which nucleons can be found not only in the ground state with spin and isotopic spin 1/2 and mass m, but also in an excited state with spin and isotopic spin 3/2 and mass M. In this theory the cloud of virtual mesons around the nucleon is phenomenologically described by an isobaric state since the nucleon can go into this state by interacting with virtual mesons as well as real mesons and electromagnetic fields. Analyses of the processes of scattering[1] and photoproduction[2] of mesons by nucleons which take into account the excited state of the nucleon have yielded satisfactory agreement with experiment. In the process of meson photoproduction the fundamental interaction is between nucleons and photons as well as mesons. The agreement between experimental and computational results on meson photoproduction permits one to hope for a similar agreement with experiment in the analysis of the scattering of photons against nucleons by including the excited state of the nucleons; here the main nucleonic interaction takes place with photons while interaction with mesons occurs only during absorption. An analysis of the scattering of photons against nucleons which takes into account the isobaric state was carried out by Minami[8]. However, in computing the absorption of the incident wave, he approximates it by the resonance term of the transition matrix element, and this substantially changes the cross section at high energies. It is therefore worthwhile to compute the scattering of photons by the same method as was used for calculating the scattering[1] and photoproduction[2] of mesons. We start from Heitler's integral equation which relates the amplitude of the scattered wave with the transition matrix elements and takes account of absorption more consistently than Ref. 8.

2. LAGRANGIAN FOR THE INTERACTION BETWEEN NUCLEONS AND PHOTONS

We shall use the same symbols to designate Lagrangians and matrices as Ref. 2. Referring to it for details, we only give the Lagrangian for the interaction between nucleons and photons:

$$\mathscr{L}_{\text{int}} = \mathscr{L}_{\text{n}\Phi} + \mathscr{L}_{\text{in}\Phi}, \quad \mathscr{L}_{\text{n}\Phi} = -\bar{e}\psi\tau_p\hat{A}\psi: \qquad (2.1)$$

$$\mathscr{L}_{\text{in}\Phi} = i\frac{e}{m}\bar{B}_\mu N F_{\mu\nu}\gamma_\nu\gamma_5\psi - i\frac{e}{m}\bar{\psi}NF_{\mu\nu}\gamma_\nu\gamma_5 B_\mu$$

$$= i\frac{e}{m}\{a^+\bar{B}_\mu^{1/2}F_{\mu\nu}\gamma_\nu\gamma_5\psi_p + b^+\bar{B}_\mu^{-1/2}F_{\mu\nu}\gamma_\nu\gamma_5\psi_n$$

$$+ a\bar{\psi}_p F_{\mu\nu}\gamma_5\gamma_\nu B_\mu^{1/2} + b\bar{\psi}_n F_{\mu\nu}\gamma_5\gamma_\nu B_\mu^{-1/2}\},$$

$$F_{\upsilon\nu} = \frac{\partial}{\partial x_\mu}A_\nu - \frac{\partial}{\partial x_\nu}A_\mu,$$

$$A_\mu = e_\mu\sum_k\sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)} + c^+e^{i(kx)}\right).$$

3. SCATTERING MATRIX ELEMENTS

The matrix elements for the transition of the system from an initial state [nucleon and photon 4-momentum $p_1\,(E_1, \mathbf{p}_1)$, $k_1\,(k_1, \mathbf{k}_1)$] to a final state [nucleon and photon 4-momentum $p_2(E_2, \mathbf{p}_2)$, $k_2\,(k_2, \mathbf{k}_2)$] are computed by the method of Feynman. Only second order matrix elements are computed. All possible matrix elements allowed by the interaction Lagrangian (2.1) are then given by four Feynman diagrams (Fig. 1). Roman numerals next to each vertex on the diagrams denote the respective interaction operators. These operators have the following forms in momentum representation:

I. Operator for transition of nucleon to isobar with absorption or emission of a photon

$$I_\mu(k) = -\frac{e}{m}N\sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)}\right.$$

$$\left.- c^+e^{i(kx)}\right)\left[e_\mu\hat{k} - k_\mu\hat{e} - \frac{1}{4}\gamma_\mu(\hat{e}\hat{k} - \hat{k}\hat{e})\right]\gamma_5.$$

II. Operator for transition of isobar to nucleon with absorption or emission of a photon

$$II_\mu(k) = -\frac{e}{m}N\sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)} - c^+e^{i(kx)}\right)$$

$$\times(e_\mu\hat{k} - k_\mu\hat{e})\gamma_5.$$

III. Operator for transition of nucleon to nucleon with absorption or emission of a photon

$$III(k) = e\sqrt{\frac{2\pi}{k}}\left(ce^{-i(kx)} + c^+e^{i(kx)}\right)\tau_p\hat{e}.$$

The inner lines on the diagrams correspond to propagators. The propagators for isobars[9] and nucleons with momentum $\hat{p}$ are given in momentum space by

$$K_{\mu\nu}(p)$$

$$= (\hat{p} - M)^{-1}\left(\delta_{\mu\nu} - \frac{\gamma_\mu\hat{p}\,p_\nu}{6M^2} - \frac{p_\mu p_\nu}{3M^2} - \frac{\gamma_\mu p_\nu}{2M}\right);$$

$$(\hat{p} - m)^{-1}.$$

The matrix elements corresponding to the diagrams in Fig. 1 are then:

$$\begin{aligned}
&\bar{\psi}_2 II_\upsilon(k_2)K_{\upsilon\nu}(p_3)I_\nu(k_1)\psi_1;\\
&\bar{\psi}_2 II_\mu(k_1)K_{\mu\nu}(p_4)I_\nu(k_2)\psi_1;\\
&\bar{\psi}_2 III(k_2)(\hat{p}_3 - m)^{-1}III(k_1)\psi_1;\\
&\bar{\psi}_2 III(k_1)(\hat{p}_4 - m)^{-1}III(k_2)\psi_1;
\end{aligned} \qquad (3.1)$$

the only terms remaining in the operators I_μ, II_μ, III are those corresponding to the absorption of a photon k_1 and the emission of a photon k_2 and $p_3 = p_1 + k_1 = p_2 + k_2$, $p_4 = p_1 - k_2 = p_2 - k_1$.

The transition matrix W for the process $\gamma + N \to \gamma' + N'$ may be written by means of (3.1) in the form

$$W = \bar{\chi}_2 NN\chi_1(A + B) + \bar{\chi}_2\tau_p\tau_p\chi_1(C + D), \qquad (3.2)$$

if ψ_1 and ψ_2 are written as $\psi = \chi u$, the product of an isotopic spin function and a mechanical spin and coordinates function u. The matrix elements for the two possible scattering processes of photons against nucleons are:

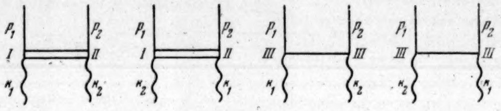

FIG. 1. Feynman diagrams of second order matrix elements for the process $\gamma + N \rightarrow \gamma' + N'$; double lines, isobars; single lines, nucleons; wavy lines, photons.

Process	$\bar{\chi}_2 N N \chi_1$	$\bar{\chi}_2 \tau_p \tau_p \chi_1$
$\gamma + p \rightarrow \gamma + p$	$\lvert a \rvert^2$	1
$\gamma + n \rightarrow \gamma + n$	$\lvert b \rvert^2$	0 .

The matrix elements

$$A = (e/m)^2\, 2\pi\, (k_1 k_2)^{-1/2}\, \bar{u}_2 \gamma_5 (e_{2\mu} \hat{k}_2 - k_{2\mu} \hat{e}_2)\, K_{\mu\nu}(p_3) \left(e_{1\nu} \hat{k}_1 - k_{1\nu} \hat{e}_1 - \frac{1}{2} \gamma_\nu \hat{e}_1 \hat{k}_1 \right) \gamma_5 u_1;$$

$$B = (e/m)^2\, 2\pi\, (k_1 k_2)^{-1/2}\, \bar{u}_2 \gamma_5 (e_{1\mu} \hat{k}_1 - k_{1\mu} \hat{e}_1)\, K_{\mu\nu}(p_4) \left(e_{2\nu} \hat{k}_2 - k_{2\nu} \hat{e}_2 - \frac{1}{2} \gamma_\nu \hat{e}_2 \hat{k}_2 \right) \gamma_5 u_1;$$

$$C = e^2 2\pi\, (k_1 k_2)^{-1/2}\, \bar{u}_2 \hat{e}_2 (\hat{p}_2 - m)^{-1} \hat{e}_1 u_1; \qquad (3.3)$$

$$D = e^2 2\pi\, (k_1 k_2)^{-1/2}\, \bar{u}_2 \hat{e}_1 (\hat{p}_4 - m)^{-1} \hat{e}_2 u_1$$

in the center-of-mass system where $\mathbf{p}_1 + \mathbf{k}_1 = \mathbf{p}_2 + \mathbf{k}_2 = 0$ (so that $k_1 = k_2 = k$ and the total energy of the system is $E_1 + k_1 = E_2 + k_2 = W$), we have the form

$$A = \left(\frac{e}{m}\right)^2 Q \sum_{i,\,k=0}^{5} a_{i,\,k} L_{i,\,k}^{\Phi'\Phi}; \quad C = e^2 Q \sum_{i,k=0}^{1} c_{i,\,k} L_{i,\,k}^{\Phi'\Phi};$$

$$\begin{aligned} B &= -\left(\frac{e}{m}\right)^2 \frac{Q}{M^2 - m^2 + 2(k_2 p_1)} \sum_{i,\,k=0}^{5} b_{i,\,k} L_{i,\,k}^{\Phi'\Phi} = \\ &= -\left(\frac{e}{m}\right)^2 \frac{Q}{M^2 - m^2 + 2k(W - k)} \sum_{i,\,k=0}^{i,\,k=\infty,} \beta_{i,\,k} L_{i,\,k}^{\Phi'\Phi}; \end{aligned} \qquad (3.4)$$

$$D = -e^2 \frac{Q}{2(k_2 p_1)} \sum_{i,\,k=0}^{5} d_{i,\,k} L_{i,\,k}^{\Phi'\Phi} = -e^2 \frac{1}{2} \frac{Q}{k(W - k)} \sum_{i,\,k=0}^{\infty} \delta_{i,\,k} L_{i,\,k}^{\Phi'\Phi},$$

$$a_{0.0} = \frac{k^2m^2\,(m+W)^2\,(W+2M)}{6M^2W^2}; \quad a_{1.1} = \frac{k^2m^2\,(m+W)^2\,(W-2M)}{6M^2W^2};$$

$$a_{2.2} = \frac{k^2\,(m+W)^2\,(m+3W)^2}{48W^2\,(W-M)};$$

$$a_{2.3} = a_{3.2} = \frac{k^3\,(m+W)\,(m+3W)}{8\sqrt{3}W\,(W-M)}; \quad a_{3.3} = \frac{k^4}{4\,(W-M)}; \quad a_{4.4} = \frac{k^2\,(m+W)^2\,(m+3W)^2}{48W^2\,(W+M)};$$

$$a_{4.5} = a_{5.4} = \frac{k^3\,(m+W)\,(m+3W)}{8\sqrt{3}W\,(W+M)}; \quad a_{5.5} = \frac{k^4}{4\,(W+M)};$$

$$b_{0.0} = [k^2\,(m+W)^2\,/\,18M^2W^2]\,(9M^2W^3 - 2m^2MW^2 - m^4W - 6m^2M^3);$$

$$b_{1.1} = [k^2\,(m+W)^2\,/\,18M^2W^2]\,(9M^2W^3 + 2m^2MW^2 - m^4W + 6m^2M^3);$$

$$b_{2.2} = [k^2\,(m+W)^2\,/\,144M^2W^2]\,[9m^2W^3 + W^2\,(6m^3 - 2m^2M + 27M^3)$$
$$+ W\,(m^4 - 12m^3M + 18mM^3) - 18m^4M + 3m^2M^3];$$

$$b_{2.3} = b_{3.2} = [k^3\,(m+W)\,/\,24\sqrt{3}M^2W]\,[3m^2W^2$$
$$+ W\,(m^3 + 2m^2M + 9M^3) + 6m^3M + 3mM^3];$$

$$b_{3,3} = (k^4\,/\,12M^2)\,(m^2W + 3M^3 - 2m^2M);$$

$$b_{4.4} = \frac{k^2\,(m+W)^2}{144M^2W^2}\,[9m^2W^3 + W^2\,(6m^3 + 2m^2M - 27M^3)$$
$$+ W\,(m^4 + 12m^3M - 18mM^3) + 18m^4M - 3m^2M^3];$$

$$b_{4.5} = b_{5.4} = \frac{k^3\,(m+W)}{24\sqrt{3}M^2W}\,[3m^2W^2 + W\,(m^3 - 2m^2M - 9M^3) - 6m^3M - 3mM^3];$$

$$b_{5.5} = (k^4\,/\,12M^2)\,(m^2W - 3M^3 + 2m^2M);$$

$$c_{0.0} = (m+W)^3\,/\,4W^2; \quad c_{1.1} = k\,(m+W)\,/\,2W;$$

$$d_{0.0} = \frac{k\,(m+W)^3\,(2m-W)}{6W^2}; \quad d_{1.1} = -\frac{k^2\,(m+W)\,(2m+W)}{3W}; \quad d_{2.2} = -\frac{k^2\,(m+W)\,(m+2W)}{6W};$$

$$d_{2\,3} = d_{3.2} = -k^2\,(m+W)\,/\,2\sqrt{3}; \quad d_{3.3} = k^2m\,(m+W)\,/\,2W;$$

$$d_{4.4} = -k\,(m+W)^4\,/\,6W^2;$$

$$d_{4.5} = d_{5.4} = -k^2\,(m+W)^2\,/\,2\sqrt{3}W; \quad d_{5.5} = 0; \quad Q = \pi\,/\,k\,(W-k)\,(m+W-k). \tag{3.5}$$

The normalized angular polynomial-matrices $L_{i,\,k}^{\Phi'\Phi}$ for the reaction $\gamma + N \rightarrow \gamma' + N'$, in terms of which the matrix elements A, B, C, D are expanded, are obtained and described in detail in Ref. 2. Each of them describes an angular distribution of the reaction $\gamma + N \rightarrow \gamma' + N'$ for given angular momenta of the photons γ and γ' and of the total system $\gamma + N$. We shall use here the first 10 polynomials where

$$
\begin{gathered}
L_{0.0}^{\Phi'\Phi}(1_{\mathrm{E}}\,{}^{1}/_{2}\,1_{\mathrm{E}}) = (\boldsymbol{\sigma}\mathbf{e}')(\boldsymbol{\sigma}\mathbf{e});\quad L_{1,1}^{\Phi'\Phi}(1_{\mathrm{M}}\,{}^{1}/_{2}\,1_{\mathrm{M}}) = (\boldsymbol{\sigma}\mathbf{s}')(\boldsymbol{\sigma}\mathbf{s});\\
L_{2.2}^{\Phi'\Phi}(1_{\mathrm{M}}\,{}^{3}/_{2}\,1_{\mathrm{M}}) = 3(\mathbf{s}'\mathbf{s}) - (\boldsymbol{\sigma}\mathbf{s}')(\boldsymbol{\sigma}\mathbf{s});\\
L_{2.3}^{\Phi'\Phi}(1_{\mathrm{M}}\,{}^{3}/_{2}\,2_{\mathrm{E}}) = -i\sqrt{3}\,[(\boldsymbol{\sigma}\mathbf{e})(\mathbf{s}'\mathbf{k}) + (\boldsymbol{\sigma}\mathbf{k})(\mathbf{s}'\mathbf{e})];\\
L_{3.2}^{\Phi'\Phi}(2_{\mathrm{E}}\,{}^{3}/_{2}\,1_{\mathrm{M}}) = i\sqrt{3}\,[(\boldsymbol{\sigma}\mathbf{e}')(\mathbf{k}'\mathbf{s}) + (\boldsymbol{\sigma}\mathbf{k}')(\mathbf{e}'\mathbf{s})];\\
L_{3,3}^{\Phi'\Phi}(2_{\mathrm{E}}\,{}^{3}/_{2}\,2_{\mathrm{E}}) = (\boldsymbol{\sigma}\mathbf{e}')(\boldsymbol{\sigma}\mathbf{e})(\mathbf{k}'\mathbf{k}) + (\boldsymbol{\sigma}\mathbf{k}')(\boldsymbol{\sigma}\mathbf{e})(\mathbf{e}'\mathbf{k}) + (\boldsymbol{\sigma}\mathbf{e}')(\boldsymbol{\sigma}\mathbf{k})(\mathbf{k}'\mathbf{e}) + (\boldsymbol{\sigma}\mathbf{k}')(\boldsymbol{\sigma}\mathbf{k})(\mathbf{e}'\mathbf{e});\\
L_{4.4}^{\Phi'\Phi}(1_{\mathrm{E}}\,{}^{3}/_{2}\,1_{\mathrm{E}}) = 3(\mathbf{e}'\mathbf{e}) - (\boldsymbol{\sigma}\mathbf{e}')(\boldsymbol{\sigma}\mathbf{e});\\
L_{4.5}^{\Phi'\Phi}(1_{\mathrm{E}}\,{}^{3}/_{2}\,2_{\mathrm{M}}) = i\sqrt{3}\,[(\boldsymbol{\sigma}\mathbf{s})(\mathbf{e}'\mathbf{k}) + (\boldsymbol{\sigma}\mathbf{k})(\mathbf{e}'\mathbf{s})];\\
L_{5,4}^{\Phi'\Phi}(2_{\mathrm{M}}\,{}^{3}/_{2}\,1_{\mathrm{E}}) = -i\sqrt{3}\,[(\boldsymbol{\sigma}\mathbf{s}')(\mathbf{k}'\mathbf{e}) + (\boldsymbol{\sigma}\mathbf{k}')(\mathbf{s}'\mathbf{e})];\\
L_{5,5}^{\Phi'\Phi}(2_{\mathrm{M}}\,{}^{3}/_{2}\,2_{\mathrm{M}}) = (\boldsymbol{\sigma}\mathbf{s}')(\boldsymbol{\sigma}\mathbf{s})(\mathbf{k}'\mathbf{k}) + (\boldsymbol{\sigma}\mathbf{k}')(\boldsymbol{\sigma}\mathbf{s})(\mathbf{s}'\mathbf{k}) + (\boldsymbol{\sigma}\mathbf{s}')(\boldsymbol{\sigma}\mathbf{k})(\mathbf{k}'\mathbf{s}) + (\boldsymbol{\sigma}\mathbf{k}')(\boldsymbol{\sigma}\mathbf{k})(\mathbf{s}'\mathbf{s}). \qquad (3.6)
\end{gathered}
$$

Here $\mathbf{s} = [\mathbf{ke}]$, $\mathbf{s}' = [\mathbf{k}'\mathbf{e}']$, and the symbols in parenthesis following the $L_{i,k}^{\Phi'\Phi}$'s denote, reading right to left, the angular momentum and multipolar character of the incident quantum, the angular momentum of the system and the angular momentum and multipolar character of the scattered quantum. The coefficients $\beta_{i,k}$ and $\delta_{i,k}$ in (3.4) are related to the coefficients $b_{i,k}$ and $d_{i,k}$ by means of formulas obtained by expanding the functions

$$
F = \sum_{i,k=0}^{5} f_{i,k} L_{i,k}^{\Phi'\Phi} / (1 + \tau\cos\theta) = \sum_{i,k=0}^{\infty} \varphi_{i,k} L_{i,k}^{\Phi'\Phi},\quad \cos\theta = (\mathbf{k}'\mathbf{k})/k'k
$$

in series of polynomials $L_{i,k}^{\Phi'\Phi}$:

$$
\begin{gathered}
\varphi_{0,0} = {}^{1}/_{4}\,\{2J_0 f_{0,0} + J_1(2f_{1,1} + f_{2,2} - \sqrt{3}f_{2,3} - \sqrt{3}f_{3,2} + 3f_{3,3})\\
+ ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(f_{4,4} - \sqrt{3}f_{4,5} - \sqrt{3}f_{5,4} + 3f_{5,5})\};\\
\varphi_{1,1} = {}^{1}/_{4}\,\{2J_0 f_{1,1} + J_1(2f_{0,0} + f_{4,4} - \sqrt{3}f_{4,5} - \sqrt{3}f_{5,4} + 3f_{5,5})\\
+ ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(f_{2,2} - \sqrt{3}f_{2,3} - \sqrt{3}f_{3,2} + 3f_{3,3})\};\\
\varphi_{2,2} = {}^{1}/_{8}\,\{4J_0 f_{2.2} + J_1(f_{0.0} + 5f_{4,4} + \sqrt{3}f_{4.5} + \sqrt{3}f_{5,4} - 3f_{5,5})\\
+ ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(f_{1.1} + f_{2.2} + \sqrt{3}f_{2\,3} + \sqrt{3}f_{3,2} + 3f_{3,3}) + 6J_3 f_{5.5}\};\\
\sqrt{3}\varphi_{2,3} = {}^{1}/_{8}\,\{4J_0\sqrt{3}f_{2,3} + 3J_1(-f_{0,0} + f_{4,4} + \sqrt{3}f_{4.5} - \sqrt{3}f_{5.4} + 3f_{5,5})\\
+ ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(-3f_{1,1} + 3f_{2,2} - \sqrt{3}f_{2,3} + 3\sqrt{3}f_{3,2} - 3f_{3,3}) - 6J_3(2f_{5,5} - \sqrt{3}f_{5,4})\};\\
\varphi_{3,3} = {}^{1}/_{8}\,\{4J_0 f_{3,3} + 3J_1(f_{0,0} - f_{4,4} + \sqrt{3}f_{4,5} + \sqrt{3}f_{5,4} - f_{5,5})\\
+ ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(3f_{1.1} + 3f_{2,2} - \sqrt{3}f_{2,3} - \sqrt{3}f_{3,2} + f_{3,3})\\
+ 2J_3(3f_{4.4} - 2\sqrt{3}f_{4.5} - 2\sqrt{3}f_{5.4} + 4f_{5,5})\};\\
\varphi_{4,4} = {}^{1}/_{8}\,\{4J_0 f_{4,4} + J_1(f_{1,1} + 5f_{2,2} + \sqrt{3}f_{2,3} + \sqrt{3}f_{3,2} - 3f_{3,3})\\
+ ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(f_{0,0} + f_{4,4} + \sqrt{3}f_{4,5} + \sqrt{3}f_{5,4} + 3f_{5,5}) + 6J_3 f_{3,3}\};\\
\sqrt{3}\varphi_{4,5} = {}^{1}/_{8}\,\{4J_0\sqrt{3}f_{4,5} - 3J_1(f_{1,1} - f_{2,2} - \sqrt{3}f_{2,3} + \sqrt{3}f_{3,2} - 3f_{3,3})\\
- ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(3f_{0,0} - 3f_{4,4} + \sqrt{3}f_{4,5} - 3\sqrt{3}f_{5,4} + 3f_{5,5}) - 6J_3(2f_{3,3} - \sqrt{3}f_{3,2})\};\\
\varphi_{5,5} = {}^{1}/_{8}\,\{4J_0 f_{5,5} + 3J_1(f_{1,1} - f_{2,2} + \sqrt{3}f_{2.3} + \sqrt{3}f_{3,2} - f_{3,3})\\
+ ({}^{3}/_{2}J_2 - {}^{1}/_{2}J_0)(3f_{0,0} + 3f_{4.4} - \sqrt{3}f_{4,5} - \sqrt{3}f_{5,4} + f_{5.5})\\
+ 2J_3(3f_{2,2} - 2\sqrt{3}f_{2,3} - 2\sqrt{3}f_{3,2} + 4f_{3,3})\}.
\end{gathered}
$$

Formulas for $\varphi_{3,2}$ and $\varphi_{5,4}$ are obtained from the formulas for $\varphi_{2,3}$ and $\varphi_{4,5}$ by permuting corresponding indices.

$$J_n(\tau) = \int\limits_{-1}^{1} \frac{x^n\, dx}{1+\tau x},$$

where

$$\text{for } \beta_{i,k} \quad \tau = \frac{2k^2}{M^2 - m^2 + 2k(W-k)},$$

$$\text{for } \delta_{i,k} \quad \tau = \frac{k}{W-k}.$$

Thus the transition matrix (3.2) for scattering of photons agains nucleons can be written with the help of tables and formulas (3.4), (3.5) in the form

$$W(\gamma' p', \gamma p) = e^2 Q \sum_{i,\,k=0}^{\infty} \left\{ \frac{a^2}{m^2} \left(a_{i,k} - \frac{\beta_{i,k}}{M^2 - m^2 + 2k(W-k)} \right) + c_{i,k} - \frac{\delta_{i,k}}{2k(W-k)} \right\} L_{i,k}^{\Phi'\Phi}. \quad (3.7)$$

4. EFFECTIVE SCATTERING CROSS SECTION

In order to obtain the effective scattering cross section, it is necessary to know the amplitude $F(\gamma', \gamma)$ of the scattered wave. As shown in Ref. 2, $F(\gamma', \gamma)$ can be obtained by solving the following system of Heitler integral equations (formula 4.5 of Ref. 2; the notations are identical):

$$F^{3/2}(\pi, \gamma) = V^{3/2}(\pi, \gamma) - i\eta_\pi \int U^{3/2}(\pi, \pi') F^{3/2}(\pi', \gamma)\, d\Omega_{\pi'} - i\eta_\gamma \int V^{3/2}(\pi, \gamma') F(\gamma', \gamma)\, d\Omega_\gamma;$$

$$F^{1/2}(\pi, \gamma) = V^{1/2}(\pi, \gamma) - i\eta_\pi \int U^{1/2}(\pi, \pi') F^{1/2}(\pi', \gamma)\, d\Omega_{\pi'} - i\eta_\gamma \int V^{1/2}(\pi, \gamma') F(\gamma', \gamma)\, d\Omega_{\gamma'};$$

$$F(\gamma', \gamma) = W(\gamma', \gamma) - i\eta_\pi \int \left\{ \frac{1}{3} [V^{3/2}(\pi, \gamma')]^+ F^{3/2}(\pi, \gamma) + \frac{2}{3} [V^{1/2}(\pi, \gamma')]^+ F^{1/2}(\pi, \gamma) \right\} d\Omega_\pi - i\eta_\gamma \int W(\gamma', \gamma'') F(\gamma'', \gamma)\, d\Omega_{\gamma''};$$

$$\eta_\pi = (1/8\pi^2)\, q^2 dq / dW, \quad \eta_\gamma = (1/8\pi^2)\, k^2\, dk / dW. \quad (4.1)$$

This system of equations can be solved by means of the angular polynomial matrices $L_k^{M'M}$, $L_{i,k}^{M\Phi}$, $L_{i,k}^{\Phi'M}$ otbained in Ref. 2. Expanding all the transition matrices and amplitudes of scattered waves in series of corresponding angular polynomials, and denoting the expansion coefficients by corresponding small script letters, we obtain the system of algebraic equations

$$f_{i,k}^{3/2} = v_{i,k}^{3/2} - i4\pi\eta_\pi u_i^{3/2} f_{i,k}^{3/2} - i4\pi\eta_\gamma \sum_l v_{i,l}^{3/2} f_{l,k};$$

$$f_{i,k}^{1/2} = v_{i,k}^{1/2} - i4\pi\eta_\pi u_i^{1/2} f_{i,k}^{1/2} - i4\pi\eta_\gamma \sum_l v_{i,l}^{1/2} f_{l,k};$$

$$f_{l,k} = w_{l,k} - i4\pi\eta_\pi \left(\frac{1}{3} v_{i,l}^{3/2} f_{i,k}^{3/2} + \frac{2}{3} v_{i,l}^{1/2} f_{i,k}^{1/2} \right) - i4\pi\eta_\gamma \sum_n w_{l,n} f_{n,k}; \quad (4.2)$$

$i = 0$ if $k = 0$ and i equals the integer given by either $(k+1)/2$ or $(k+2)/2$ for the remaining values of k. The system (4.2) can be solved exactly. An exact solution is cumbersome, however. Neglecting quantities of the order of g^2 and higher with respect to 1, we obtain in the first approximation the following solution to system (4.2):

$$f_{l,k} = \frac{w_{l,k}}{1 + i4\pi\eta_\pi \left(u_i^{3/2} + u_i^{1/2} \right)};$$

$$f_{i,k}^{3/2,\,1/2} = \frac{v_{i,k}^{3/2,\,1/2}}{1 + i4\pi\eta_\pi u_i^{3/2,\,1/2}}, \quad (4.3)$$

where i depends on k as in (4.2). Therefore,

$$F(\gamma', \gamma) = \sum_{l,\,k=0}^{\infty} f_{l,k} L_{l,k}^{\Phi'\Phi} = \sum_{l,\,k} \frac{w_{l,k}}{1 + i4\pi\eta_\pi \left(u_i^{3/2} + u_i^{1/2} \right)} L_{l,k}^{\Phi'\Phi}, \quad (4.4)$$

where the $w_{l,k}$ are obtained from Eqs. (3.8), (3.5) and (3.7), and $u_i^{3/2}$, $u_i^{1/2}$ are related to $\tan\delta_i^{3/2}$, $\tan\delta_i^{1/2}$ of Ref. 1 by the formulas

$$\operatorname{tg}\delta_{i+1}^{3/2} = -4\pi\eta_\pi u_i^{3/2}, \quad \operatorname{tg}\delta_{i+1}^{1/2} = -4\pi\eta_\pi u_i^{1/2}.$$

We have limited ourselves to the first ten terms of the series in computing the amplitude $F(\gamma', \gamma)$. This implies that we consider all possible scatterings of dipolar and quadrupolar quanta against nucleons, assuming that the angular momentum of the system is no higher than 3/2. Mesonic absorption is included in the first six terms of the series (processes $1_E \rightarrow {}^1/_2 \rightarrow 1_E$, $1_M \rightarrow {}^1/_2 \rightarrow 1_M$, $1_M \rightarrow {}^3/_2 \rightarrow 1_M$, $1_M \rightleftarrows {}^3/_2 \rightleftarrows 2_E$, $2_E \rightarrow {}^3/_2 \rightarrow 2_E$). This means that absorption is considered in conjunction with s and p meson waves.

In order to determine the effective cross section, it is necessary to obtain $|F|^2$, average it over the spins of the initial states of the nucleon, sum it over the spins of the final states of the nucleon (these operations are equivalent to $1/2\,\mathrm{Sp}|F|^2$), average it over the polarizations of the incident photons and sum over the polarizations of the scattered photons. The last two operations are equivalent to an averaging over polarizations of both incident and scattered photons and multiplication by two. It is therefore necessary to obtain $\mathrm{Sp}\,\overline{|F|^2}$ where the bar denotes averaging over the polarizations of both photons.

In the center-of-mass system, we then have

$$d\sigma / d\Omega = (1/4\pi^2)\,k^2 (W-k)^2 W^{-2}\,\mathrm{Sp}\,\overline{|F|^2};$$

$$\mathrm{Sp}\,\overline{|F|^2} = M_0 + M_1\cos\theta + M_2({}^3/_2\cos^2\theta - {}^1/_2) + M_3\cos^3\theta;$$

$$\begin{aligned} M_0 = {} & 2\,[(0,0;\ 0,0) + (1,1;\ 1,1)] + 4\,[(2,2;\ 2,2) + 2\,(2,3;\ 2,3) + (3,3;\ 3,3) \\ & + (4,4;\ 4,4) + 2\,(4,5;\ 4,5) + (5,5;\ 5,5)]; \end{aligned} \tag{4.5}$$

$$\begin{aligned} M_1 = {} & 2\,[2\,(0,0;\ 1,1) + (0,0;\ 2,2) - 2\sqrt{3}(0,0;\ 2,3) + 3\,(0,0;\ 3,3) + (1,1;\ 4,4) - \\ & - 2\sqrt{3}\,(1,1;\ 4,5) + 3\,(1,1;\ 5,5) + 5\,(2,2;\ 4,4) + \\ & + 2\sqrt{3}\,(2,2;\ 4,5) - 3\,(2,2;\ 5,5) + 2\sqrt{3}\,(2,3;\ 4,4) + 6\sqrt{3}\,(2,3;\ 5,5) - \\ & - 3(3,3;\ 4,4) + 6\sqrt{3}\,(3,3;\ 4,5) - 3\,(3,3;\ 5,5)]; \end{aligned} \tag{4.6}$$

$$\begin{aligned} M_2 = {} & (2,2;\ 2,2) + 4\,(2,3;\ 2,3) + (3,3;\ 3,3) + (4,4;\ 4,4) + 4\,(4,5;\ 4,5) + \\ & + (5,5;\ 5,5) + 2\,[(0,0;\ 4,4) - 2\sqrt{3}\,(0,0;\ 4,5) + 3\,(0,0;\ 5,5) + (1,1;\ 2,2) - \\ & - 2\sqrt{3}(1,1;\ 2,3) + 3(1,1;\ 3,3) + 2\sqrt{3}(2,2;\ 2,3) + 3\,(2,2;\ 3,3) - 2\sqrt{3}(2,3;\ 3,3) + \\ & + 2\sqrt{3}\,(4,4;\ 4,5) + 3\,(4,4;\ 5,5) - 2\sqrt{3}\,(4,5;\ 5,5)]; \end{aligned}$$

$$\begin{aligned} M_3 = {} & 4\,[3\,(2,2;\ 5,5) + 6\,(2,3;\ 4,5) + 3\,(3,3;\ 4,4) - 4\sqrt{3}\,(2,3;\ 5,5) \\ & - 4\sqrt{3}\,(3,3;\ 4,5) + 4\,(3,3;\ 5,5)], \end{aligned}$$

$$2\,(i,\ k;\ l,\ m) = f_{i,k}^{+} f_{l,m} + f_{l,m}^{+} f_{i,k}; \qquad f_{i,k} = f_{k,i}. \tag{4.6}$$

The total cross section is

$$\sigma = (k^2 (W-k)^2 / \pi W^2)\,M_0. \tag{4.7}$$

5. RESULTS

We present in Figs. 2 and 3 the total effective cross section σ as a function of the energy of the incident photon k_0 in the laboratory system, and the differential cross section $d\sigma/d\Omega$ in the center-of-mass system for various energies k_0. The cross sections were computed assuming*

$$a = 1.61: \quad G^2 = 0.12; \quad g^2 = 0.10;$$

$$f^2 = 0.529; \quad M = 1.316m. \tag{5.1}$$

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* The meson mass is assumed to be exactly $\mu = 276\,m_e$.

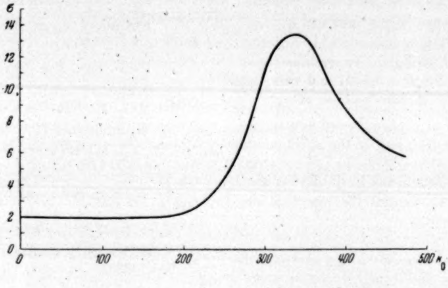

FIG. 2. Total effective cross section $\sigma(10^{-31}\,\mathrm{cm}^2)$ for the scattering of photons agains protons as a function of the photon energy k_0 (mev) in the laboratory system.

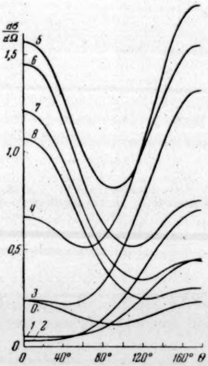

FIG. 2. Differential cross section $d\sigma/d\Omega\,(10^{-31}/\mathrm{cm}^2/$steradian) for the scattering of photons against protons as a function of the scattering angle θ in the center-of-mass system. Energy of the photons: *0*--0, *1*--192, *2*--230, *3*--272, *4*--296, *5*--344, *6*--392, *7*--430, *8*--470 mev.

The fundamental constants in this computation are a, G^2 and M, and these are taken as in Ref. 2; the constants g^2 and f^2 differ from those assumed in Ref. 2, but do not play an essential role here as they only enter in secondary terms in the absorption**; therefore, the cross section does not change very much if one adopts the constants given in Eq. (5.1) of Ref. 2 instead of the constants (5.1) of the present article.

For small energies, the total cross section equals the Thompson cross section $(8\pi/3) \times (e^2/mc^2)^2 = 1.973 \times 10^{-31}$ cm², for as $k \to 0$, isobaric terms and terms due to absorption go to zero. For $k_0 \sim 180$ mev the cross section begins to grow, and for $k_0 = 344$ mev*** it reaches a maximum of 1.34×10^{-30} cm², exceeding the Thompson cross section by a factor of almost 7. Upon further increase in k_0, the cross section falls. For small energies the angular distribution of scattered photons coincides with $1 + \cos^2\theta$; then as the energy increases, a preferential backward scattering appears. At resonance ($k_0 = 344$ mev.) the angular distribution is again symmetrical with respect to $\theta = \pi/2$, and for energies beyond resonance a preferential forward scattering appears, the asymmetry increasing with energy. The change in the position of the maximum from backward to forward angles takes place extremely rapidly in the resonance region.

Comparison of these results with those of Sachs and Foldy[6] and Minami[8] shows that the width of the curve obtained for the total cross section is several times larger while its maximum is several times smaller than for curves of Refs. 6

** In distinction from Ref. 2 where g^2 and f^2 enter in the transition matrix as well as in the absorption.

*** When $k_0 = 344$ mev, the energy of the system W equals the mass M of the isobar in the center-of-mass system.

and 8* obtained with the same parameter values. Furthermore, the maximum is not found at $k_0 = \mu c^2$ as in Ref. 6, but at a considerably higher energy. The angular distribution resembles those of Refs. 6 and 8. only for very small and very large k_0, and strongly differs at other values of k_0; this is especially true of the backward scattering in the pre-resonance region and of the minimum for $\theta = \pi/2$ at resonance.

In conclusion, I wish to thank O. P. Ryibalkin, V. I. Petukhov and A. G. Trunov for carrying out the numerical computations.

Note added in proof: After completing this analysis, the author discovered that an analogous calculation was carried out independently by R. Gurzhi.

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On the *S*-Matrix for Particles with Arbitrary Spin

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This is a discussion of the perturbation theory for particles with arbitrary spin. The properties of the singular functions for such particles are discussed. It is shown that the elements of the *S*-matrix may be found by the Feynman rules.

THE present theory of wave fields permits us to consider fields corresponding to particles with any integer or half integer spin or with any number of integer or half integer spin states.[1-4] In spite of the fact that particles with spin larger than unity are not observed experimentally, the theory of such particles can present some interest in the interpretation of the properties of the newly discovered mesons and in the study of the isobaric states of nucleons. For instance, the theory of particles with spin 3/2 has been applied to the construction of the partly phenomenological theory of interaction of π-mesons with nucleons[4] - and the results obtained compare satisfactorily with the experimental data.

The basic problem in the study of the interaction of particles with arbitrary spin with other fields amounts to the construction of the scattering matrix S. It has already been assumed previously that the elements of the S-matrix for particles with arbitrary spin may be obtained by the Feynman rules.[5-6], [7] in analogy to the quantum electrodynamics case. However, until now no basis was given to this assumption.

A difficulty arises in the construction of the S-matrix for particles with arbitrary spin. The difficulty is related to the fact that for higher spins not all the components of the wave function $\psi(x)$ are dynamically independent.[1] This fact is the main impediment in the attempt to generalize, for the case of arbitrary spin, the S-matrix theory; the latter was derived by Dyson[7] for quantum electrodynamics (spin 1/2 in the interaction representation. (For more details on these difficulties see Refs. 5, 6).

In the present paper, it will be shown how one can obtain the elements of the S-matrix for particles with arbitrary spin in the Heisenberg representation. We follow the Yang-Feldman method.[8] For the sake of simplicity, we will consider the interaction with an electromagnetic field (the interaction with other fields can be treated analogously).

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* The constants used in this article are almost identical to those of Ref. 8.

1. The interaction of a particle with arbitrary spin[*] with the electromagnetic field can be written in the form[3]

$$(L_k\nabla_k + i\varkappa)\psi = ieA_kL_k\psi;$$

$$\Box A_k = j_k;\ j_k = e(\bar{\psi}L_k\psi)$$

$$(h = c = 1;\ k = 0,\ 1,\ 2,\ 3;$$

$$\nabla_k = \partial/\partial x_k;\ \Box = \nabla_k^2), \quad (1)$$

where A_k is the quantized 4-vector potential of the electromagnetic field and L_k are matrices the properties of which are studied in Ref. 3.

It has been shown [9,10] that the matrices L_k subject to the conditions stated in the earlier footnote satsify the identity

$$(L_kp_k)^n[(L_kp_k)^2 - (\varkappa^2/\mu_1^2)p_k^2]\cdots$$

$$[(L_kp_k)^2 - (\varkappa^2/\mu_s^2)p_k^2] = 0, \quad (2)$$

where p_k is an arbitrary 4-vector, $\mu_1, \mu_2, \dots, \mu_s$ is the rest mass spectrum of the particle[**], and n is an integer dependent on the properties of the matrix L_0. For particles with spin ½: $n = 0$, $s = 1$; for particles with spin 0, 1: $n = 1$, $s = 1$. In the first case, (2) gives a known relation for the Dirac matrices, and in the second case, (2) gives the Deffin-Kemmer relations.

2. Let us first consider the basic singular functions satisfying equation (1).

It has been shown[9] that, in the absence of interaction the components of the wave function ψ satisfy the relation:

$$[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\pm = -R_{\rho\sigma}(x - x'), \quad (3)$$

$$R(x) = (-1)^s\left(\frac{i}{\varkappa}\right)^n(L_k\nabla_k - i\varkappa)\sum_{r=1}^{s}\left\{(L_k\nabla_k)^n \times \prod_{\substack{i=1\\ i\neq r}}^{s}\left[(L_k\nabla_k)^2 - \frac{\varkappa^2}{\mu_i^2}\Box\right]\right\}b_r\Delta(x,\ \mu_r), \quad (4)$$

$$b_r = \prod_{i=1}^{s}(\mu_i^2/\varkappa^2)\Big/\prod_{\substack{i=1\\ i\neq r}}^{s}(\mu_i^2 - \mu_r^2), \quad (5)$$

where n and s are defined the same way as for (2), and $\Delta(x, \mu_s)$ is the Pauli singular function corresponding to the particle with a rest mass μ_s.

$$\Delta(x,\ \mu_r) = (2\pi)^{-3}\int e^{i\mathbf{px}}\frac{\sin p_0x_0}{p_0}d^3p;$$

$$p_0 = \sqrt{p^2 + \mu_r^2}.$$

In (3) the minus sign is taken for integer spin and the plus sign for half integer spin. The vacuum expectation value (in the absence of interaction) has the form[6]

$$\langle[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\mp\rangle_0 = iR_{\rho\sigma}^{(1)}(x - x') \quad (6)$$

(the plus sign is taken for integer spin, minus for half integer spin), where $R^{(1)}(x)$ is obtained from the expression of $R(x)$ [Eq. (4)] by substituting for $\Delta(x, \mu_r)$:

$$\Delta^{(1)}(x,\ \mu_r) = (2\pi)^{-3}\int e^{i\mathbf{px}}\frac{\cos p_0x_0}{p_0}d^3p;\ p_0 = \sqrt{p^2 + \mu_r^2}.$$

The Green function of the equation $(L_k\nabla_k + ik) \times \psi(x) = 0$ can be presented in the form[6]:

$$S_G(x - x') = S(\nabla)\sum_{r=1}^{s}b_rG(x - x';\ \mu_r), \quad (7)$$

$$G(x, \mu_r) = -(2\pi)^{-4}\int e^{ipx}\frac{d^4p}{p^2 - \mu_r^2}, \quad (8)$$

where b_r may be obtained from (5) and $S(\nabla)$ is a differential operator of the form

$$S(\nabla) = -\sum_{m=0}^{n+2s-3}\left(\frac{i}{\varkappa}\right)^{m+1}(L_k\nabla_k)^m\prod_{i=1}^{s}(\Box + \mu_i^2)(\varkappa/\mu_i)^2 + \left(\frac{i}{\varkappa}\right)^n(L_k\nabla_k - i\varkappa)(L_k\nabla_k)^n \times \sum_{r=1}^{s}\prod_{\substack{i=1\\ i\neq r}}^{s}\frac{\left[(L_k\nabla_k)^2 - \frac{\varkappa^2}{\mu_i^2}\Box\right]}{\mu_r^2 - \mu_i^2}(\Box + \mu_i^2). \quad (9)$$

---

[*] We are considering only equations with finite dimensions (ψ has a finite number of components). Such equations describe particles, the spin and the rest mass of which are taking a finite number of values. Furthermore, it is assumed that the energy (in the case of integer spin) or the charge (in the half-integer case) are definite.

[**] the mass spectrum of a particle is determined by the relationship $\mu_i = |\varkappa/\lambda_i|$ where λ_i are[3] the non-zero eigenvalues of the matrix L_0

In particular, in the Dirac equation ($n = 0$, $s = 1$), $S(\nabla)$ takes the form $L_k\nabla_k - ik$; in the case of the Deffin–Kemmer equation ($n = 1$, $s = 1$), one obtains from (9) the known formula

$$S(\nabla) = (i/\varkappa)\,[(L_k\nabla_k)^2 - i\varkappa(L_k\nabla_k) - (\Box + \varkappa^2)].$$

To define completely the Green's function $S_{\sim}(x - x')$ one has furthermore to define the integration path in (8). Let us denote by $\Delta_r(x - x'; \mu_i)$ and by $\Delta_a(x - x'; \mu_i)$ the Green's function $G(x - x'; \mu_i)$ for the Klein-Fock equation $(\quad + \mu_i^2)\,\psi(x) = 0$, giving solutions in the form of retarded and advanced potential, respectively; by $\Delta_F(x - x'; \mu_i)$ the causal Feynman Δ-function, and by $\bar{\Delta}(x - x'; \mu_i)$ the Green's function corresponding to the principal value of the integral in (8).

The corresponding Green's function for particles with arbitrary spin will have the form

$$\bar{S}(x - x') = S(\nabla)\sum_{i=1}^{s}\bar{\Delta}(x - x';\ \mu_i);$$

$$S_r(x - x') = S(\nabla)\sum_{i=1}^{s}\Delta_r(x - x';\ \mu_i);$$

$$S_F(x - x') = S(\nabla)\sum_{i=1}^{s}\Delta_F(x - x';\ \mu_i);$$

$$S_a(x - x') = S(\nabla)\sum_{i=1}^{s}\Delta_a(x - x';\ \mu_i). \quad (10)$$

Furthermore, let us define two more functions

$$S(x - x') = S(\nabla)\sum_{i=1}^{s} b_i\Delta(x - x';\ \mu_i);$$

$$S^{(1)}(x - x') = S(\nabla)\sum_{i=1}^{s} b_i\Delta^{(1)}(x - x';\ \mu_i). \quad (11)$$

Using formula (9) and also the fact that $\Delta(x - x')$ and $\Delta^{(1)}(x - x')$ satisfy the Klein-Fock equation, it is easy to see that

$$S(x - x') \equiv -R(x - x');$$

$$S^{(1)}(x - x') \equiv R^{(1)}(x - x'),$$

where $R(x - x')$ and $R^{(1)}(x - x')$ are the functions defined in (3) and (6). In this fashion, Eqs. (3) and (6) can be replaced by

$$[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\pm = S(x - x'),\ \langle[\psi_\rho(x),\ \bar{\psi}_\sigma(x')]_\mp\rangle_0 = -iS^{(1)}(x - x'). \quad (12)$$

From (12) one sees that, in the Green's functions as well as in the permutation relations and in the vacuum expectation values, the only operator operating on the singular Δ-function is the differential operator $S(\nabla)$; this is the same situation as in quantum electrodynamics.*** This situation will be considerably taken advantage of in our work.

It is also important to note that the relationships between the Green's functions S_r, S_a, S_F, and S are the same is in quantum electrodynamics, namely:

$$S_F(x) = \bar{S}(x) - (i/2)\,S^{(1)}(x), \quad (13a)$$

$$\bar{S}(x) = {}^1/_2\,\{S_a(x) + S_r(x)\}, \quad (13b)$$

$$S(x) = S_r(x) - S_a(x), \quad (13c)$$

$$S_r(x - x') = 0 \quad (x_0 < x_0');$$

$$S_a(x - x') = 0 \quad (x_0 > x_0'). \quad (13d)$$

However, the case of particles with spin larger than ½ differs considerably from that of quantum electrodynamics. This difference is that, in general, Eqs. (10) and (11) defining the functions $S(x)$ and $S(x)$ do not imply the equality

$$\bar{S}(x) = {}^1/_2\,\varepsilon(x)\,S(x),$$

$$\varepsilon(x) = \begin{cases} 1 & x_0 > 0 \\ -1 & x_0 < 0, \end{cases}$$

as they do in quantum electrodynamics.

Indeed, from the definition of $S(x)$ and $S(x)$ it follows that

$$\bar{S}(x) = S(\nabla)\,\{{}^1/_2\,\varepsilon(x)\,\Delta(x)\}$$

$$= {}^1/_2\,\varepsilon(x)\,S(x) + {}^1/_2\,[S(\nabla),\ \varepsilon(x)]_-\,\Delta(x). \quad (14)$$

***See, for instance, Refs. 11 and 18. The proof of this statement given in Ref. 12 is erroneous, as it is based on the assumption that the matrix $(L_k p_k + K)^{-1}$ can be diagonalized. But all the relativistically invariant equations satisfying the conditions stated in the footnote *, except the Dirac and the Deffin-Kemmer equations, cannot be diagonalized (see Ref. 3, p. 726).

If the differential operator $S(\nabla)$ contains derivatives of order not higher than one, as it does only in the case of particles described by the Dirac equation, then $[S(\nabla),\ \epsilon(x)]\,\Delta(x) \equiv 0$, because $\epsilon(x)$ depends only on x_0 and

$$\tfrac{1}{2}[\partial/\partial x_0,\ \varepsilon(x)]_-\Delta(x) = \delta(x_0)\,\Delta(x) \equiv 0.$$

It is easy to see that in the general case

$$\tfrac{1}{2}[S(\nabla),\ \varepsilon(x-x')]_-\Delta(x-x')$$

$$= f(L,\ \nabla)\,\delta(x-x'), \qquad (15)$$

where $F(L, \Delta)$ is some polynomial in the matrix L_k and in the differential operators $\nabla_k = \partial/\partial x_k$, and $\partial(x-x')$ is a four-dimensional ∂-function. Insofar as $\epsilon(x-x')$ depends only on time coordinates, the function F has a non-invariant form. To give a relativistically invariant form to all the formulas, it is necessary to change somewhat the definition of $\epsilon(x-x')$. For this purpose let us introduce the ensemble of space-like surfaces $\sigma(x)$, in such a way that one and only one of the surfaces would go through each point x of the universe. Let us denote by n_μ the unit vectors orthogonal to these surfaces. Let us introduce the notation: $x > x'$ if $\sigma(x)$ lies in the "future" with respect to $\sigma(x')$ and $x < x'$ in the opposite case. By the symbol $\epsilon(x, x')$ we will mean

$$\varepsilon(x,\ x') = \begin{cases} 1 & x > x' \\ -1 & x < x'. \end{cases}$$

with this definition, the relationship between $\Delta(x-x')$ and $\Delta(x-x')$ retains its form

$$\overline{\Delta}(x-x') = \varepsilon(x,\ x')\,\Delta(x-x').$$

Instead of (15) and (14) we get:

$$\tfrac{1}{2}[S(\nabla),\ \varepsilon(x,\ x')]_-\Delta(x-x')$$

$$= F(L_\mu n_\mu,\ \nabla_\nu n_\nu,\ \Box)\,\delta(x-x'), \qquad (16a)$$

$$\overline{S}(x-x') = \tfrac{1}{2}\varepsilon(x,\ x')\,S(x-x')$$

$$+ F(L_\mu n_\mu,\ \nabla_\nu n_\nu,\ \Box)\,\delta\ (x-x'), \qquad (16b)$$

where F is some polynomial in $= \nabla_\lambda \nabla_\lambda$, in $\nabla_\lambda n_\lambda$, and in the matrices $L_\mu n_\mu$. It is important to note that the quantities n_λ are involved in F in such a manner that if one puts $n_\lambda \equiv 0$, then $F=0$.

In particular, it follows from (16b) that $< P'(\psi(x), \psi(x') >_0$ [++] is in general not equal to $S_F\ (x-x')$, as it is in the case of quantum electrodynamics. Indeed, it follows from (12) that:

$$\langle P'(\psi(x),\ \overline{\psi}(x'))\rangle_0$$

$$= \tfrac{1}{2}\varepsilon(x,\ x')\,S(x-x') - (i/2)\,S^{(1)}(x-x').$$

Using Eqs. (16b) and (13a), we get

$$\langle P'(\psi(x),\ \overline{\psi}(x'))\rangle_0 = S_F(x-x')$$

$$- F(L_\mu n_\mu,\ \nabla_\nu n_\nu,\ \Box)\,\delta(x-x'). \qquad (17)$$

3. Knowing the Green's function, one can solve Eq. (1) by the method of successive approximations. For this purpose it is easier to replace (1) by their equivalent integral expressions. If one uses the retarded potential, the latter will have the form

$$\psi(x) = \psi^{(i)}(x)$$

$$- ie\int S_r(x-x')\,L_\mu A_\mu(x')\,\psi(x')\,d^4x',$$

$$A_\mu(x) = A_\mu^{(i)}(x)$$

$$- e\int D_r(x-x')\,j_\mu(x')\,d^4x'. \qquad (18)$$

If, on the other hand, one uses the advanced potential, one will get

$$\psi(x) = \psi^{(f)}(x)$$

$$- ie\int S_a(x-x')\,L_\mu A_\mu(x')\,\psi(x')\,d^4x', \qquad (19)$$

$$A_\mu(x) = A_\mu^{(f)}(x) - e\int D_a(x-x')\,j_\mu(x')\,d^4x',$$

where $\psi^{(i)}$, $A^{(i)}$ and $\psi^{(f)}$, $A^{(f)}$ satisy the equations and the commutation relations of the free fields. Let us assume that the interaction is adiabatically turned on at $t = x_0 = -\infty$ and turned off at $t = +\infty$. Then, according to (13d)

[++] $$P'(\psi(x),\ \overline{\psi}(x')) = \begin{cases} \psi(x)\,\overline{\psi}(x') & x > x' \\ \alpha\,\overline{\psi}(x')\,\psi(x) & x < x' \end{cases}$$

$\alpha = \pm 1$; plus sign for integer spin, minus – for half integer.

$$\psi(x)\to\psi^{(i)}(x),$$

$$A_\mu(x)\to A_\mu^{(i)}(x)\quad\text{for}\quad t\to-\infty,$$

$$\psi(x)\to\psi^{(f)}(x),$$

$$A_\mu(x)\to A_\mu^{(f)}(x)\quad\text{for}\quad t\to+\infty.\qquad(20)$$

This leads to the physical interpretation of $\psi^{(i)}$, $A^{(i)}$ and of $\psi^{(f)}$, $A^{(f)}$; $\psi^{(i)}$ and $A^{(i)}$ describe the corresponding fields when $t\to-\infty$ "before the interaction has been turned on", and $\psi^{(f)}$ and $A^{(f)}$ describes them when $t\to+\infty$, "after the interaction has been turned off." These operators have to satisfy the relations [8]

$$\psi^{(f)}(x)=S^{-1}\psi^{(i)}(x)\,S;$$

$$A_\mu^{(f)}(x)=S^{-1}A_\mu^{(i)}(x)\,S,\qquad(21)$$

where S is the scattering matrix.

On the other hand, using Eqs. (18) and (19), one can express $\psi^{(f)}(x)$ and $A^{(f)}(x)$ in terms of $\psi^{(i)}(x)$ and $A^{(i)}(x)$, as a power series in e. For this purpose let us subtract (18) from (19) and use equation (13c). We get

$$\psi^{(f)}(x)=\psi^{(i)}(x)$$

$$+ie\int S(x-x')\,L_\mu A_\mu(x')\,\psi(x')\,d^4x',\qquad(22)$$

$$A_\mu^{(f)}(x)=A_\mu^{(i)}(x)+e\int D(x-x')\,j_\mu(x')\,d^4x'.$$

It is easy to get the desired expressions of $\psi^{(f)}$ and $A^{(f)}$ in terms of $\psi^{(i)}$ and $A^{(i)}$. First, one has to apply an iteration method to (18) to get an expression of $\psi(x)$ and $A(x)$ in terms of $\psi^{(i)}(x)$ and $A^{(i)}(x)$, and then substitute these expressions in Eqs. (22).

4. Let us now determine the elements of the S-matrix. Let us first note that $\psi^{(f)}(x)$ and $\psi^{(i)}(x)$ describe free fields, and that they can therefore be presented in the form***

$$\psi^{(f)}(x)=V^{-1/2}$$

$$\times\sum_p\{c_p^{(f)}\varphi^+(p)\,e^{ipx}+c_p^{(f)*}\varphi^-(p)\,e^{-ipx}\},$$

$$\psi^{(i)}(x)=V^{-1/2}$$

$$\times\sum_p\{c_p^{(i)}\varphi^+(p)\,e^{ipx}+c_p^{(f)*}\varphi^-(p)\,e^{-ipx}\},\qquad(23)$$

where $C_p^{(i)*}$ and $C_p^{(i)}$ are the creation and annihilation operators in the occupation number representation "before the interaction has been turned on", and $C_p^{(f)*}$ and $C_p^{(f)}$ – the same operators, "after the interaction has been turned off". It follows from (21) that

$$c_p^{(f)}=S^{-1}c_p^{(i)}\,S.\qquad(24)$$

Analogously, one can write

$$A_\mu^{(f)}(x)=V^{-1/2}$$

$$\times\sum_k\{a_k^{(f)}A_\mu(k)\,e^{ikx}+a_k^{(f)*}A_\mu^*(k)\,e^{-ikx}\},$$

$$A_\mu^{(i)}(x)=V^{-1/2}$$

$$\times\sum_k\{a_k^{(i)}A_\mu(k)\,e^{ikx}+a_k^{(i)*}A_\mu^*(k)\,e^{-ikx}\}.\qquad(25)$$

The physical interpretation of the operators $a_k^{(f)*}$, $a_k^{(f)}$ and $a_k^{(i)*}$, $a_k^{(i)}$ corresponds to the interpretation of $C^{(F)}$ and $C^{(i)}$. It follows from (21) that

$$a_k^{(f)}=S^{-1}a_k^{(i)}\,S.\qquad(24')$$

Let us denote by ϕ_0 the vacuum wave function of the system. One can write

$$S\Phi_0=\Phi_0.\qquad(26)$$

The element of the S-matrix corresponding to the transition from the state (i) at $t\to-\infty$(which involved the considered particles in the states p_1, p_2, . . . and photons in the states k_1, k_2, . . .) to the state (F) $\sim(p_1', p_2' \ldots;\ k_1, k_2', \ldots\ldots)$

***In equation (23), the symbol ρ denotes the whole set of quantum numbers which define the state of the considered particle. In analogy, in Eq. (25), k denotes the set of momentum 4-vectors and the polarization of the photon; px and kx represent the scalar products of the 4-momentum and of the 4-vector of the point x.

at $t \to +\infty$ is given by the formula

$$S_{(i)}^{(f)} = \langle c_{p_1}^{(i)} \cdots a_{k_1}^{(i)} \cdots S c_{p_1}^{(i)*} \cdots a_{k_1}^{(i)*} \cdots \rangle_0, \quad (27)$$

where $\langle \ \rangle_0$ is the expectation value over the vacuum ϕ_0. Expressing $C_p^{(i)}$ and $a_k^{(i)}$ through $C_p^{(f)}$ and $a_p^{(f)}$ with the help of (24), and taking into account (26), let us rewrite Eq. (27) in the form

$$S_{(i)}^{(f)} = \langle c_{p_1}^{(f)} \cdots a_{k_1}^{(f)} \cdots c_{p_1}^{(i)*} \cdots a_{1k}^{(i)*} \cdots \rangle_0. \quad (28)$$

To calculate this quantity, it is necessary to express the operators $C,^{(f)}\, a^{(f)}$ through $c,^{(i)}\, a^{(i)}$ with the required accuracy with respect to e, This can be done using the expressions of $\psi^{(f)}\ (x)$ $A^{(f)}(x)$ in terms of $\psi^{(i)}(x)$, $A^{(i)}(x)$ [obtained with the help of Eqs. (18) and (19) by the iteration method]. After this is done, it is in principle possible to calculate the vacuum expectation value of the right hand side of (28). The procedure can be applied with relative ease to get the matrix elements to the first and second order in e. It happens that these elements are such as required by the Feynman-Dyson rules. To the vertices of the diagrams correspond the matrices L_μ [involved in (1)], and, to the solid line, the function

$$S_F(x - x') = S(\nabla) \sum_{i=1}^{s} b_i \Delta_F(x - x';\ \mu_i), \quad (29)$$

where $S(\nabla)$ is given by (9),

The method given above becomes quite complicated when applied to compute the matrix elements $S_{(i)}^{(f)}$ to any order in e. It is however possible to show that even in this case, the matrix elements will have a form corresponding to the Dyson-Feynman rules.

Indeed, it follows from what was said above, that the calculation of $S_{(i)}^{(f)}$ involves only the expression of the operators $\psi^{(f)}$, $A^{(f)}$ through $\psi^{(i)}$, $A^{(i)}$ the formulas (12), the Green's functions (10) and the relations (13), (16b) between them. All of these are absolutely analogous to their equivalents in quantum electrodynamics – with only one exception: the formula (16b) has a second term, depending on the normals to the space-like surfaces, which vanishes identically in the quantum electrodynamic case. The effect of this form is such that the expression of the matrix element in the general case has the form

$$S_{(i)}^{(f)} = S_{(i)}^{\prime(f)} + S_{(i)}^{\prime\prime(f)}, \quad (30)$$

where $S_{(i)}^{\prime(F)}$ is the term which would be obtained if one set $F(L_\mu n_\mu, \nabla_\nu n_\nu, \quad) = 0$ in (16b), and $S_{(i)}^{\prime\prime(F)}$ is the term containing $F(L_\mu n_\mu, \nabla_\nu n_\nu, \quad)$, $S_{(i)}^{\prime\prime(F)}$ vanishing if one sets $S = 0$. In quantum electrodynamics $S = S'$, and S' as is well known, is given by the Feynman-Dyson rules. Hence, in general, S' has also to be constructed according to these rules. Let us now show that $S'' = 0$ in the general case as well as for spin ½ particles. For this purpose, let us note that the elements of the S-matrix cannot depend on n_μ – as it can be seen, for instance, from Eq. (21). Hence S'' also does not depend on n_μ and therefore does not depend on $F(L_\mu n_\mu, \nabla_\nu n_\nu, \quad)$ (because this function vanishes if $n_\mu = 0$). This is possible if, and only if $S'' = 0$.

In the matrix elements of any order in e, the terms depending on the normal to space-like surfaces have to compensate each other, and these elements are constructed according to the Feynman-Dyson rules – in the sense given above. For matrix elements of the first and second order, this was verified directly.[6]

After the completion of this work **** a paper[13] was published in which the author considers the commutation relations, the Green's function and the S-matrix for spin 3/2 particles, described by the Pauli-Fierz equation.[1] The results of this work are contained in ours, as a special case.

The author wishes to express his deep gratefulness to Prof. V. L. Ginzburg, and also to E. S. Fradkin and V. Ia. Fainberg for valuable discussion of the results of this work.

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On the Scattering of Photons by Nucleons

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The crosss section for scattering of photons against nucleons is computed on the basis of a semiphenomenological isobaric theory including absorption.

THE fact that a nucleon is surrounded by a meson field leads to a scattering cross section for photons by nucleons, which differs from the well-known Klein-Nishina formula. By analogy with the scattering and photoproduction of mesons by nucleons, one may expect that in this case, too, the cross section will exhibit a resonance character as a function of energy. Yet, attempts to carry out corresponding calculations meet with the principal difficulties inherent in all present meson theories. But if one includes the meson field in a semiphenomenological way by introducing isobaric states[1,2], then the computations can be carried to completion, as indeed is done in the present note*.

The Lagrangian for the interaction between the nucleon and the photons has the form**:

$$L = \sqrt{4\pi} e \bar{\psi} \tau \hat{A} \psi - \sqrt{4\pi} \frac{e}{M} \bar{B}_\mu N F_{\mu\nu} \gamma_\nu \gamma_5 \psi + \sqrt{4\pi} \frac{e}{M} \bar{\psi} N^+ F_{\mu\nu} \gamma_\nu \gamma_5 B_\mu; \qquad (1)$$

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu};$$

$$A_\mu = \frac{e_\mu}{\sqrt{2\omega}} (c e^{ikx} + c^+ e^{-ikx}),$$

where $\tau = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ is the nucleonic charge operator,

$$N = \left(\begin{array}{cc|cccc} \multicolumn{2}{c|}{\multirow{2}{*}{0}} & 0 & a & 0 & 0 \\ & & 0 & 0 & b & 0 \\ \hline 0 & 0 & & & & \\ a^+ & 0 & \multicolumn{4}{c}{0} \\ 0 & b^+ & & & & \\ 0 & 0 & & & & \end{array} \right)$$

is a Hermitian matrix in isotopic spin space describing transitions from ground to isobaric states, and vice versa.

The incident photon shall be characterized by the polarization vector **e** and the 4-momentum $k(\mathbf{k}, i\omega)$; we shall denote the 4-momentum of the nucleon in the initial state by $p(\mathbf{p}, iE)$. The corresponding quantities in the final state shall be denoted by $\mathbf{e}'$, k' and p'. The computations shall be carried out in the center-of-mass system. The second order matrix elements U are sums of four terms corresponding to the four possible scattering reactions (two of these are the usual ones, and the other two include an isobar in the intermediate state). We represent the nucleon wave function ψ as a product of u_p, a function of the coordinates and the mechanical spin and χ, a function of the isotopic spin; then:

$$U = \chi'^+ \tau^+ \tau \chi (A + B) + \chi'^+ N^+ N \chi (C + D); \qquad (2)$$

$$A = (\sqrt{4\pi} e)^2 (2\omega)^{-1} \bar{u}_{p'} \hat{e}' (\hat{p} + \hat{k} + M)^{-1} \hat{e} u_p;$$

$$B = (\sqrt{4\pi} e)^2 (2\omega)^{-1} \bar{u}_{p'} \hat{e} (\hat{p} - \hat{k}' + M)^{-1} \hat{e}' u_p;$$

$$C = \left(\sqrt{4\pi} \frac{e}{M} \right)^2 \frac{1}{2\omega} \bar{u}_{p'} (\hat{e}' k'_\mu - e'_\mu \hat{k}') \times K_{\mu\nu} (p + k) (\hat{e} k_\nu - e_\nu \hat{k}) u_p;$$

$$D = \left(\sqrt{4\pi} \frac{e}{M} \right)^2 \frac{1}{2\omega} \bar{u}_{p'} (\hat{e} k_\mu - e_\mu \hat{k}) \times K_{\mu\nu} (p - k') (\hat{e}' k'_\nu - e'_\nu \hat{k}') u_p.$$

The isobaric propagator $K_{\mu\nu}$ has the form[5]

$$K_{\mu\nu}(p) = (\hat{p} - \varkappa)^{-1} \left\{ [-i\gamma_\mu (\hat{p} + 3\varkappa) + 2p_\mu] \frac{p_l}{6\varkappa^2} + \delta_{\mu l} \right\} \left(\delta_{l\nu} - \frac{1}{4} \gamma_l \gamma_\nu \right),$$

where $\varkappa$ is the mass of the isobar.

Inasmuch as the expressions for A, B, C and D

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* A similar calculation has already been carried out in Ref. 3. However, the author treats absorption in an approximate fashion by including it only at resonance. In the present article, absorption is treated according to the method of Heitler (see Ref. 4), following the scheme developed in Ref. 2.

** We use here Pauli's notation.

are very cumbersome, let us introduce here coefficients for expanding these quantities into the polynomials $L_{ik}^{\Phi'\Phi}$ of Ref. 6:

$$A=(\sqrt{4\pi}e)^2\sum_{i,\,k=0}^{5}a_{ik}L_{ik}^{\Phi'\Phi}. \qquad (3)$$

$$B=(\sqrt{4\pi}e)^2\sum_{i,\,k=0}^{5}b_{ik}L_{ik}^{\Phi'\Phi},$$

$$C=\left(\sqrt{4\pi}\frac{e}{M}\right)^2\frac{\omega}{4(\varepsilon-\varkappa)}\sum_{i,\,k=0}^{5}c_{ik}L_{ik}^{\Phi'\Phi};$$

$$D=\left(\sqrt{4\pi}\frac{e}{M}\right)^2\frac{E+M}{8E}\sum_{i,k=0}^{5}d_{ik}L_{ik}^{\Phi'\Phi}.$$

where $E=\sqrt{M^2+\omega^2}$ is the energy of the nucleon, and $\epsilon=E+\omega$ is the total energy of the system. The nonzero coefficients of A and C are:

$$a_{00}=-\frac{E+M}{4\omega E(\varepsilon+M)},\quad a_{11}=-\frac{1}{4E(\varepsilon+M)},$$

$$c_{00}=\frac{M^2(\varepsilon-\varkappa)(\varepsilon+2\varkappa)}{3\varkappa^2\varepsilon E};$$

$$c_{11}=\frac{M^2(\varepsilon-\varkappa)(\varepsilon-2\varkappa)}{3\varkappa^2\varepsilon E};$$

$$c_{22}=\frac{5E+3M+4\omega}{12E};\quad c_{33}=\frac{\omega(\varepsilon-M)}{4E(\varepsilon+\varkappa)};$$

$$c_{44}=\frac{(\varepsilon-\varkappa)(5E+3M+4\omega)}{12E(\varepsilon+\varkappa)};$$

$$c_{55}=\frac{\omega^2(\varepsilon-\varkappa)}{4E(\varepsilon+\varkappa)(E+M)};$$

$$c_{23}=c_{32}=\frac{\omega(\varepsilon+E+2M)}{4\sqrt{3}E(E+M)};$$

$$c_{45}=c_{54}=-\frac{\omega(\varepsilon-\varkappa)(\varepsilon+E+2M)}{4\sqrt{3}E(\varepsilon+\varkappa)(E+M)}.$$

The coefficients b_{ik} are related as follows to the auxiliary quantities $\beta_1, \beta_2, \ldots, \beta_{10}$:

$$b_{00}=4f_0(\beta_1-\alpha\beta_2-\alpha\beta_3+\beta_5+{}^1/_2\beta_7+{}^1/_2\beta_{10})+{}^1/_2\ln x(\beta_4-\beta_3)+f_2(\beta_8+\beta_9);$$

$$b_{11}=4f_0(\beta_2+\beta_4-\alpha\beta_5-\alpha\beta_6+{}^1/_2\beta_8+{}^1/_2\beta_9)+{}^1/_2\ln x(\beta_1-\beta_6)+f_2(\beta_7+\beta_{10});$$

$$b_{22}=4f_0(\beta_4-\alpha\beta_5-2\alpha\beta_7+2\beta_8+{}^1/_2\beta_9-\alpha\beta_{10})-{}^1/_2f_1(2\beta_6+\beta_7)$$
$$+{}^1/_8\ln x\beta_{10}+f_3(-\beta_1+\alpha\beta_2)+f_5\beta_9;$$

$$b_{33}=3f_0(-2\beta_3+\beta_4-\alpha\beta_5+{}^1/_2\beta_9)+3f_1\beta_6+f_3(-3\beta_1-3\alpha\beta_2-2\beta_{10})$$
$$+f_4\beta_7+f_5(4\beta_3+{}^4/_3\beta_8+{}^1/_3\beta_9);$$

$$b_{44}=f_0(\beta_1-\alpha\beta_2-3\beta_6-2\alpha\beta_9+{}^1/_2\beta_{10})+f_1(2\beta_3-{}^1/_2\beta_9)+f_2\beta_8$$
$$+f_3(-\beta_4+\alpha\beta_5)-\alpha f_4\beta_7+2f_5\beta_6+{}^1/_4\beta_7;$$

$$b_{55}=3f_0(\beta_1-\alpha\beta_2-\beta_6+{}^1/_2\beta_7)+f_3(-3\beta_4+3\alpha\beta_5-2\beta_8)+f_4\beta_9$$
$$+f_5(2\beta_6+{}^1/_3\beta_7+{}^4/_3\beta_{10});$$

$$b_{23}=b_{32}=\sqrt{3}f_0(\beta_3-\beta_4+\alpha\beta_5+{}^1/_2\beta_9)+\frac{\sqrt{3}}{2}f_1\beta_7+\sqrt{3}f_3(\beta_1-\alpha\beta_2)$$
$$-\frac{1}{\sqrt{3}}f_5(2\beta_3+\beta_9);$$

$$b_{45}=b_{54}=\sqrt{3}f_0(\beta_1-\alpha\beta_2-2\beta_6-{}^1/_2\beta_7)+\frac{\sqrt{3}}{2}f_1(2\beta_3-\beta_9)$$
$$+\sqrt{3}f_3(-\beta_4+\alpha\beta_5)+\frac{1}{\sqrt{3}}f_5(4\beta_6+\beta_7),$$

where

$$f_0={}^1/_4\left(1-\frac{\alpha}{2}\ln x\right);\quad f_1=\alpha f_0+{}^1/_8\ln x;\quad f_2=-\alpha f_0+{}^1/_8\ln x;$$
$$f_3={}^3/_2\,\alpha f_0+{}^1/_{16}\ln x;$$
$$f_4=-\frac{3}{2}\alpha f_0+\frac{1}{16}\ln x;\quad f_5=\frac{3}{2}\alpha^2 f_0+\frac{1}{8};\qquad x=\frac{\alpha+1}{\alpha-1}.$$

Therefore,

$$\alpha = E/\omega; \quad \beta_1 = 1 + \beta^{-1}; \quad \beta_3 = 2;$$

$$\beta_4 = \beta + 1; \quad \beta_6 = 2;$$

$$\beta_7 = -2\beta_1; \quad \beta_8 = -2(\beta + 1);$$

$$\beta_2 = \beta_5 = \beta_9 = \beta_{10} = 0; \quad \beta = (E + M)/\omega.$$

The coefficients d_{ik} are analogously related to the auxiliary quantities $\delta_1, \delta_2, \ldots, \delta_{10}$. Thus,

$$\alpha = (\varkappa^2 - M^2 + 2E\omega)/2\omega^2;$$

$$\delta_1 = \frac{1}{\omega}\left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{6\varkappa^2}(-M^2 + 3\varkappa M + 6\varkappa^2) - \frac{\varepsilon + M}{(E + M)^2}\left(E - \omega - \frac{ME\varepsilon}{3\varkappa^2}\right)$$
$$- \frac{\omega}{3\varkappa^2 (E + M)^2}(M\varepsilon^2 + 6\varkappa^2 M + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_2 = \frac{\omega}{3\varkappa^2 (E + M)^2}(M^2\varepsilon - 6\varkappa^2\varepsilon + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_3 = -\frac{\varepsilon + M}{(E + M)^2}\frac{M + \varkappa}{3\varkappa^2}(-M^2 + 3M\varkappa + 3\varkappa^2) + \frac{2\varepsilon}{E + M};$$

$$\delta_4 = -\frac{1}{\omega}\left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{6\varkappa^2}(-M^2 + 3M\varkappa + 6\varkappa^2) - \frac{\varepsilon + M}{\omega (E + M)}\left(E - \omega + \frac{ME\varepsilon}{3\varkappa^2}\right)$$
$$+ \frac{1}{3\varkappa^2\omega}(M\varepsilon^2 + 6\varkappa^2 M + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_5 = -\frac{\varepsilon + M}{E + M}\left(2 + \frac{M\varepsilon}{3\varkappa^2}\right) - \frac{1}{3\varkappa^2\omega}(-M\varepsilon^2 + 6\varkappa^2 M + 2\varkappa M^2 + 3\varkappa^3);$$

$$\delta_6 = \frac{\varepsilon + M}{E + M}\frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M + 3\varkappa^2) + \frac{2\varepsilon}{E + M};$$

$$\delta_7 = -\left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M + 3\varkappa^2)$$
$$- \frac{2}{3\varkappa^2 (E + M)^2}[(\varepsilon + M)(M^2 E + 3\varkappa^2\omega) - \omega (M + \varkappa)(-M^2 + 3\varkappa M)];$$

$$\delta_8 = \left(\frac{\varepsilon + M}{E + M}\right)^2 \frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M + 3\varkappa^2)$$
$$- \frac{\varepsilon + M}{E + M}\frac{2}{3\varkappa^2\omega}(M^2 E + 3\varkappa^2 M) - 2\,\frac{M + \varkappa}{3\varkappa^2\omega}(-M^2 + 3\varkappa M);$$

$$\delta_9 = \frac{2\omega}{3\varkappa^2}\,\frac{(\varepsilon - \varkappa)(-M^2 + 3\varkappa M)}{(E + M)^2}; \quad \delta_{10} = \frac{2}{3\varkappa^2\omega}(\varepsilon + \varkappa)(-M^2 + 3\varkappa M).$$

In order to determine the scattering cross section including absorption, we apply Heitler's integral equation[4] which yields for the scattering of light by nucleons the following equations:

$$F(p\gamma) = U(p\gamma, p\gamma)$$
$$- i\eta_q \int U(p\gamma, n^+) F(n^+)\, d\Omega$$
$$- i\eta_q \int U(p\gamma, p^0) F(p^0)\, d\Omega$$
$$- i\eta_\gamma \int U(p\gamma, p\gamma) F(p\gamma)\, d\Omega;$$

$$F(n^+) = U(n^+, p\gamma) - i\eta_q \int U(n^+, n^+) F(n^+)\, d\Omega$$
$$- i\eta_q \int U(n^+, p^0) F(p^0)\, d\Omega$$
$$- i\eta_\gamma \int U(n^+, p\gamma) F(p\gamma)\, d\Omega;$$

$$F(p^0) = U(p^0, p\gamma) - i\eta_q \int U(p_0, n^+) F(n^+)\, d\Omega$$
$$- i\eta_q \int U(p^0, p^0) F(p^0)\, d\Omega$$
$$- i\eta_\gamma \int U(p^0, p\gamma) F(p\gamma)\, d\Omega,$$

where

$$\eta_q = \frac{q\sqrt{q^2 + \mu^2}\sqrt{q^2 + M^2}}{8\pi^2(\sqrt{q^2 + \mu^2} + \sqrt{q^2 + M^2})},$$

$$\eta_\gamma = \frac{\omega^2 E}{8\pi^2(\omega + E)};$$

and where q is the momentum of the meson.

Here, for example, $U(n^+, p\gamma)$ and $F(n^+)$ are the matrix element and amplitude for the photoproduction process $\gamma + p \to n + \pi^+$. These equations can be solved by expanding the F's and the U's in the orthogonal polynomials $L_{ik}^{\Phi'\Phi}$, $L_{ik}^{M\Phi}$ and $L_{k}^{M'M}$ of Refs. 2 and 6. A simple calculation yields the result

$$F(p\gamma) = \sum_{i,\,k=0}^{5} f_{ik} L_{ik}^{\Phi'\Phi};$$

$$f_{ik} = \frac{u_{ik}}{1 + 4\pi\eta_q\left(u_{(k)}^{1/2} + u_{(k)}^{3/2}\right)},$$

where (k) is an integer equal to either $(k+1)/2$ or $(k+2)/2$, and by means of Eqs. (1)-(3),

$$u_{ik} = (\sqrt{4\pi}e)^2\Big[a_{ik} + b_{ik}$$
$$+ \left(\frac{a}{M}\right)^2\left(\frac{\omega}{4(\varepsilon - \varkappa)}c_{ik} + \frac{E + M}{8E}d_{ik}\right)\Big];$$

where $u_k^{1/2}$ and $u_k^{3/2}$ are the expansion coefficients of the meson scattering matrix for total isotopic spin $I = 1/2$ and $I = 3/2$ (see Ref. 2). The differential cross section in the center-of-mass system is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{4\pi^2}\frac{\omega^2 E^2}{\varepsilon^2}\frac{1}{4}\sum_{e'e}\mathrm{Sp}\,|F|^2,$$

where $$\frac{1}{4}\sum_{e'e}\mathrm{Sp}\,|F|^2 = M_0 + M_1\cos\vartheta$$
$$+ M_2\frac{1}{2}(3\cos^2\vartheta - 1) + M_3\cos^3\vartheta,$$

and where ϑ is the angle between **k** and **k**′, and where

$$M_0 = 2(\varphi_{00,00} + \varphi_{11,11}) + 4(\varphi_{22,22} + \varphi_{33,33} + \varphi_{44,44} + \varphi_{55,55}) + 8(\varphi_{23,23} + \varphi_{45,45})$$
$$M_1 = 2(2\varphi_{00,11} + \varphi_{00,22} + 3\varphi_{00,33} + \varphi_{11,44} + 3\varphi_{11,55} + 5\varphi_{22,44} - 3\varphi_{22,55}$$
$$- 3\varphi_{33,44} - 3\varphi_{33,55}) + 2\sqrt{3}(\varphi_{00,23} - \varphi_{22,45} + \varphi_{11,45} - 3\varphi_{44,54} - \varphi_{44,23} - 3\varphi_{55,23});$$
$$M_2 = \varphi_{22,22} + \varphi_{33,33} + \varphi_{44,44} + \varphi_{55,55} + 4(\varphi_{23,23} + \varphi_{45,45}) + 2(\varphi_{00,44} + 3\varphi_{00,55}$$
$$+ \varphi_{11,22} + 3\varphi_{11,33} + 3\varphi_{22,33} + 3\varphi_{44,55}) + 4\sqrt{3}(\varphi_{00,45} + \varphi_{11,23} - \varphi_{22,23}$$
$$+ \varphi_{33,23} - \varphi_{44,45} + \varphi_{55,45});$$
$$M_3 = 4(3\varphi_{22,55} + 3\varphi_{33,44} + 4\varphi_{33,55} + 6\varphi_{23,45}) + 16\sqrt{3}(\varphi_{33,45} + \varphi_{55,23});$$
$$\varphi_{ik,\,lm} = \frac{1}{2}(f_{ik}^{+}f_{lm} + f_{lm}^{+}f_{ik}).$$

Integrating over angles, we finally obtain the total scattering cross section for the scattering of light against protons.

$$\sigma = (\omega^2 E^2 / \pi\varepsilon^2)\,M_0.$$

Numerical computations were carried out with this formula. The values of $u_k^{1/2,\,3/2}$ were taken from Ref. 2, and $a = 1.61$ in accordance with Ref. 6. The following values of σ were obtained as a function of ω (in the laboratory system):

ω (mev)	280	340	400
$\sigma\cdot 10^{-31}$	11.6	21.3	20.3

As could be expected, the cross section has a maximum at $\epsilon = \varkappa$ in the center-of-mass system, corresponding to $\omega = 340$ mev in the laboratory system.

After obtaining the above results, the author found out that an analogous computation had been carried out by V. I. Ritus. The methods of solution differ, however, and so do the numerical results (which may be due to the choice of constants).

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Letters to the Editor

The Role of Radiational Losses in Cyclic Accelerators

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WE showed in Refs. (1,2) that during the movement of relativistic electrons in cyclic accelerators, there appears a strong damping of betatron oscillations. In the present work we give the generalization of the results of Refs. 1 and 2.

We start with the classical relativistic equation of Dirac-Lorentz for the motion of an electron (Ref. 3).

$$mc\frac{du_i}{ds} = \frac{e}{c}F_{ik}u_k + \frac{2e^2}{3e}\left[\frac{d^2u_i}{ds^2} + u_iu_k\frac{d^2u_k}{ds^2}\right], \quad (1)$$

where u_i are components of 4-velocity of an electron, F_{ik} — components of a tensor of the electromagnetic field. The sum in the parenthesis is the self-action force due to the presence of radiation.

Let us consider the movement of an electron in an axially symmetric magnetic field $H(r)$ in the neighborhood of an orbit of a constant radius R with an accelerating electric field $\mathcal{E}_r = \mathcal{E}_z = 0$, $\mathcal{E}_\theta = \mathcal{E}(\theta, t)$. In a linear approximation we obtain equations from Eq. (1) which are necessary for the description of the radial and azimuthal motion:

$$\ddot{\rho} + \left(\frac{\dot{E}}{E} + \frac{W}{E}\right)\dot{\rho} + (1-n)\frac{c^2}{R^2}\rho \quad (2)$$

$$= \frac{c^2}{R^2}\frac{E - eH_0R}{eH_0R} + \frac{R^2}{c^2}\left(\frac{mc^2}{E}\right)^2\frac{W}{E}\dddot{\rho},$$

$$\dot{E} = e\mathcal{E}c - W + eH_0R\rho, \quad (3)$$

$$\dot{\theta} = (c/R)(1-\rho) = c/R - \dot{\varphi}, \quad (4)$$

where the following dependence was used: $H = H_0(1 - n\rho)$, $\rho = r/R - 1$, E = energy of an electron, $W = \left(\frac{2e^2c}{3r^2}\right)\left(\frac{E}{mc^2}\right)^4$ = magnitude of the radiational losses.

The term containing $\dddot{\rho}$ in Eq. (2) describes the "proper" attenuation of the betatron oscillations. Practically, it does not influence the motion in comparison with the main dissipation $\sim W\rho/E$, if $|1-n| \ll (E/mc^2)^2$, which is always fulfilled. In the main orbit of an accelerator, the radiational losses are compensated, and the energy of the electrons grows in time in correspondence with the growth of the magnetic field. The solution of equations (2) – (4) appears as follows: $\rho = \rho_M + \rho_{bet}$, with

$$\rho_M = (E - eH_0R)/eH_0R(1-n), \quad (5)$$

where ρ_{bet} are betatron oscillations, with the frequency $(c/R)\sqrt{1-n}$, which take place near the instantaneous orbit with the radius $R(1 + \rho_M)$. From Eqs. (2) – (5) we obtain an equation of the synchrotron and betatron oscillations (Ref. 4), whose amplitude changes according to the following law *

$$a_{bet} \sim E^{-1/2}\exp\left[-1/2\,\frac{n}{(1-n)}\int_{t_0}^{t}\frac{W}{E}\,dt\right]; \quad (6)$$

$$a_M \sim E^{-3/4}\left[e\dot{H}_0R + W\right]^{1/4}\exp\left[-\frac{1}{2}\frac{3-4n}{1-n}\int_{t_0}^{t}\frac{W}{E}\,dt\right].$$

These expressions are the generalization of the laws of variation of $a_{bet} \sim E^{-1/2}$, $a_M \sim E^{-3/4}$ for the case when the radiation losses, which are compensated by the external field (accelerator), are present.

If the acceleration takes place in the curl field (betatron), then, for the presence of radiation, the condition $\dot{\bar{H}} = 2H$ is changed in the corresponding manner so that the condition R = const. holds. In this case, for ρ_{bet} we obtain the same law (Eq. 6) and

$$\rho_M \sim E^{-1}\exp\left[-\frac{3-4n}{(1-n)}\int_{t_0}^{t}\frac{W}{E}\,dt\right]. \quad (6a)$$

We note that damping of the type (6) is in fact not connected with the actual form of radiation losses, which are compensated by the external field. An analogous result could be obtained, for instance, in the case of losses for the bremsstrahlung on the nuclei of the residual gas. The quantity W, which appears in Eq. (6), can have the interpretation of power spent by the accelerating system for the compensation of all similar losses.

Until now all our results followed completely from the classical equation of Dirac-Lorentz (Eq. 1). We want to consider the corrections due to the consideration of quantum effects. For the description of the motion of the electron up to the energies $\dot{E} \ll E_{1/3} = mc^2(Rmc/\hbar)^{1/3}$($\approx 10^{15}$ ev for usual conditions) one can, with good reason, use classical concepts and consider only the statistical (quantum) character of radiation. For this purpose the following term should be added to Eq. (3):

$$W(R) - \sum_i \varepsilon_i \delta(t - t_i), \tag{7}$$

where ϵ_i is the energy of a separate photon, emitted at the moment t_i, δ is the delta function. The physical meaning of the changes introduced in the motion because of the introduction of (7) into (3), was considered by us in Refs. 1 and 2. The solution of Eqs. (2), (7), (4) gives for the average square values of $\bar{\rho}^2_{\text{bet}}$ and $\bar{\rho}^2_{\text{M}}$ very similar expressions which also hold for the synchrotron (Refs. 5,6) and for the betatron (Refs. 1, 2).

$$\overline{\rho^2_{\text{bet}}} \lessapprox \frac{55\sqrt{3}}{96} \frac{\Lambda}{Rn(1-n)} \left(\frac{E}{mc^2}\right)^2; \tag{8}$$

$$\rho_{\text{M}} \lessapprox \frac{55\sqrt{3}}{96} \frac{\Lambda}{R(1-n)(3-4n)} \left(\frac{E}{mc^2}\right)^2,$$

where $\Lambda = \hbar / \nu c$ is the ${}_2$Compton wavelength.

The expression for $\bar{\rho}_{\text{bet}}$ was found also by a different, more complicated, method in the works of Sokolov and Ternov.[7] However, since they did not consider the damping (6), their results are erroneous.

It is interesting to follow the mechanism of damping, z_{bet}, in more detail for the case when the statistical character of losses is taken into consideration. At the time of emission of the i th quantum, a vertical recoil force acts on the electron

$$(d/dt)(m\dot{z}) = (\varepsilon_i/c^2)\,\dot{z}\delta(t - t_i)$$

(we omit quasi-elastic forces). On the other hand, $(d/dt)(m\dot{z}) = \dot{z}\dot{m} + m\ddot{z}$. The change of mass can be broken up into two parts: loss of energy during the radiation, and its receipt from the accelerating field

$$\dot{m} = -(\varepsilon_i/c^2)\,\delta(t - t_i) + \dot{m}_{\text{accel}}$$

$$m_{\text{accel}} = \frac{1}{c^2}(W + e\dot{H}_0 R).$$

It is not difficult to see that the increase of the oscillation amplitude at the expense of the mass at the time of radiation is fully compensated by the radiation friction. Consequently only the increase of the mass m_{accel} influences the amplitude of the betatron oscillations. This increase is due to the compensating field. From this, there immediately follows an equation of the type (6).

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*Here we generalize our result from Refs. 1,2.

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Superconductivity of Barides, Carbides, Nitrides and Silicides of Transition Metals

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DORFMAN and Kikoin[1] concerned themselves with the interesting fact that the transition point to the superconducting state increases in the series TiB —TiC—TiN; the same increase was noted in the series VC—VN and ZrB—Zr—C—ZrN.

The data available in the literature on the transition temperatures of similar compounds are given in Refs. 2–4. In spite of the wide spread of the transition points obtained by different investigators, one can make the following preliminary observations. The value of T_k is connected with the electron density distribution, i.e., it depends on the acceptor capability of the atom of the transition metal $1/Nn^5$ and of the ionization potential of the non metal (φ).*

In titanium compounds, the transition points are very low, and although some increase of T_k probably takes place in the series TiB_2 — TiC—TiN, i.e., with increase in the ionization potential of the metalloid, the probabilities of realization of high acceptor probability of the 3d-level of Ti are lessened. This appears more clearly for the compounds of Zr. A significant increase in the transition points takes place for compounds of Ta and Nb, but not of V, although all these elements belong to the subgroup Va. There is a high scattering capacity in vanadium, yielding only to Ti and Zr, which also brings about a lowering of the transition point.

In compounds of Nb and Ta, there is evidently a very favorable relation of the value of 1/Nn of the metal and φ of the metalloid; therefore in the series Me (V) B → Me(V) C → Me (V) N there is a more clearly expressed rise in T_k ; moreover, the transition points are high in absolute value.

For a transition to compounds of W and Mo, the values of T_k are much higher than for compounds of Ti, Zr, Hf and V, but lower than for compounds of Nb and Ta; this is possible as a consequence of the increase of screening of the d-band of these metals by the natural excitations of the electrons,[6] which makes difficult the excitation of the valence electrons of the metalloid. In each case, it is characteristic that the sharp decrease in the number 1/Nn from 0.167 – 0.100 for Ti, Zr, V, Hf to 0.5 – 0.67 for Ta, Nb, W and Mo_{+}, is accompanied by such a sharp increase in the value of T_k .

It should also be noted that in a number of cases the value T_k increases with increase in the metallic content, for example, for Nb_2N, $T_k = 9.5°$ K, but for NbN, $T_k = 15°$ K, Mo_2C, $T_k = 2{,}9°$ K, and for MoC –– 8° K, for Mo_2N, $T_k = 5°$ K, for MoN –– 12° K, for W_2C, $T_k = 2.74°$ K, for WC, 2.5 ––4.21 ° K.

The relatively lower transition values for all borides in comparison with carbides and nitrides are probably explained by the presence of strong covalent bonds between the boron atoms , which leads to the formation of the characteristic structure of the elements — little chains, lattices, shells of boron atoms in boride crystals.[7] In this connection, the fraction of electrons capable of completing the electron deficiency of the atoms of the transition metals is not large and T_k is correspondingly decreased.

Actually, in the case of all metallic compounds, especially Nb, Ta, W and Mo, the increase of T_k in the transition from Me — B to Me —C, is, as a rule, much sharper than for the transition from Me —C to Me—N. Therefore the borides are essentially different from the carbides and nitrides. The latter are close to each other in values of T_k. but the smaller ionization potential of carbon in comparison with nitrogen enhances the effect of the increase of T_k in series MeB —MeC —MeN, especially for compounds of the transition metals with low electron deficiencies. Compounds of silicon, which have still lower ionization potentials than boron ought, from this point of view, to possess still lower transition temperatures, i.e., there ought to be the series: MeSi →MeB → MeC →MeN, which actually takes place in most cases, with the exception of some silicides (V_3 Si, TaSi, W_s Si_2) whose superconductivity is related chiefly to purely structured factors, for example, in V_3 Si, which has the structure β —W).

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*N is the principal quantum number, n is the number of electrons of the incomplete d-level.

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Production of Nuclear Stars by γ-Quanta

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IN this article is investigated that particular mechanism of the production of nuclear stars by γ- quanta in which the γ-quantum creates a virtual π-meson pair at a large distance from a nucleus. This pair is thereupon absorbed by this nucleus and a star is produced. All considerations are conducted in the region of high energies $\omega >> \mu$* (where ω is the frequency of the quantum, μ is the meson rest mass).

In Ref. 1 there was considered a process in which only one member of the pion pair created by the γ-quantum is absorbed by that nucleus, creating a star, and the remaining pion carried away an energy of the order of the total energy of the star. The method used in the calculation of this process, we also use in the present case, i.e., the cross section can be found with the help of the matrix element of the radiative transition for which a form of the ψ-function describing the absorbed meson was determined in Ref. 1.

We will consider the nucleus to be perfectly black to the pion, and subsequently generalize the result to semi-transparent nuclei.

The matrix element is

$$M=\frac{2e}{i}\sqrt{\frac{2\pi}{\omega}}\int[\psi_1^*(\mathbf{j}\nabla)\,\psi_2^*]\,e^{i\omega\mathbf{r}}\,d\mathbf{r} \tag{1}$$

(we set $\hbar = c = 1$), where j is the polarization of the quantum, ψ_1 (ψ_2) is a spherical wave collapsing to an arbitrary point on the cross section of the nucleus $\mathbf{s}_1$ ($\mathbf{s}_2$).

$$\psi_1=\frac{\sqrt{p_1}}{4\pi^{3/2}}\frac{e^{-ip_1|\mathbf{r}-\mathbf{s}_1|}}{|\mathbf{r}-\mathbf{s}_1|} \tag{2}$$

$$=\frac{\sqrt{p_1}}{8\pi^{7/2}}\int\frac{e^{-i\mathbf{q},\,\mathbf{r}-\mathbf{s}_1}}{q^2-p_1^2+i\varepsilon}\,d\mathbf{q};$$

$$\psi_2=\frac{\sqrt{p_2}}{4\pi^{3/2}}\frac{e^{-ip_2|\mathbf{r}-\mathbf{s}_2|}}{|\mathbf{r}-\mathbf{s}_2|}$$

$$=\frac{\sqrt{p_2}}{8\pi^{7/2}}\int\frac{e^{-i\mathbf{q}',\,\mathbf{r}-\mathbf{s}_2}}{q'^2-p_2^2+i\varepsilon}\,d\mathbf{q}',\quad \varepsilon\to 0.$$

Setting $\mathbf{q}=\mathbf{q}_\omega+\mathbf{g}$, $\omega\cdot\mathbf{g}=0$, and carrying out the integration, we obtain

$$M=\frac{e}{i}\sqrt{\frac{p_1p_2}{32\pi^5\omega^3}} \tag{3}$$

$$\times\int\frac{d\mathbf{g}(\mathbf{j}\times\mathbf{g})e^{i\mathbf{g},\,\mathbf{s}_2-\mathbf{s}_1}}{\mu^2+g^2}\left(\frac{E_1}{\sqrt{p_1^2-g^2}}+\frac{E_2}{\sqrt{p_2^2-g^2}}\right),$$

where the integration over g is taken over the entire plane perpendicular to ω. The effective cross section of the process will be:

$$\sigma=2\pi\int|M|^2\,d\mathbf{s}_1\,d\mathbf{s}_2\,dE_1\,dE_2\delta \tag{4}$$

$$\times(\omega-E_1-E_2)\,|\overline{F}(\omega)|^2.$$

Here the integration over $\mathbf{s}_1$, $\mathbf{s}_2$ are taken over the cross section of a nucleus of radius R, where πR^2 is equal to the cross section of all the inelastic processes in the collision of a pion with a nucleus. The integration over E is taken from 0 to ω.

We used the conservation law $\omega = E_1 + E_2$ in view of the fact that in the present process, as also in creation processes of free pion pairs,[2] the mesons are created in the main at large distances from the nucleus $r_{\text{eff}} >> R > 1/\mu$ (this is related to the small transfer of longitudinal momentum to the nucleus in the pair creation process). But then there is an indeterminacy in the energy (Because the pions are virtual) $\Delta E \sim 1/\Delta t \sim 1/r_{\text{eff}} << \mu$, and one can disregard this quantity. In the expression for the cross section there also entered a factor $\overline{F}$, a pion form factor averaged over angle and energy,[2] dependent on the possible finiteness of the pions and on their mutual interaction.

Carrying out the integration, we obtain, to logarithmic order,

$$\sigma=(e^2R^2/12)\ln(\omega^3R/\mu^2)\,|\overline{F}|^2. \tag{5}$$

If we stipulate that $3\ln(\omega/\mu) >> \ln\mu R$, then

$$\sigma=(e^2R^2/4)\ln(\omega/\mu)\,|\overline{F}|^2. \tag{6}$$

If we set $\overline{F} = 1$, then

$$\sigma=(e^2R^2)\ln(\omega/\mu). \tag{7}$$

In this case the cross section increases logarithmically with the energy of the γ-quantum. One should keep in mind, however, that the logarithm is determined in the region of large angles between the momenta of the mesons and the γ-quantum. In this region neglecting the form factor may appear to be invalid, since the form factor can appreciably lower the contribution of this region to the total cross section. If, as a consequence of this, we limit ourselves to the consideration of the region of small angles, then in Eq. (3) for the matrix element we must cut off the integration over g at some $g_{\text{max}} \sim \mu$.

For the effective cross section of the process, then, we obtain, considering $R >> 1/\mu$.

$$\sigma=\frac{e^2R^2}{12}\left[\ln\frac{\mu^2+g_{\text{max}}^2}{\mu^2}-\frac{g_{\text{max}}^2}{\mu^2+g_{\text{max}}^2}\right], \tag{8}$$

i.e., the cross section in this case does not depend on the energy of the γ-quantum. The obtained cross section is consistent with the total cross section for the creation of free pion pairs.[2]

An answer to the question as to how significant the form factor of the pion is, can be obtained from a comparison of the theoretical results (for example Refs. 2, 1) with the experimental data.

The obtained results may be generalized using a semi-transparent model of the nucleus.[3] Introducing into the expression for the ψ-function a factor characterizing the absorption of the meson,

$$\psi_i=\frac{\sqrt{p_i}}{4\pi^{3/2}}\left[1-\exp\left(-2\varkappa_i\sqrt{R^2-s_i^2}\right)\right]^{1/2} \tag{9}$$

$$\times\frac{e^{-ip_i|\mathbf{r}-\mathbf{s}_i|}}{|\mathbf{r}-\mathbf{s}_i|},$$

($i = 1, 2$), where $\varkappa_i$ is the absorption coefficient of the i th meson, we obtain the following expression for the effective cross section:

$$\sigma = \frac{e^2}{2\pi} |\overline{F}(\omega)|^2 \int_0^{\omega} \frac{E_1(\omega - E_1)}{\omega^3} \tag{10}$$

$$\times \ln \frac{E_1^2(\omega - E_1)^2 R}{\mu^2 |2E_1 - \omega|} (\sigma_{\varkappa_1} + \sigma_{\varkappa_2} - \sigma_{\varkappa_1 + \varkappa_2})\, dE_1,$$

where $\sigma_{\varkappa}$ is the cross section for the capture, by a nucleus of radius R, of a pion with an absorption coefficient $\varkappa$.

The integration cannot be carried out in general, since the dependence of $\sigma_{\varkappa}$ on energy is unknown. If one takes $\varkappa$ to be independent of energy, $\varkappa_1 = \varkappa_2 = \varkappa$, then we obtain

$$\sigma = (e^2/4\pi) \ln(\omega/\mu) |\overline{F}|^2 (2\sigma_{\varkappa} - \sigma_{2\varkappa}). \tag{11}$$

If we introduce a cut-off in angle, then under these conditions the cross section becomes

$$\sigma = \frac{e^2}{12\pi}\left[\ln \frac{\mu^2 + g_{\max}^2}{\mu^2} \right. \tag{12}$$

$$\left. - \frac{g_{\max}^2}{\mu^2 + g_{\max}^2}\right] (2\sigma_{\varkappa} - \sigma_{2\varkappa}).$$

In addition to the one considered, there are possible a series of other processes for the formation of nuclear stars by γ-quanta. However, in view of the fact that in this process an effective role is played by a region large in comparison to nuclear dimensions, one can expect that the considered mechanism is the fundamental one at high energies $\omega >> \mu$.

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The Influence of the Proximity of an External Resonance on the Magnitude of the Transition Energy in a Strong Focussing Accelerator

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THE magnitude of the transition energy in a strong focussing accelerator is determined by the formula

$$E_{\mathrm{cr}} = mc^2 \alpha^{-1/2}; \quad \alpha = d \ln L / d \ln p, \tag{1}$$

where L is the length of the orbit, p is the momentum of the particle. Usually, only the direct dependence of the orbit length on the momentum is taken into account, resulting from the equation

$$\frac{d^2 r}{d\theta} - \left(\frac{l}{2\pi}\right)^2 \frac{\partial H / \partial r}{H_0 \rho} r = -\left(\frac{l}{2\pi}\right)^2 \frac{\Delta p}{p} \frac{1}{\rho}. \tag{2}$$

where $\rho = \rho(\theta)$ is the radius of curvature of the unperturbed trajectory (for $(\Delta p / p)_{\mathrm{synch}} = 0$)*, l is the length of a periodic sector, $\theta = 2\pi s / l$, s is the coordinate along the unperturbed trajectory, H is the magnetic field, r is the horizontal deviation from the equilibrium orbit. However, in the vicinity of resonances, L obviously depends on the distance from the resonances, ϵ_r, ϵ_z, and these distances depend sharply on $(\Delta p / p)_{\mathrm{synch}}$. Therefore

$$\alpha = \frac{\partial \ln L}{\partial \ln p} + \frac{\partial \ln L}{\partial \varepsilon_{0r}} \frac{d\varepsilon_{0z}}{d \ln p} \tag{3}$$

$$+ \frac{\partial \ln L}{\partial \varepsilon_{0z}} \frac{d\varepsilon_{0z}}{d \ln p} = \alpha_p + \alpha_z,$$

$$\varepsilon_r = \nu_r - \nu_{\mathrm{res}} \quad \varepsilon_z = \nu_z - \nu_{\mathrm{res}} \tag{4}$$

where $\nu_{r,z}$ are the betatron quasi-frequencies of the transverse oscillations. The derivatives in Eq. (3) are taken at $(\Delta p / p)_{\mathrm{synch}} = 0$, i.e. ϵ_{0r}, ϵ_{0z} correspond to the mid-position of the synchrotron oscillations. The quantity α corresponds to a high energy $mc^2 \alpha^{-1/2}$. At such energies, the betatron oscillations about the equilibrium orbit are already sufficiently small; therefore parametric resonance, generally speaking, plays a weak role in the effect. **

Thus we have to understand L to be the length of the perturbed equilibrium periodic orbit. Clearly,

$$L \approx L_0 + \int_0^{L_0} \frac{r}{\rho_r}\,ds + \int_0^{L_0} \frac{z}{\rho_z}\,ds + \frac{1}{2}\int_0^{L_0} ds\left[\left(\frac{dr}{ds}\right)^2 + \left(\frac{dz}{ds}\right)^2\right]. \tag{5}$$

$$\frac{1}{\rho_r} = \frac{H_z}{H_0\rho} \approx \frac{1}{\rho}\left(1 + \frac{\Delta H_z}{H_0}\right), \quad \frac{1}{\rho_z} = -\frac{H_r}{H_0}\frac{1}{\rho}, \tag{6}$$

where ΔH_z and H_r are the perturbations of the magnetic field along the unperturbed equilibrium orbit. In order to calculate α_ϵ, one should substitute in Eq. (5), in place of r and z, r_ϵ and z_ϵ, which describe the perturbed equilibrium orbit in the vicinity of an external resonance for $(\Delta p / p)_{\text{synch}} = 0$. They are found from the formulas

$$r_\varepsilon \approx \frac{1}{|\varphi|_{\max}} \frac{h_r}{\varepsilon_{0r}} \operatorname{Re}\left[\varphi_r^* \exp\left(i\varepsilon_{0r}\,\theta - i\gamma_r\right)\right]; \tag{7}$$

$$\frac{dr_\varepsilon}{d\theta} \approx \frac{1}{|\varphi|_{\max}} \frac{h_r}{\varepsilon_{0r}} \operatorname{Re}\left[\frac{d\varphi_r^*}{d\theta} \exp\left(i\varepsilon_{0r}\,\theta - i\gamma_r\right)\right] \tag{8}$$

and similarly for z; $\varphi_{r,z}$ are Floquet functions,

$$\varphi(\theta) = f(\theta)\exp(i\nu\varepsilon); \tag{9}$$

$$f(\theta) = f(\theta + 2\pi), \ \varphi\varphi^{*\prime} - \varphi^*\varphi' = -iw.$$

For ΔH_z and H_r, it is necessary to substitute Eq. (5) into Eq. (6)

$$\Delta H_z \approx (\partial H_z/\partial r)\, r_\varepsilon, \ H_r \approx (\partial H_z/\partial r)\, z_\varepsilon. \tag{10}$$

It is easy to see that only the terms quadratic in r_ϵ, z_ϵ, r'_ϵ, z'_ϵ of Eq. (5) contribute to α_ϵ.

Usually an operating point is selected such that $\nu_r \approx \nu_z$, therefore justifying the approximate equation

$$\int_0^{2\pi} \frac{\partial H/\partial r}{H_0\rho}\,|\varphi_r|^2\,d\theta \approx -\int_0^{2\pi} \frac{\partial H/\partial r}{H_0\rho}\,|\varphi_z|^2\,d\theta; \tag{11}$$

$$\int_0^{2\pi} |\varphi'|^2\,d\theta \approx \frac{l^2}{4\pi^2 a\rho_0}\left(\int_{\text{foc}} |\varphi|^2\,d\theta - \int_{\text{def}} |\varphi|^2\,d\theta\right); \tag{12}$$

$$a = \frac{H_z}{\partial H_z/dr},$$

where $\int_{\text{foc}}$ and $\int_{\text{def}}$ are integrals respectively over a focussing and defocussing sector, ρ_0 is the unperturbed radius of curvature within an ordinary magnet. In addition we make use of the dependence of the frequency on the momentum

$$\Delta\nu_{r,z} = \pm\frac{1}{2\pi w}\left(\frac{l}{2\pi}\right)^2 \int_0^{2\pi} |\varphi|^2\,\frac{\partial H/\partial r}{H_0\rho}\,d\theta\left(\frac{\Delta p}{p}\right). \tag{13}$$

The upper sign refers to ν_r, the lower to ν_z. Then we obtain

$$\alpha_\varepsilon = \frac{1}{w}\left(\frac{A_{0r}^2/a^2}{\varepsilon_{0z}} + \frac{A_{0z}^2/a^2}{\varepsilon_{0z}}\right) \times\left[\frac{l}{4\pi^2\rho_0\,|\varphi|_{\max}}\left(\int_{\text{foc}} |\varphi|^2\,d\theta - \int_{\text{def}} |\varphi|^2\,d\theta\right)\right]^2, \tag{14}$$

where A_{0r}, A_{0z} are the amplitudes of the transverse oscillations for $(\Delta p / p)_{\text{synch}} = 0$:

$$A_{0r,z} = |h_{0r,z}/\varepsilon_{0r,z}|. \tag{15}$$

If $A_{0r} \sim A_{0z}$, $\epsilon_{0r} \sim \epsilon_{0z}$, then generally

$$\alpha_\varepsilon \sim (0.5\text{—}1)\,A_0^2/a^2\varepsilon_0 M^2, \quad 1/8M \leqslant \varepsilon_0 < 1/M, \tag{16}$$

where M is the number of periodic sectors, $L = Ml$.

It should be remarked that even in the absence of oscillations about the equilibrium orbit, the influence of parametric resonance can be told in that in place of ϵ_0 in Eqs. (15), (16), at the worst $\epsilon_0 - g$ enters, where g is the half width of the region of parametric resonance.

As can be seen from Eq. (8), α_ϵ is a strong function of the quantity ϵ_0 ($\alpha_\epsilon \sim \epsilon_0^{-3}$). From Eq. (16) it is seen that α_ϵ can easily attain a magnitude $\sim (1-5) \times 10^{-3}$. The sign of α_ϵ is determined by the sign of ϵ_0 (we note that in accelerators $\epsilon_r\,\epsilon_z > 0$ always). From this follows the important result that in accelerators with compensation for the transition energy according to the method of Vladimirsky – Tarasov[1], one should always choose the operating point in the (ν_r, ν_z) plane such that the distances from neighboring external resonances satisfy the inequalities $\epsilon_r < 0$, $\epsilon_z < 0$.

In accelerators without compensation for the transition energy the effect can introduce a large uncertainty at the moment of the phase flip. It would be possible to avoid this, if it were possible to control the deviations ϵ_{0r}, ϵ_{0z} during the course of the accelerating cycle.

However, if control of the deviations $\epsilon_{0r,z}$ is possible, then one should apply the effect toward the elimination of the transition energy [by means of a slow reduction of (ϵ_0)]. Equation (16) shows that under standard conditions this is quite realistic. It is necessary to take into account that at energies ~ 5 – 10 mev the free betatron and synchrotron oscillations are already sufficiently attenuated. This would be the cheapest way of eliminating the transition energy.

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*It is necessary to remark that for the calculation of α, only that part of $\Delta p / p$ is important which corresponds to an oscillation of the momentum about some equilibrium value. We denote it by $(\Delta p / p)_{\text{synch.}}$

**By parametric resonance we mean one due to a perturbation of the gradient $\partial H_z \partial r$; by an external resonance, one due to a perturbation of the field H_z.

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Relaxation Times T_1 and T_2 in Anthracite

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THE authors were the first to measure the electronic para-magnetic resonance in anthracite (Ref. 1). It was found that the half-width of the absorption line in anthracite is $\Delta H = 0.7$ oersted i.e., considerably smaller than in other types of stone coals. The value $\Delta H = 0.3$ oersted was obtained for anthracite in Ref. 2. Probably the half-width varies somewhat for the different kinds of anthracite. Our last measurements on the samples of Kuzbask anthracite for the frequencies 12.25 and 22 mc gave $\Delta H = 0.5$ oersted. We wanted to determine for anthracite the time of spin-lattice relaxation, T_1. For this purpose, with the above mentioned frequencies, measurements of the degree of saturation (Ref. 3) were made for different amplitudes of the oscillating magnetic field. The magnitude of the amplitude was determined with the method previously used in Ref. 4. The method was checked on $\alpha\alpha$-diphenyl – β – picrylhydrazyl, for which $T_1 = 6.6 \times 10^8$ sec; moreover, the parameter of the half-width T_2 was taken equal to 6.0×10^8 sec in correspondence with the halfwidth of the line $\Delta H = 0.95$ oersted found for the monocrystal of the above-named free radical (Ref. 5). The magnitude of T_1 is in good agreement with the researches of Refs. 3 and 6. For the Kuzbask anthracite sample the time T_1 was equal to 12×10^{-8} sec for the core $T_2 = 11.4 \times 10^{-8}$ sec.

The theory of paramagnetic resonance in systems with large exchange interaction (Ref. 5) demands that $T_1 \approx T_2$; therefore, our result confirms the presence of strong exchange in anthracite, noted in Ref. 1.

In conclusion, we point out that for the temperature of liquid air, the relaxation time for anthracite is somewhat longer, since the saturation occurs for smaller amplitudes of the oscillating field. This is in agreement with the concept that the carriers of paramagnetism in anthracite are "broken bonds" between the carbon atoms.

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Concerning the Blatt, Butler, and Shafroth Paper on Superfluidity and Superconductivity Theory

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IN a series of papers, Blatt, Butler and Shafroth[1-6] concern themselves with the theory of superfluidity and superconductivity, and come forth with some far-reaching conclusions, with which it is impossible to agree. Two points stand out.[1-6] The first, associated with a consideration of the superfluidity and superconductivity of an ideal Bose gas in a vessel, has already been discussed,[7] and has only methodological significance. The second essential point – the statement concerning the finiteness of the correlation length Λ for the momenta of a pair of particles in all real systems, in contrast to an ideal Bose gas, is incorrect. The momentum correlation coefficient is introduced[3] in such a way that it is not directly applicable for momenta equal to zero, and furthermore, the problem of the correlation length is not connected in any explicit way with the properties of the matrix density (a mixed representation of this matrix, the so-called quantum distribution function is actually used,[3] see for example Ref. 8). In this connection, consideration of the matrix density

$$\rho(\mathbf{r}', \mathbf{r}) = \int \Psi^*(\mathbf{r}', q)\,\Psi(\mathbf{r}, q)\,dq$$

(see Ref. 9) brings a greater clarity to the problem. Actually, for an isotropic body (liquid)

$$\rho(\mathbf{r}', \mathbf{r}) = \rho(|\mathbf{r}' - \mathbf{r}|) \equiv \rho(R)$$

and in the usual liquids $\rho\ (R \to \infty) \to 0$. In this case the corresponding correlation length Λ' is finite (Λ' is the distance R, beginning from which one can say that $\rho = 0$). An infinite correlation length corresponds to the case where $\rho\,(R \to \infty) \equiv \rho_{\infty} \neq 0$, which occurs (at temperatures below the critical temperature)for an ideal Bose gas, and as follows from a series of considerations, for helium II also and for electrons in superconductors [9,10]. In the case for which $\rho_{\infty} \neq 0$, the Fourier-representation

$$\rho(\mathbf{k}) = \int \rho(\mathbf{R})\, e^{i\mathbf{kR}}\, d\mathbf{R}$$

contains a term $w_0\,\delta\,(k)$, which corresponds to the presence in the system of a number of particles not equal to zero, possessing momenta exactly equal to zero (we assume that the volume of the system $V \to \infty$). A difference of ρ_{∞} and w_0 from zero appears as that property of a degenerate ideal Bose-gas, which establishes its superfluidity and superconductivity in the sense as given in Refs. 1, 2, 7. Thus, the statements contained in Ref. 3 denote in essence that in real systems we always find $\rho_{\infty} = 0$ or $w_0 = 0$. All the corresponding arguments of Ref. 3 reduce, however, to the observation that for very much larger systems it is improbable that there is present a correlation between particles at opposite ends of the system. However for any monocrystal, for example, there is a correlation between the particles independent of the dimensions so that the actual boundedness of the latter in the plane is clearly not essential; the same pertains to the "remote order" in ferromagnetics etc. Finally, it follows from Ref. 12, in a direct contradiction to Ref. 3, that the consideration weak interaction in a Bose-gas does not lead to the disappearance in ρ (k) of a term of the type $w_0\,\delta$ (k). Therefore, the existence of an analogous situation in helium II and in superconductors, although not strictly shown, is still quite possible and even almost certain (or, in any case, very probable). On the strength of what is shown above, the statement[4] on the non-equilibrium character of the superfluidity of helium II is likewise clearly unfounded, not to mention the fact that such a representation encounters other serious objections. In Ref. 6, no basis or justification is made for, nor any changes brought about, from the basic work[3,5] on the theory of superconductivity.

Comparisons between theory and experiment [6] do not change the conclusions, in particular observations[13] concerning changes in the depth of penetration of the field are linked for (no discernible reason) to a change in frequency (see Refs. 13, 14 ; in these works, experiments[13] interpreted from a different point of view do not agree with those of Ref. 6).

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The Spin of the Λ-Particle

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THERE has recently been discussion in the literature on the spin of the Λ-particle; in particular, a model has been considered in which the Λ-particle represents a system of a nucleon + π-meson, which has sufficiently small dimensions [$a \sim (M\mu)^{-1/2}$ (here M is the mass of the nucleon, μ the mass of the meson)] and sufficiently large angular momentum ($l = 5$). Such a model would enable us to explain: 1) the long lifetime of the Λ-particle, 2) the results of correlation experiments, 3) the simultaneous creation of Λ- and θ-particles. A series of attempts have been undertaken to obtain such a model from the equations of Bethe-Salpeter[1,2].

We shall show that the model of large spin does not agree with the experimental facts connected with the existence of Λ-nuclei.

As is known[3], the Λ-nuclei represent nuclear shells in which one of the neutrons is replaced by a Λ-particle. These shells last not less than 10^{-12} sec and decay in two ways: with the emission of a π-meson (with the release of energy of the order $\epsilon = 37$ mev) and without the emission of a π-meson, wherein the energy release is of the order of $\epsilon + \mu \sim \mu$ and only nucleons emerge.

The latter process is similar to internal conversion in atoms. The role of the radiation nucleus is played by the decaying Λ-particle, the role of the γ-quantum by the π-meson, and the role of the conversion electron by one of the nucleons in the nucleus, which absorbs this π-meson. We calculate the probability of such conversion, making the following assumptions:

1) The Λ-particle is an infinitely heavy particle located at the origin of the coordinates.

2) The wave function of the π-meson outside of the effective dimensions of the λ-particle ($r > a$) is a diverging wave with angular momentum l, so normalized that one particle is emitted in time:

$$\varphi_\pi = (v\tau_0)^{-1/2} R_l Y_{l_0}.$$

Here v is the velocity of the meson, Y_{l_0} is the normalized spherical harmonic, R_l is the normalized radial wave function.

$$R_l = k(\pi / 2kr)^{1/2} H_{l+1/2}(kr),$$

$$R_l = \frac{(2l-1)!!}{k^l r^{l+1}} \; (r \ll l/k), \quad R_l = \frac{1}{r} e^{i(kr-\pi l/2)} \; (r \to \infty),$$

where k is the propagation vector of the meson.

3) The wave function of the nucleon in the initial and final states is

$$\psi_1 = \begin{cases} u_1/\sqrt{\Omega} & \text{for } r < r_\text{я} \\ 0 & \text{for } r > r_\text{я} \end{cases} \quad \psi_2 = (u_2/\sqrt{V})\, e^{i\mathbf{pr}}.$$

Here u is the spinor part of the wave function, Ω, r_n are the volume and radius of the nucleus, V is the normalized volume, p is the momentum of the nucleon in its final state.

The probability of conversion with the emission of a nucleon is equal to

$$1/\tau = 2\pi |U|^2 \rho_E, \tag{1}$$

$$U = g\sqrt{\frac{2\pi}{\mu}} \int \psi_1^* \left(\frac{\sigma\nabla}{2M} \varphi_\pi\right) \psi_2 \, dr.$$

Carrying out the computation by the usual method[4], we obtain

$$\frac{1}{\tau} = 2\pi g^2 \frac{2\pi}{\mu} \frac{1}{v\tau_0} \frac{1}{\Omega} \frac{k^2}{4M^2} \frac{Mp}{(2\pi)^3} \times \left\{\frac{l+1}{2l+1} I_{l+1}^2 + \frac{l}{2l+1} I_{l+1}^2\right\}, \tag{2}$$

where

$$I_l = \int R_l(kr)\, g_l(pr)\, r^2 \, dr, \tag{3}$$

$$g_l = (2\pi)^{3/2} (pr)^{-1/2} J_{l+1/2}(pr),$$

$$g_l = 4\pi \frac{p^l r^l}{(2l+1)!!} \; (\text{for } r \ll l/p),$$

$$g_l = \frac{4\pi}{r} \sin\left(pr - \frac{\pi l}{2}\right) \; (\text{for } r > l/p).$$

Upon integration over r for $r_\text{n} \gg l/\text{p}$, the upper limit can be set equal to ∞, since the chief contribution is biven by the inverval $r < l/p$; for $r_\text{n} < l/p$, we can use r_n as the upper limit. So far as the lower limit of integration is concerned, it can be set equal to zero for large l and small a, since the contribution of the interval $0 < r < a$ to the integral is small. In fact, using the expressions for R_l and g_l for small r, we get

$$I_l^a = \int_0^a R_l(kr)\, g_l(pr)\, r^2 dr = \frac{2\pi}{2l+1} \left(\frac{p}{k}\right)^l a^2,$$

while

$$I_l = \int_0^\infty R_l(kr)\, g_l(pr)\, r^2 dr = 4\pi \left(\frac{p}{k}\right)^l \frac{1}{p^2 - k^2}.$$

For $\alpha \leq 1/p$, which is the case in this model, $I_l^a / I_l \leq 1/2\,(2l+1)$ and is small for large l. It is easy to see that if r_n is used as the upper limit, then $I_l^a / I_l = a^2 / r_n^2 \ll 1$.

For large l, $I_{l-1} \ll I_{l+1}$; Substituting I_{l+1} in Eq. (2) and multiplying by the number of nucleons in the nucleus, we obtain

$$\frac{1}{\tau} = \frac{3(l+1)}{2(2l+1)} g^2 \frac{\mu^3 k}{p^3 M} \left(\frac{p}{k}\right)^{2l+2} \frac{1}{\tau_0}. \qquad (4)$$

for $g^2 = 10$, $l = 5$, $\epsilon = 37$ mev, $k = \sqrt{2\mu\epsilon}$, $p = \sqrt{2M\mu}$, we get for the conversion coefficient $\tau_0 / \tau \sim 5 \times 10^6$. For $p = \sqrt{M\mu}$, we get $\tau_0 / \tau \sim 2 \times 10^5$.

If $r_n < l/p$, then we put I_{l+1} in place of I_{l+1}^a, with $a = r_n$. The value of the conversion coefficient in this case is also $\sim 10^6 - 10^5$. The large values of the conversion coefficient are explained by the large l, on the one hand, and on the other, by the fact that the momentum of the nucleon which absorbed the meson is larger than the momentum of the meson ($p/k \gg 1$), so that the centrifugal barrier is more transparent.

Thus the lifetime of the bound Λ-particle is approximately 10^6 times smaller than for the free Λ-particles, and the decay ought to be almost exclusively conversion, which contradicts experiment. Our conclusions do not depend on the character of the internal region, since only the small dimension of the Λ-particle is essential for it. It appears improbable that the contribution from the internal region could compensate the contribution of the external region, since such a compensation would have to be one of extraordinarily great exactness.

The problem of the spin of the Λ-particle is of interest also, aside from any dependence on the model considered by us. Suppose that the origins of the metastability of the Λ-particle are not due to a large angular momentum, but to some sort of forbidden principle[5,6] connected, for example, with the isotopic spin. In this case, the proof carried out above that there can be no large spin associated with the Λ-particle, is not valid. This is connected with the fact that now the effective dimensions of the Λ-particle can be of the order of $l \geq 4$, and in this case we cannot draw any conclusions on the magnitude of conversion in this interval. However, even in this case, it can be shown that the presence of a very large spin in the Λ-particle leads to an anomalously large "conversion coefficient". In fact, it follows from Eqs. (3) and (4) that the region of integration from $1/\mu$ to ∞, in the case of $l \geq 4$, gives a contribution which is of the order of magnitude of the integral from 0 to ∞ and, consequently, all our estimates maintain their force.

We express our gratitude to Prof. I. Ia. Pomeranchuk for stating the problem and for his guidance, and to B. L. Ioffe for discussion of the problem.

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Binding Energy of Hyper-Nuclei

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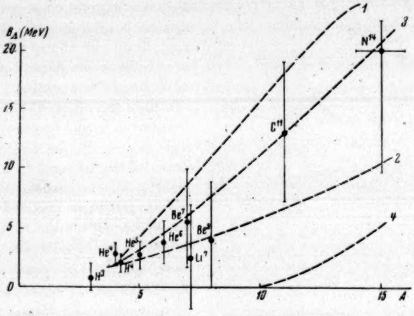
ONE of the fundamental discoveries of recent times has been the discovery of Danisz and Pinewski[1] of new types of nuclei–the so-called hyper- or Λ-nuclei, which contain a Λ^0-particle in addition to neutrons and protons. It has been shown that in many cases the binding energy of the Λ^0-particle to the nuclei is positive and that the comparatively short life of the Λ-nuclei ($\tau \sim 10^{-10}$ sec) is connected with the instability of the Λ^0-particle itself[1-12]. The character of the interaction of the Λ^0-particles with nucleons (N) has not yet been established[3,7,13]. Observations have permitted us to establish, although with low accuracy, the binding energy of a series of light nuclei[1-12] We have plotted the most trustworthy values for the binding energy (see Figure), which illustrates the dependence of the binding energy



of Λ^0-particles (to the nucleus B_Λ) on the number of nucleons A. As is evident from the graph, the fundamental regularities can be expressed in the following fashion: a) B_Λ increases with the number of nucleons in the nucleus: $B_\Lambda = -2.7 + 1.15 A$ mev(A = mass number); b) the binding energy is independent of the charge and also of the spin of the nuclear part (Λ-nucleus minus Λ^0-particle); c) hyper-nuclei lighter than Λ-tritium (${}_\Lambda H^3$) are absent.

The absence of saturation of the binding energy, so clearly evident in ordinary nuclei, is rather astonishing. It creates the impression that the Λ^0-particle interacts on an equal basis with all the nucleons of the nucleus, in such a way that the mean binding energy of each nucleus is equal to ~ 1 mev, i.e., considerably less energy per nucleon than in ordinary nuclei. We assume that the forces of interaction of Λ^0-particles and nucleons are identical to the nuclear forces $N - N$, in particular, they have, for example, the same radius of action[3-13]. We can then introduce a self-consistent potential for the Λ--nucleus. This potential must differ slightly from the potential of the bar nucleus, in view of the very weak interaction of the Λ^0-particle with the nucleus. A single hyperon, like particles of other types (such as protons or neutrons) will be found at the 1s level in the ground state of the Λ-nucleus. The energy of this level in the square well model with infinitely steep walls is given by

$$\varepsilon_{1s} = V_0 - \pi^2\hbar^2 / 2M_\Lambda R_0^2, \qquad (1)$$

where V_0 is the depth of the well, $R_0 = r_0 A^{1/3}$ is the radius of the well, M_Λ is the mass of the Λ^0-particle ($M_\Lambda = 2182\, m_e$). We take $R_0 = 1.4 \times 10^{-13} A^{1/3}$ as the radius of the well. This allows us to obtain the correct energy for the light nuclei. If we consider V_0 to be the same for all nuclei and equal to 43 mev, then the binding energy of Λ-nuclei increases rapidly with increase in A, as calculation shows (see the broken line *1* in the Figure). A similar calculation has also been carried out for the more realistic case of a rectangular well of finite depth. In this case[15,16]

$$\varepsilon_{1s} = V_0 - z_{1s}^2\hbar^2 / 2M_\Lambda r_0^2 A^{2/3}, \qquad (2)$$

where z is a quantity dependent on the product $V_0 R_0^2$. If we now use values of r_0 and V_0 in accord with the binding energy of the nuclei[15,16] ($r_0 = 1.38 \times 10^{-13}$ for all nuclei under consideration, with the exception of the very light, where r_0 increases slightly) and consider the depth of the well for the Λ^0-particle to be half that for nucleons (i.e., above 25 mev for all nuclei with the exception of the very light, where V_0 is somewhat smaller), then we get rather satisfactory agreement with the experimental data (see curve *3* in the drawing). Curve *2* corresponds to a well with V_0 = const = 15 mev and r_0 = const = 1.38×10^{-13}. Curve *3* should reach saturation for $A \sim 25$-30, which would be interesting to verify experimentally. (A similar saturation sets in for curve *1* at somewhat smaller A, and for curve *2* at somewhat larger A.) The results of the calculations agree with the available data[3,13] in this regard, that the forces $\Lambda^0 - N$ are weaker (although of the same order) than the forces $N - N$[3,13]. Curve *4* in the Figure portrays the calculation of the value of the binding energy of excited Λ-nuclei (for the same V_0 and r_0 as for curve *3*), when the Λ^0-particle is in the $1p$ state. In view of the large excitation energy, it is to be expected that the perturbed hyper-nuclei would undergo transitions to the ground state by means of the emission of a γ-quantum or would emit nucleons before the Λ^0-particle would decay. The other possibility of excitation of hyper-nuclei is connected with the transition of nucleons to higher levels.

The Λ-nucleus can exist if its decay to lighter hyper-nuclei (in particle, to the Λ^0-particle) and to nuclei (or to nucleons) is not energetically possible, i.e., if the mass of the initial hyper-nucleus is less than the sum of the masses of the possible products:

$$M(A,\ Z,\ \Lambda) - M(A;\ Z_1;\ \Lambda) - M(A - A_1;\ Z - Z_1) < 0,$$

where A is the total number of neutrons + protons + Λ^0-particles. If $M(A, Z, \Lambda) = ZM_p + NM_n + M_\Lambda - E(A, Z, \Lambda)$ [$E(A, Z, \Lambda)$ is the total binding energy of the given Λ-nucleus, wherein $E(A, Z, \Lambda) = E(A - 1, Z) + B_\Lambda$], then, as is easy to work out, the condition for stability of the Λ-nucleus with regard to neutron emission will be (in mev): $E(A - 1, Z) + 1.15 - E(A - 2, Z) > 0$. The conditions for stability of the Λ-nucleus relative to decay with emission of a proton and Λ-particle will be, correspondingly:

$$E(A - 1; Z) + 1{,}15 - E(A - 2; Z - 1) > 0$$

and $$E(A - 1, Z) - 23{,}6 - E(A - 5; Z - 2) > 0.$$

Testing, by means of this formula, shows that the nuclei ${}_\Lambda\mathrm{HE}^4$, ${}_\Lambda\mathrm{He}^5$, ${}_\Lambda\mathrm{Be}^8$ are sufficiently stable. At the same time, ${}_\Lambda\mathrm{He}^6$ and especially ${}_\Lambda\mathrm{H}^5$ are slightly stable: the first relative to neutron emission, the second, to proton emission. If $E(\mathrm{H}^4) + 1.15 > E(\mathrm{H}^3)$, then ${}_\Lambda\mathrm{H}^5$ could be regarded as stable. ${}_\Lambda\mathrm{H}^3$, ${}_\Lambda n^3$ and $({}_\Lambda\mathrm{H}^3)^*$ evidently ought to be unstable and have actually not been discovered experimentally. However, there does exist the nucleus ${}_\Lambda\mathrm{H}^3$, in which, evidently, the spins of the neutron and proton are antiparallel as in the deuteron. From the viewpoint of isotopic spin, the existence of only one stable Λ-nucleus with mass number 3 is connected with the fact that the isotopic spin of ${}_\Lambda\mathrm{H}^3$ is equal to zero; the latter is possible if $T = 0$ for the Λ^0-particle itself. This also agrees with the empirical data on the absence of $\Lambda^\pm$-hyperons. From the viewpoint of meson theory, the interaction of the Λ^0-particle with a nucleon can occur[3] only be means of the emission of pions ($\Lambda^0 \to \Lambda^0 + \pi^+ + \pi^-$; $\Lambda^0 \to \Lambda^0 + \pi^0$), or by emission of K-mesons, $\Lambda^0 \to N^{0+}\ K^{0-}$, which is permitted by the rule of conservation of "strangeness"[6,14]. However, if $T = 0$ for the Λ^0-particle, then the possibility of exchange of π^0-mesons is excluded by the law of conservation of isotopic spin. Evidently this is associated with the very weak interaction of the Λ^0-particles with nucleons. In view of this, the existence of Λ-nuclei consisting of two particles (for example, ${}_\Lambda\mathrm{H}^2$ and ${}_\Lambda n^2$) is highly improbable, in agreement with the estimates based on the conclusion developed above on the approximately half strength interaction of Λ^0-particles with nucleons.

Note added in proof: A recent consideration of the idea that the binding energy of Λ-nuclei ought to saturate at sufficiently high mass numbers was reported in a research carried out simultaneously and just published[17].

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The Uncertainty Relation and Fluctuation Theory

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AS is well known, one of the fundamental bases of quantum mechanics is the uncertainty principle, first obtained by Heisenberg. In operator notation, it has the form

$$p_x x - x p_x = \hbar / i. \tag{1}$$

The attempts of the representatives of the Copenhagen school of physicists (Bohr, Heisenberg and others) to consider the uncertainty principle as the result of uncontrollable interaction between the object and the measuring apparatus did not develop the physical significance of this important relation, not to mention the fact that such an interpretation led to a subjective, idealistic understanding of the phenomenon of the microworld (for details, see Ref. 1, p. 183).

As was pointed out by Blokhintsev (see Ref. 2, p. 154), the connection between corpuscular and wave properties, for the presence of many particles, can be correctly established with the aid of the introduction of statistical ensembles. However, statistical ensembles are not suitable for the interpretation of the uncertainty principle in its application to an isolated electron.

We attempt to construct a theory of electron motion (in isolated examples, for the time being), in which the quantum effects would be considered as the result of the singular effect of a collection of virtual particles which form the vacuum. An analogy can be established between the quantum effects and the theory of fluctuations, for example, in the investigation of radial vibrations in the theory of the "radiating" electron.

Making use of quantum theory, one of us, together with Ternov[3], succeeded in showing that the square of the amplitude of excitations (due to radiation in the direction of an incident photon with energy $\Delta E = \hbar\omega$) of macroscopic radial vibrations (which make up the "macro-atom") will increase according to the law

$$(\Delta R)^2 = R^2 (1-q)^2 (\Delta E / E)^2. \tag{2}$$

On the other hand, this same quantum formula can be obtained in semi-classical fashion. For this purpose we must consider, in the classical equations of the vibration of the electron (radial component), the fluctuating forces which are statistically independent:

$$F_{\text{fluct}} = [R \Delta E / (1-q) c^2] \, \delta' (t - t_i), \tag{3}$$

where q is the power of decay of the magnetic field in the neighborhood of the equilibrium orbit ($H \sim R^{-q}$, R = radius). In the identity of the two methods, we are inclined to see the connection (including even the quantitative side of the question) between the quantum method and the theory of fluctuations, where the so-called Markov chains occur, i.e., the statistical independence of consecutive processes.

We shall attempt to connect the quantum character of the motion of an electron in the microworld with the fluctuations of virtual photons. Similar attempts have already been made by a series of authors (see, for example, Welton, Ref. 4 and Kalitsin, Ref. 5).

Let us consider the equation of motion of a harmonic oscillator in the field of virtual photons;

$$m\ddot{x} = -m\omega_0^2 x - e(E_x + E_x^i), \tag{4}$$

where

$$E_x^i = -\frac{1}{c}\frac{\partial A_x^i}{\partial t} = -\frac{2}{3}\frac{e}{c^3}\dddot{x}, \quad E = -\frac{1}{c}\frac{\partial A_x}{\partial t} \tag{5}$$

is the self-acting electric field, i.e., unradiated longitudinal photons.

Then, with the help of division of the operator, we find the following expression for x

$$x = -L^{-3/2} \sum_{\vec{\varkappa}} \frac{e\omega}{mc} \sqrt{\frac{2\pi c\hbar}{\varkappa}} \tag{6}$$

$$\times \left(\frac{i a_x \exp\{-i\omega t + i\vec{\varkappa}\mathbf{r}\}}{\omega_0^2 - \omega^2 - i\gamma\omega^3} + \text{complex conjugate} \right),$$

where $\gamma = 2e^2/3mc^3$, $\omega = c\varkappa = \omega_0 y$, and a_x is the quantum amplitude of the vector-potential.

In the given problem, the momentum of the particles must be equated to

$$p_x = m\dot{x} - \frac{e}{c}(A_x + A_x^i) \tag{7}$$

$$= -L^{-3/2} \sum_{\vec{\varkappa}} \frac{e\omega_0^2}{c} \sqrt{\frac{2\pi c\hbar}{\varkappa}}$$

$$\times \left(\frac{a_x \exp\{-i\omega t + i\vec{\varkappa}\mathbf{r}\}}{\omega_0^2 - \omega^2 - i\gamma\omega^3} + \text{complex conjugate} \right).$$

It is evident from (6) and (7) that, thanks to a consideration of the field of virtual photons, the classical quantities x and p_x are operators which do not commute with each other:

$$p_x x - x p_x = \frac{\hbar}{i}\frac{2}{\pi}\gamma\omega_0 J, \tag{8}$$

$$J = \int_0^\infty \frac{y^2 dy}{(y^2 - 1)^2 + \gamma^2\omega_0^2 y^6}.$$

In deriving the last formula, we took into consideration the permutation relations

$$[a_s a_{s'}^+]_- = \delta_{ss'} - \varkappa_s^0 \varkappa_{s'}^0.$$

The integral J does not diverge and has the value

$$J = \pi/2\gamma\omega_0 + O(\gamma\omega_0). \tag{9}$$

Therefore, in first approximation, we obtain permutation relations which coincide with the permutation relations (1) of wave theory.

Making use of the operator expressions (6) and (7) for x and p_x, we can obtain the energy levels for the harmonic oscillator. If the momentum is given, not in the form (8), but in the form

$$p_x = m\dot{x}, \tag{10}$$

as was done, for example, in Ref. 8, then an additional factor of 2 appears on the right side of Eq. 1. It is of interest to note that when photons are absent ($a_s^+ a_{s'} = 0$) it is better to use Eq. (7) for the momentum in the expression of zero point energy

$$E_0 = (p_x^2/2m) + \tfrac{1}{2} m\omega_0^2 x^2, \tag{11}$$

rather than Eq. (10) as would have been more natural in the given case.

Giving the momentum by Eq. (7), we find that the zero-point energy will automatically contain the necessary subtraction terms, leaving the finite quantity

$$E_0 = \frac{\hbar\omega_0}{2} + \frac{\hbar\omega_0^2 e^2}{3\pi c^3 m}\left(\ln\frac{3c^3 m}{2e^2\omega_0} - 1\right). \tag{12}$$

The first term is the well-known expression for the zero-point energy without vacuum terms, and the second is an additional energy caused by the vacuum action.

A strict quantum electrodynamical derivation of the corresponding formula gives an expression very close to Eq. (12):

$$E_0^{\text{qm}} = \frac{\hbar\omega_0}{2} + \frac{\hbar\omega_0^2 e^2}{3\pi c^3 m}\left(\ln\frac{mc^2}{\hbar\omega_0} - 0{,}2\right). \tag{13}$$

It is of interest to observe that if we limit the integral of type (9) (which is also encountered in the calculation of the zero-point energy), to relativistic frequencies of vibration $\omega_{\max} = 2mc^2/\hbar$, as was done by Weisskopf[9], then we obtain Eq. (13) for the additional energy, with the help of the fluctuation method.

Thus a classical system which describes the motion of an electron in its interaction with the second quantized field of photons (actually radiated, or only virtual) is the same as in quantum mechanics.

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Radiation Resonance in Synchrotrons

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THE present research deals with a resonance which can take place under certain conditions in synchrotrons, due to the presence of radiation, and which can bring about amplification of the amplitude of betatron oscillations. The main points are the following:

1. In the absence of radiation, the betatron oscillations in synchrotrons are described by an equation of the form

$$(d^2x / d\zeta^2) + F(\zeta)x = 0, \tag{1}$$

where x is the departure from the equilibrium orbit, $F(\zeta)$ is some periodic function which is determined by the nature of the change in the magnetic field, ζ is a parameter which characterizes the position of a particle in the synchrotron. We shall consider that the synchrotron consists of N identical sectors; in each of the sectors, the parameters ζ changes by 2π. Then one revolution of the particle in the synchrotron corresponds to a change of ζ by $2N\pi$, $F(\zeta + 2\pi) = F(\zeta)$.

As is known from the theory of Flock, solutions of Eq. (1) take the form

$$x(\zeta) = e^{i\mu\zeta}g(\zeta), \quad g(\zeta + 2\pi) = g(\zeta), \tag{2}$$

where μ is a parameter which is determined by the behavior of $F(\zeta)$. The problem of the investigation of the stability of the motion described by Eq. (1) consists of the calculation of the value of μ. Choosing μ in the form of a function of the parameters which characterize the synchrotron, we obtain the conditions for which the motion is stable, i.e,. is suitable for practical use.

Equation (1) does not hold in the presence of radiation. In such a case we must consider the effect on the system of the radiation and of the accelerating interval and subsequently, investigate the motion of the system towards stability.

2. We assume that we can neglect the quantum character of the radiation and consider the radiation as a continuous process of energy loss. Then Eq. (1) is somewhat modified by the replacement of $F(\zeta)$ by another function $F_1(\zeta)$ with the same period, and a periodic function of ζ with period $2\pi N$ (for definiteness, we consider the case of a single accelerating interval) appears on the right-hand side. If, for the modified homogeneous equation, the parameter of stability is denoted by μ_1 (μ_1 is always close to μ), then for

$$\mu_1 N = p, \quad p = 0, \pm 1, \pm 2, \ldots \tag{3}$$

in the solution of the equation with the right-hand side, there appear long-lasting terms, i.e., resonance takes place. The amplitude of the betatron oscillations begins to increase rapidly.

By way of an example, we consider a synchrotron with four sections with soft focussing. We denote by l the lengths of the rectalinear sections, by R the radius of curvature of the curved sections and by $n = -(r/H)(dH/dr)$ the power of the decay of the magnetic field. Then the condition for stability of motion for the calculation without consideration of radiation is given by the following inequality:

$$\frac{l}{R} < 2(1-n)^{-1/2} \times \left[1 + \cos\frac{\pi}{2}\sqrt{1-n}\right] \Big/ \sin\frac{\pi}{2}\sqrt{1-n}. \tag{4}$$

Radiation resonance occurs when

$$\frac{l}{R} = 2(1-n+\varepsilon)^{-1/2} \times \cos\left(\frac{\pi}{2}\sqrt{1-n+\varepsilon}\right) \Big/ \sin\left(\frac{\pi}{2}\sqrt{1-n+\varepsilon}\right). \tag{5}$$

In this equality, ϵ is a very small quantity which changes slowly in the process of the acceleration cycle. Thus at different stages of the acceleration, different values of l/R are dangerous, i.e., there is a region of resonance values. This region of resonance values lies within the region of stability.

3. What has been demonstrated above is valid when the quantum character of the radiation is neglected. It is now clear what changes in this treatment are brought about by consideration of the quantum character of the radiation, because it is known[1] that the latter influences the motion of the electron.

The physical picture of the resonance described above is that the particle passes successively through accelerating regions in one and the same phase of its betatron oscillation. It can be calculated that in one revolution the mean phase change $<(\Delta\varphi^2)>^{1/2}_{\mathrm{av}}$ is in the thousandths of a radian. Thus for $E = 1$ bev, $A = 1$ cm and $n = 0.6$, $<(\Delta\varphi)^2>^{1/2}_{\mathrm{av}} \approx 1.2 \times 10^{-3}$ radian. This means that during one thousand revolutions, the quantum character of the radiation does not lead to the departure of the particle from resonance.

On the basis of all that has been said, it can be concluded that the resonance described above is dangerous for the operation of the synchrotron. In practice, one should stay as far away from this resonance as possible. We have called it radiative resonance because it is determined by radiation and is more intense the more intense the radiation.

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On the Letter to the Editor of I. M. Tsidil'kovskii and F. G. Bass[1]

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IN a recent letter, Tsidil'kovskii and Bass made several critical comments on our researches[2,3] We consider it necessary to answer these comments.

General formulas were obtained in Refs. 2 and 3 for the thermoelectric, galvanometric, thermo- and photomagnetic effects in semiconductors, which are valid for a very wide class of the stationary, spherically symmetric part of the distribution function (for sufficiently weak external magnetic field H and temperature gradient ΔT), in particular, they are valid in the presence of initial heating, and also, in a whole series of cases, for ionization by the field E. Here there appeared certain peculiarities of the phenomenon of transfer under these conditions. Inasmuch as this type of work has not been published up to the present, we feel that the work possessed some new content bearing on the problem of the behavior of semiconductors in strong electric fields.

As is known, the majority of transfer phenomena arise under definite, artificially created (or, using the expression of Tsidil'kovskii and Bass, "far-fetched"), experimental conditions. Therefore, we presumed that, where it was necessary, artificial initial heating of the electrons, in the investigation of transfer phenomena under conditions of initial heating, was no more far-fetched than, let us say, the artificial imposition of a magnetic field on the semiconductor in the investigation of electric conductivity, which is usually done. The purpose in the two cases is the same: by creating artificial conditions, it is possible to discover physical properties of the semiconductors. Incidentally, the artificial initial heating assumed by us is the natural phenomenon in the operation of certain semiconductor devices.

Further, we do not agree with the statement on the existence of a contradiction in our calculation of the coefficient of electron heat conductivity k_e. The case $j = 0$ was used by us as a method which permitted the separation of the heat current, associated with the flow, from heat current which is proportional to the temperature gradient. Calculation of the coefficient of proportionality k itself is carried out under general conditions, which is evident from our Eqs. (3) and (8)[2].

Tsidil'kovskii and Bass attributed to us the statement that there is no necessity "of solving the equations of Davydov for the case in which f_0 depends on E, H and r." Such a statement by us, in such an unrestricted form, was not made. After the statmement on p. 675[3], "The solution of this system (the discussion is about a system of kinetic equations) in the general form is at least very difficult; however, there is no necessity of this," there is added, "The fact is that for sufficiently large dimensions of the semiconductor, and for sufficiently small temperature, concentration and field gradients along all three directions x, y and z (which usually is the case), one can neglect products of the spatial distributions functions in Eqs. (2.3) and (2.4) of Ref. 1.

"Further, inasmuch as we, in our calculations, restricted ourselves to weak magnetic fields, then, in the determination of the moments χ^{ν}, it suffices to find f_0 for the case in which the magnetic field is equal to zero."

In particular, let us see why we can look for f_0 by setting $H = 0$ in our case. The following expression was obtained on p. 565 of Ref. 2 for the asymmetrical part of the distribution function ($\Omega = eH/mc$):

$$\mathbf{f}_1 = \frac{-(l/v)\,\mathbf{R} + (l^2/v^2)\,[\mathbf{R}, \vec{\Omega}] + [l^3/v^3]\,\vec{\Omega}\,(\vec{\Omega}\cdot\mathbf{R})}{1 + (l^2/v^2)\,\Omega^2} \qquad (1)$$

It was shown on p. 566 that all our calculations are derived under the assumption that

$$(l\Omega/v)^2 \ll 1. \qquad (2)$$

Consequently, f_0 [for f_1 taken from Eq. (3)] is determined on p. 675 from Eq. (3.3)[3], which do not contain the field H. Thus, if f_0 were to depend essentially on H, then this would be taking place under the conditions unlike those assumed by us [the inequality (2)].

$$\mathbf{f}_1 = l\mathbf{R}/v.$$

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Coulomb Excitation of Nuclei by Heavy Ions

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THE cross section of Coulomb excitation of the nucleus in the case of quadrupole excitation is expressed by the formula

$$\sigma = (m^2 v_f^2 / z_2^2 e^2 \hbar^2)\, B_e(2)\, f_2(\xi), \tag{1}$$

where

$$\xi = (z_1 z_2 e^2 / \hbar)(1/v_f - 1/v_i), \tag{2}$$

m is the reduced mass of the bombarding particle, z_1 is the number of protons in its nucleus, v_i and v_f are its relative velocity before and after the collision, z_2 is the number of protons in the nucleus of the target atom, $B_e(2)$ is the reduced probability of electric quadrupole transition of this nucleus from the ground state to a given excited state, $f_2(\xi)$ is the Coulomb excitation function[1-3]. The graph of the function $(25/2\pi^2) \times f_2(\xi)$ is drawn in Ref. 2. This function increases rapidly with decreasing ξ.

In cases in which the cross section σ is small, experimenters have attempted to step up the energy of the bombarding particle. However, this can lead to the appearance of nuclear reactions (especially in research with protons and for small z_2), which make very difficult the possibility of correctly calculating the quantity σ from the experimental data. One can step up the energy of the particles significantly without risking the excitation of nuclear reactions if accelerated heavy ions are used instead of protons or α-particles for Coulomb excitation of the nucleus.

Let us estimate the value of the cross section σ_T of Coulomb excitation of the nucleus by heavy ions. It is appropriate to replace the calculation of σ_T with the calculation of the relative value (σ_T/σ_P) in its dependence on ξ_P (the subscript p refers to the proton). The parameter ξ_P can be represented in the form

$$\xi_P = 0.1575 z_1 z_2 \sqrt{\mu_P}\left((E_P - \Delta E)^{-1/2} - E_P^{-1/2}\right). \tag{3}$$

In Eq. (3) and later, the collision energy E and the excitation energy ΔE are expressed in mev; μ is the reduced mass of the particle, expressed in nuclear mass units. As follows from Eq. (1),

$$\frac{\sigma_T}{\sigma_P} = \frac{\mu_T}{\mu_P}\frac{(E_T - \Delta E)}{(F_P - \Delta E)}\frac{f_2(\xi_T)}{f_2(\xi_P)}. \tag{4}$$

In the calculation of the values of σ_T/σ it is assumed that $E_T = Z_T E_P$. The parameter ξ_T is computed by a formula analogous to Eq. (3). The results of the calculations, relative to the use of nitrogen ions, are plotted in Fig. 1 in the form of graphs computed for tantalum ($z = 73$, $A = 181$) and manganese ($z = 25$, $A = 55$) for different values of the parameter $k = \Delta E / E_P$. For convenience in estimating the energy of the particles from the graph, we have also provided the values of E_P which correspond to the given ξ_P for different values of the parameter k. Consideration of the graphs permits us to draw the following conclusions:

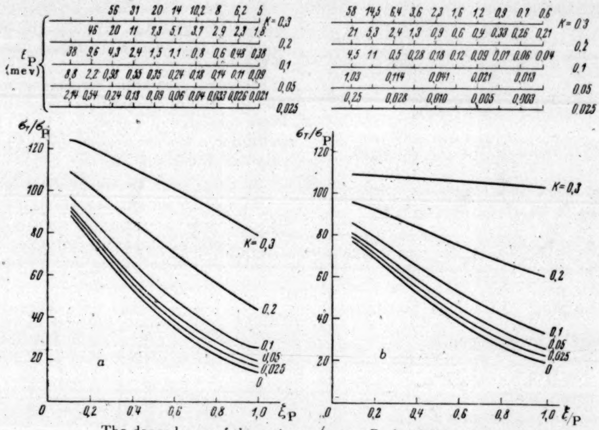
1) With decrease in E_P, the ratio σ_T/σ_P decreases. The maximum value of σ_T/σ_P for a given k occurs for $\xi_P \to 0$, i.e., for large E_P;

2) For ξ_P in the region from 0 to 1, the values of σ_T/σ_P are much greater than unity;

3) The values of σ_T/σ_P increase with increase in k, especially for $\xi_P \to 1$.

In cyclotrons of average size, one can accelerate triply ionized nitrogen ions up to energies of 10-30 mev. In Ref. 4, which pertains to the investigation of Coulomb excitation nitrogen ions were investigated with energies of 15.6 mev. For ions with such energy we find the value of σ_T/σ_P for the excitation of the first and second levels of Ta^{181} ($\Delta E_1 = 0.137$ mev and $\Delta E_2 = 0.303$ mev). In this case $E = 14.5$ mev, $E_P = 14.5/7 \approx 2.1$ mev*, $k_1 = 0.065$, $k_2 = 0.15$. Interpolating the data shown in Fig. 1, we get for the first and second excited levels of Ta^{181} the valuse of σ_T/σ_P equal to 75 and 50, respectively.

Calculation shows that for nuclei close to the A and z for Ta^{181} in the excitation of the level with $\Delta E = 0.5$ mev, the equality $\sigma_P = \sigma_T$ (14.5 mev) will hold for $E_P = 5.3$ mev; similarly, if $\Delta E = 0.2$ mev, then $\sigma_P = \sigma_T$ only for $E_P \gg 7$ mev. For nuclei of the type Mn_{25}^{55}, for all values of ΔE lower than 0.7 mev, $\sigma_T > \sigma_P$ if $E_P < 7$ mev.



The dependence of the ratio σ_T/σ_P on E_P for different values of $k = \Delta E/E_P$. The curve 0 was computed according to the formula of semiclassical theory[1,2] which holds as $k \to 0$; a -- $z_2 = 73$; $A = 181$; b -- $z_2 = 25$, $A = 55$.

To increase the yield of γ-photons, the experimental investigations of the coulomb excitation are frequently carried out with thick targets. The ratio of the yield of γ-rays due to the Coulomb excitation of some state is given by the formula

$$\frac{Q_T}{Q_P} = \int_{E_T}^{0} \sigma_T(E) \frac{dE}{(dE/dx)_T} \Bigg/ \int_{E_P}^{0} \sigma_P(E) \frac{dE}{(dE/dx)_P} \quad (5)$$

Equation (5) assumes an equal number of protons and heavy ions falling on the target per second.

In application to the excitation of the first level of the nucleus Ta^{181} by nitrogen ions with $E_T = 14.5$ mev, calculation from Eq. (5) gives $Q_T/Q_p = 12$. The value of $(dE/dx)_T$ which is necessary for the calculation for the retardation of nitrogen ions in Ta is obtained by the method of recalculation of the range-energy curve for α-particles and tantalum according to the method set up by Longchamp.

Then, the ratio Q_Y/Q_p for a thick target is still greater than 1 although it is appreciably smaller than σ_T/σ_p (12 instead of 75).

In the passage of charged particles through matter, there arises characteristic x-radiation. In some cases, the energy of the x-ray K-quanta is close to the energy of γ-photons which are emitted as a result of Coulomb excitation. In such cases, the K-radiation forms an interfering noise. It was shown by Henneberg[7] that the cross section for the formation of x-ray K-quanta in the ionization of atoms by slow charged particles is approximately proportional to $z_1^2 (E_1/A_1)^4$. Making use of this relation and the data given above, it is possible to calcluate that in the case of a thick Ta target, the ratio of the number of x-ray K-quanta, arising as a result of the ionization of the atoms of Ta by the incident particles, to the number of K-quanta with energies to 137 kev, in the use of nitrogen ions with energies of 15.6 mev, is 15 times smaller than in the use of protons with energies of 2.1 mev.

We thank G. N. Flerov for calling our attention to the possibility of the use of heavy ions for the investigation of Coulomb excitation and also give thanks to K. A. Ter-Martirosian who put at our disposal tables and graphs of the function $f_2(\xi)$, kindly sent him by Alder and Winther.

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* We note that the protons used for Coulomb excitations of Ta^{181} in Ref. 5 have almost the same energy.

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Experimental Study of Coulomb Excitation of Nuclei by Nitrogen Ions

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THEORETICAL estimates[1] indicate some advantage can be gained if heavy ions are used as bombarding particles in the study of Coulomb excitation of nuclei. We have carried out an experimental investigation of Coulomb excitation of the nuclei of 21 elements by nitrogen ions. The purpose of the measurements was the determination of the lowest excited levels of the nuclei under investigation and the value of the reduced probabilities $B_e(2)$ for the corresponding transitions.

A beam of triply ionized nitrogen ions, accelerated in a cyclotron to 15.6 mev, was brought into a vacuum tube by means of the usual deflector and passed through a system of two magnetic quadrupole lenses, which focussed the beam on the target. To rid the beam of singly charged ions, the beam was passed through a plane condenser with horizontal plates, located at the exit of the deflector unit. A constant voltage of ~ 14 kv was applied across the plates.

The target was pressed into the bottom of an isolated metallic vessel which served as a Faraday cylinder. The beam spot on the target had a height of ~ 5, and a depth of ~ 14 mm. Use of the magnetic lenses permitted an increase in the intensity of the ion beam falling on the target (by a factor of about 5) and also separated the deflecting arrangement by 1.8 m. The quantity of electricity in the incident beam was measured by integration of the current.

The γ-radiation of the target was investigated with the help of a scintillator γ-spectrometer. The latter consisted of a crystal of NaJ(Tl), a photomultiplier, a linear amplifier and a 50 channel pulse-amplitude analyzer*.

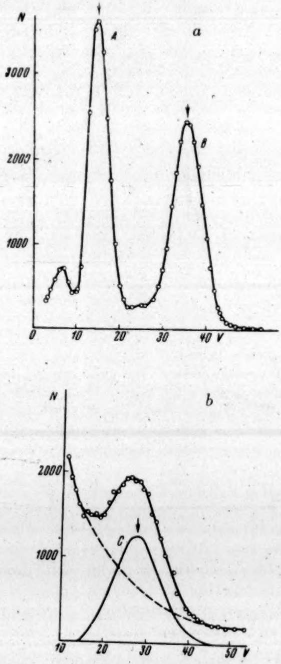

FIG. 1. Amplitude spectrum of pulses in the Coulomb excitation of Ta ^{181}: a = x-ray K-line of Ta; $B = \gamma$-line corresponding to the decay of the second excited level of Ta ^{181} with $E = 301$ kev (the curve was obtained after subtraction of the noise, indicated by the broken curve).

The form of the spectrum of Coulomb excitation obtained for radiation of tantalum foil (of 100μ thickness) by nitrogen ions is shown in Fig. 1. The peak A corresponds to the x-ray K-radiation of the atoms Ta with $E = 57.2$ kev. According to the data of Huus and Zupancic[2] the ratio $(I_K/I_\gamma)_p$, the number of K-quanta which are produced by the ionization of the atoms of Ta by protons with energy 2.1 mev, to the number of γ-photons with energy 137 kev, is equal to 12.4. Therefore, in accord with Ref. 1, we should expect in our case that $(I_K/I_\gamma)_T = 12.4/15 \approx 0.83$; experiment gives $(I_K/I_\gamma)_T \approx 0.65$ (this figure was obtained after subtraction from the total number of K-quanta of that part which is connected with the process of internal conversion for the transition, with $\Delta E = 137$ kev). The result confirms the theory of Henneberg on which the calculation in Ref. 1 is based.

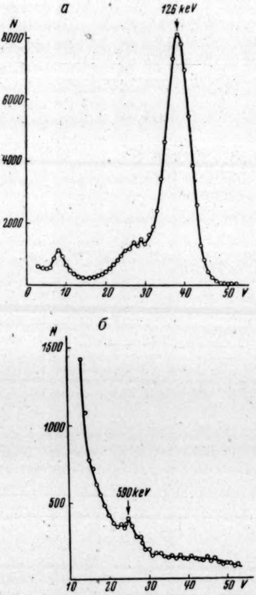

FIG. 2. Amplitude spectrum of pulses for Coulomb excitation of Mn^{55}; $a = \gamma$-line corresponding to the decay of the first excited level of Mn^{55} with $E = 126$ kev; b = energy region from 300 to 1200 kev.

The peak B corresponds to Coulomb excitation of the first excited level of Ta^{181} with $\Delta E = 137$ kev. The energy of the corresponding γ-rays is 138 ± 2 kev according to our measurements.

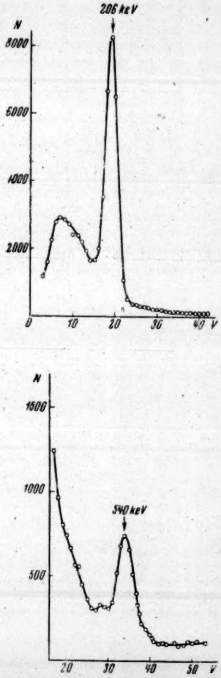

FIG. 3. Amplitude spectrum of pulse for Coulomb excitation of molybdenum; γ-line with energy 206 kev refers to Mo^{95}, γ - line with energy 540 kev, probably to $Mo.^{100}$

The probability of excitation of the second excited level of Ta^{181} with $\Delta E = 303$ kev is much less than the probability of excitation of its first level. In Fig. 1b we have plotted the γ-spectrum which corresponds to the Coulomb excitation of the second excited level of Ta^{181}. The spectrum was taken with a separation which prevents the charging and blocking of the analyzer with a large number of small pulses. In this case, one can obtain the necessary readings which correspond to γ-photons with $E = 303$ kev after a very short time. According to our data, the energy of the second excited level is equal to 301 ± 9 kev.

Irradiated nuclei and the energy of γ-photons in kev.

F_9^{19}	Na_{11}^{23}	V_{23}^{51}	Mn_{25}^{55}	Ge_{32}^{73}	Se_{34}^{77}	Mo_{42}	Rh_{45}^{103}	Ag_{47}	Cd_{48}	In_{49}^{115}	J_{53}^{127}	Ta_{73}^{181}	W_{74}	Au_{79}^{197}	Pb_{82}^{207}
125	435	320	126	72	238	206	295	310	297	562	60	138	112	286	580
205			590		452	540	358	409	325		205	301		580	
									543						

Figure 2 shows the amplitude spectrum obtained in the irradiation of manganese by nitrogen ions. We have plotted in Fig. 2*b* the spectrum in the energy region from 300 to 1200 kev. As is evident from the drawing, only one line is observed in this region, with energy ~ 590 kev. In Ref. 3, in which protons are used for the Coulomb excitation of Mn, a number of lines are observed in the given energy region. From comparision of the data of Ref. 3 with ours, it follows that these lines are connected with nuclear reactions, and not with Coulomb excitation. The spectra of Coulomb excitation of molybdenum are plotted in Fig. 3.

In bombardment of K, Ni, Cu, Sn, Bi Coulomb excitations was not observed.

Data are given in the Table of the energy of the excited levels of the nuclei under investigation.

At present, calculations are being carried out on the value of $B_e(2)$ and on treatment of the data on the bremmstrahlung of nitrogen ions.

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* The amplitude analyzer was constructed by L. N. Gal'perin.

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One-Meson and Zero-Meson Annihilation of Antinucleons

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IN connection with the extremely interesting communication that appeared recently[1] on the creation of antinucleons in the collisions of protons of high energy with nuclei, certain processes of "extraordinary" annihilation of antinucleons are considered in the present note.

In the collision of antinucleons with free nucleons, the annihilation is evidently connected with the release of not less than two π-mesons (or K-mesons). This process ("extraordinary" annihilation), in which, in all probability, several mesons are emitted, also takes place in the collision of antinucleons with nuclei. However, in the collisions of antinucleons with nucleons bound to the nucleus, there is the possibliity of other processes ("extraordinary" annihilation) in which the number of π-mesons emitted is less than or equal to one.

Annihilation with the emission of a single π-meson can take place in the collisions of an antinucleon with a nucleus of atomic mass $A \geq 2$. Annihilation which is not accompanied by emission of even a single meson is possible only in the collisions of an antinucleon with a nucleus of atomic mass $A \geq 3$. It is not difficult to see that the processes of one-meson and zero-meson annihilation of antinucleons occur in processes inverse to those in which antinucleons are created in the collisions of π-mesons and nucleons with nucleons.

Keeping in mind the possibility of setting up experiments, we have considered below several processes of "extraordinary" annihilation of antinucleons which are characterized by the fact that the number of particles in the final state is equal to 2.

In the case of collisions with deuterons, the following reactions are possible:

$$\text{a)}\quad \tilde{p} + d \rightleftarrows \pi^0 + n;\quad \text{a}')\quad \tilde{n} + d \rightleftarrows \pi^0 + p;$$

$$\text{b)}\quad \tilde{p} + d \rightleftarrows \pi^- + p;\quad \text{b}')\quad \tilde{n} + d \rightleftarrows \pi^+ + n.$$

According to the principle of charge symmetry, the cross sections of reactions of type a) are equal; the cross sections of reactions of type b) are also equal.

It is not difficult to show that charge independence requires that the cross sections of type b) be twice those of reactions of type a). From the experimental point of view, the reactions b) are especially interesting. Here four charged particles take part. The ratio of the cross sections of the direct and inverse reactions of b), for the conditions of identical energy in the center-of-mass system, is equal to

$$\frac{\sigma(\tilde{p} + d \rightarrow \pi^- + p}{\sigma(\pi^- + p \rightarrow \tilde{p} + d)} = \frac{(2S_\pi + 1)(2S_p + 1)}{(2S_{\tilde{p}} + 1)(2S_d + 1)}\frac{k^2}{q^2} = \frac{k^2}{3q^2},$$

where $S_{\tilde{p}}$, S_d are the magnitudes of the spins of the antiproton, the deuteron, etc.; k and q are the momenta of the π-mesons and the antiprotons in the center-of-mass system. Investigation of the direct and reverse reactions of b) give the possibility of verifying the correctness of the assumption that the spin of a negative particle with proton mass is equal to one-half. For example, the cross section of the reaction b) of the annihilation into a deuteron of an antiproton with kinetic energy 500 mev ought to be 1.6 times greater than the cross section of the reaction of the creation of a deuteron and an antiproton in the collision with a proton of a π-meson with energy 4.6 bev.

Let us consider processes of single meson annihilation of an antinucleon in a nucleus with $A = 3$:

$$\text{c)}\quad \tilde{p} + \mathrm{He}^3 \rightleftarrows d + \pi^0;\quad \text{c}')\quad \tilde{n} + \mathrm{H}^3 \rightleftarrows d + \pi^0;$$

$$\text{d)}\quad \tilde{p} + \mathrm{H}^3 \rightleftarrows d + \pi^-;\quad \text{d}')\quad \tilde{n} + \mathrm{He}^3 \rightleftarrows d + \pi^+.$$

The ratio of the cross section of reactions of types c) and d), according to charge independence, is equal to 2. From the experimental viewpoint, the reverse reactions c′) and d′) present interest.

Zero-meson annihilation of an antinucleon is illustrated by the following reactions:

$$\text{e)}\quad \tilde{p} + \mathrm{H}^3 \rightleftarrows n + n;\quad \text{e}')\quad \tilde{n} + \mathrm{He}^3 \rightleftarrows p + p;$$

$$\text{f)}\quad \tilde{p} + \mathrm{He}^3 \rightleftarrows n + p;\quad \text{f}')\quad \tilde{n} + \mathrm{H}^3 \rightleftarrows p + n.$$

Here the reverse reaction to e′)--the formation of He^3 in the collision of two protons--is of experimental interest.

Experimental investigation of the above-mentioned processes is of fundamental significance. It is reasonable to expect that the processes of single-meson and zero-meson annihilation are significantly less probable than the process of multiple meson annihilation. This follows, for example, from Fermi's statistical theory of multiple production of mesons.

It should be emphasized that the probability of processes of "extraordinary" annihilation of antinucleons could be increased if one could have especially strong nucleon-antinucleon interactions of the type assumed in the Fermi-Yang model.

The author thanks L. I. Lapidus for his discussions on the subject.

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Solution of the Schwinger Equation in the Bloch-Nordsieck Model

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(May, 1956)

IN the consideration of the scattering of an electron in an external field, Bloch and Nordsieck[1] assumed a method of approximate solution of the

Dirac equation, which is valid in the region of low frequencies. The zeroth approximation of this method is equivalent to substitution in the initial equations of the c-numbers for the Dirac matrices. In the determination of the Green's function, there is definite methodological interest in making the same replacement of the matrices γ^{α} by the c-numbers u^{α} ($\alpha = 0, 1, 2, 3$) in the corresponding equations. After such a substitution the resultant equations are solved exactly.

The equation for the electronic Green's function[2] has the form

$$\left\{ iu^{\alpha} \frac{\partial}{\partial x^{\alpha}} - m - \sqrt{4\pi}\, eu^{\alpha} A_{\alpha}(x) - \sqrt{4\pi}\, ieu^{\alpha} \int D_{\alpha\beta}(\xi, x|A) \frac{\delta}{\delta A_{\beta}(\xi)} d\xi \right\} G(x, y \mid A) = -\delta(x-y), \tag{1}$$

where $D_{\alpha\beta}(\xi, x \mid A)$ is the photon Green's function. In the given model, as can be easily shown, vacuum polarization is absent; consequently, we have, in momentum representation,

$$D_{\alpha\beta}(p) = \frac{g^{\alpha\beta}}{\lambda^2 - p^2 - i\varepsilon} - \frac{g^{\alpha\beta}}{M^2 - p^2 - i\varepsilon},$$

$$M \to \infty,\ g^{00} = 1,\ g^{ii} = -1.$$

Equation (1) takes the form

$$\left\{ (uk) - m - \int (up) A_{\alpha}(p) \frac{\delta}{\delta A_{\alpha}(p)} dp - \sqrt{4\pi} \frac{e}{(2\pi)^2} \int u^{\alpha} A_{\alpha}(p)\, dp - i\sqrt{4\pi} \frac{e}{(2\pi)^2} \times \int u^{\alpha} D_{\alpha\beta}(p) \frac{\delta}{\delta A_{\beta}(p)} dp \right\} G(k \mid A) = -1, \tag{2}$$

in momentum representation, where $(uk) = u^{\alpha} k_{\alpha}$; $m \to m - i\epsilon$. Here we have used the invariance of the Green's function under the transformation

$$G(x, y \mid A) = G(y - x \mid T_x A);$$

$$D(\xi, x \mid A) = D(\xi - x \mid T_x A),$$

where T_x is the displacement operator $T_x A(\xi) = A(\xi + x)$.

For the solution of Eq. (2) we make use of the method of proper time of Fock[3]:

$$G(k \mid A) = i \int_0^{\infty} \Phi(\nu, k \mid A)\, d\nu.$$

We then have for the function $\Phi(\nu, k \mid A)$ the equation

$$-i \frac{\partial}{\partial \nu} \Phi(\nu, k \mid A) = H\Phi(\nu, k \mid A); \quad \Phi(0, k \mid A) = 1,$$

where H is an operator which appears on the left side of Eq. (2). Here the relation

$$\frac{\delta}{\delta A_{\alpha}(p)} \Phi(\nu, k \mid A) = -\sqrt{4\pi} \frac{eu^2}{(2\pi)^2} \frac{e^{i(up)\nu} - 1}{(up)} \Phi(\nu, k \mid A)$$

holds. As a result, we can write

$$G(k \mid A) = i \int_0^{\infty} d\nu e^{-i[m-(uk)]\nu + f(\nu)} \times \exp\left\{ \sqrt{4\pi} \frac{e}{(2\pi)^2} \int \frac{e^{-i(up)\nu} - 1}{(up)} u^{\alpha} A_{\alpha}(p)\, dp \right\}; \tag{3}$$

$$f(\nu) = \frac{e^2}{4\pi^3} u^{\alpha} u^{\beta} \int_0^{\nu} d\nu \int \frac{e^{-i(up)\nu} - 1}{(up)} D_{\alpha\beta}(p)\, dp.$$

Integrating over p-space, we get

$$f(\nu) = -\frac{i}{2} e^2 M\nu + \frac{e^2}{\pi} \ln \frac{M}{\lambda} - \frac{e^2}{\pi} \int_0^{\infty} \frac{\exp\{-i\sqrt{\lambda^2 + p^2}\,\nu\}}{(\lambda^2 + p^2)^{3/2}} p^2\, dp. \tag{4}$$

For $\lambda = 0$ we have

$$f(\nu) = -\frac{i}{2} e^2 M\nu + \frac{e^2}{\pi} \ln \frac{M}{m'} + \frac{e^2}{\pi} \ln m'\nu. \tag{5}$$

The first two terms in Eqs. (4) and (5) are removed by renormalization of the electron mass and by the Z-factor of the electronic Green's function.

On the basis of Eq. (5), we have

$$m' = m + {}^1/_2 e^2 M;$$

$$G'(k \mid A) = Z^{-1} G(k \mid A), \quad Z = (M/m')^{e^2/\pi},$$

where m' is the experimental mass of the electron, $G'(k \mid A)$ is the renormalized Green's function. Finally, renormalization of the electronic Green's function for $\lambda = 0$ takes the form

$$G'(k \mid A) = i \int_0^{\infty} d\nu e^{-i(m' - (uk))\nu} (m'\nu)^{e^2/\pi} \times \exp\left\{ \sqrt{4\pi} \frac{e}{(2\pi)^2} \int \frac{e^{-i(up)\nu} - 1}{(up)} u^{\alpha} A_{\alpha}(p)\, dp \right\}. \tag{6}$$

In particular,

$$G'(k \mid 0) = \frac{1}{m' - |k|} \left| 1 - \frac{|k|}{m'} \right|^{-e^2/\pi}; \tag{7}$$

here, we have set $u^{\alpha} = k^{\alpha}/|k|$. A formula similar to Eq. (7) has been obtained by Abrikosov[4].

Applying the Green's function (6), it is not difficult to obtain the following formula for the effective cross section of the scattering of an electron in an external field with radiation of n photons having energy in the interval from E_1 to E_2, independent of the number of longitudinal photons radiated in this case:

$$\sigma_n = \sigma_0 \frac{1}{n!}\left\{\frac{2e^2}{\pi}\ln\frac{E_2}{E_1}\right\}^n \quad \text{при} \quad \varepsilon_i \ll E_{\text{эл}},\ \mathbf{p}_i \ll \Delta \mathbf{P}_{\text{эл}}.$$

In conclusion, I express my deep gratitude to Acad. N. N. Bogoliubov for his direction of the work.

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On the Theory of Hyperons

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THE proposed idea consists in attributing to the nucleon an internal structure very suggestive of the structure of the hydrogen atom (see Ref. 1). We shall consider the nucleon as a system of two hypothetical particles ("m-particles"), which interact by means of a certain strong field χ. The essential difference of this model from the hydrogen atom is that the strong field is not Coulombic and is different from potentials considered earlier (the Yukawa potential and others).

In Ref. 2, the author has shown that, within the framework of the basic principles of the existing theory of elementary particles, a consideration of the relativistic field is possible, which gives (for a point source) a potential which falls off very rapidly with distance. The potential can be represented by the approximate formula

$$V(r) = -(\pi a / \sqrt{2}r)\, e^{-r^2/2\lambda^2}, \tag{1}$$

where r is the distance from the source, a is the χ-charge of the source, λ is some positive constant-the "elementary length" (we limit ourselves to the case $\varkappa = 0$[2], which presents the greatest interest). The field which leads in the final count to the potential (1) is defined by the equation

$$(\Box - \lambda^{-4}(x^{\mu} - x_0^{\mu})(x^{\mu} - x_0^{\mu}))\,\chi = -4\pi a \delta (x - x_0). \tag{2}$$

It is important, from the physical point of view, to emphasize that 1) for the χ-field, the case of a free field makes no sense, i.e., a field without sources does not exist; further, assuming formally that $a = 0$, we get on the left side of Eq. (2) an isolated point x_0 which corresponds to the position of a "virtual" source; 2) the quantization of Eq. (2) cannot give a state with definite 4-momentum, only states with definite 4-angular momentum. In particular, there does not exist for Eq. (2) a state with definite energy, i.e., there is no stationary state. Therefore, the particles of the χ-field, if they exist in nature, cannot be observed by experiments with counters, Wilson chambers, etc. One can show that, from the experimental viewpoint, these would not be particles, and in this sense, we have come across a possible limit of the applicability of the corpuscular-wave dynamics.

The potential (1) can be understood in dual fashion (limiting ourselves to fields of the Bose type with spin not greater than 1): either as the fourth component of a 4-vector, analogous to the Coulomb potential, or as a scalar or pseudoscalar. The first possibility presents the greater interest, since in this case all the constants of the problem can be determined up to the determination of the mass spectrum (but not by the mass spectrum).

Neglecting the spin of the particles which make up the nucleon by our hypothesis in first approximation, we obtain the wave equation (after separating out the motion of the center-of-mass):

$$\left\{-\left(v - i\hbar\frac{\partial}{\partial t} + aV\right)^2 + c^2\left(-i\hbar\nabla - \frac{a}{c}\mathbf{V}\right)^2 + c^2\left(mc + \frac{\gamma}{c}\Phi\right)^2\right\}\psi = 0, \tag{3}$$

where $(V, \mathbf{V})$ are the vector, Φ the scalar, potential, a, γ are corresponding binding constants, m is the reduced mass for the v of internal motion. Let us assume that in our system of coordinates $\mathbf{V} = 0$, V is given by Eq. (1) and that Φ is the potential of the scalar meson field. For simplicity we assume that both particles of our system have the same mass m and possess unit χ-charges and meson charges γ, in which $\gamma = g/2$ (g is the meson charge of the nucleon as a whole)*. In view of this,

$$\gamma^2/\hbar c = g^2/4\hbar c \approx (^{25}/_4)\, e^2/\hbar c \approx {}^6/_{137} \ll 1, \tag{4}$$

which validates the application of perturbation theory to the problem of the interaction with the meson field. We neglect the interaction with the electromagnetic field because of its weakness in comparison with the meson and χ-inter-over intra-nucleonic distances.

As Werle has shown[3], step by step conversion from Eq. (3) to the nonrelativistic approximation leads to the equation ($V = 0$)

$$i\hbar\, \partial\psi/\partial t = [-(\hbar^2/2m)\nabla^2 + V_v + V_s]\,\psi, \quad (5)$$

$$V_v = aV - a^2V^2/2mc^2, \quad V_s = \gamma\Phi + \gamma^2\Phi^2/2mc^2. \quad (6)$$

As can be seen from Eqs. (6) and (1), the *m*-particles do not fall on one another unless (see, for example, Ref. 4)

$$a^2/c\hbar < {}^1\!/_2, \text{ т. е } a < \sqrt{c\hbar/2} \approx 3\cdot 10^{-8}\,\mathrm{CGS}. \quad (7)$$

If we assume that $a \approx 10^{-8}$ cgs units, we can see in Eq. (7) the reason why there is only one type of "$(m-\chi)$-atom" nucleons, since the higher χ-charges contradict the inequality (7). In our problem there are the still undetermined constants λ and m. They are connected with a and $\hbar$ by the relation

$$\lambda a^2 \approx \hbar^2/m, \quad (8)$$

which follows from (see Ref. 5)

$$\int_0^\lambda V(r)\, r dr \approx -\frac{2\hbar^2}{M},$$

where M is the mass of the nucleon $\approx 1.6 \times 10^{-24}$ gm. Further, we have the relation[5]

$$2mc^2 - \varepsilon = Mc^2, \quad \hbar\sqrt{2/m\varepsilon} = r_0, \quad (9)$$

where ϵ is the binding energy of the system, r_0 is the radius of the nucleon $\approx 10^{-13}$ cm. We then find $m \approx 1400\, m_e v$ (m_e = mass of the electron), $\epsilon \approx 450$ mev and then $\lambda = \hbar^2/ma^2 \approx 0.8 \times 10^{-14}$ cm, which is a reasonable order of magnitude for the radius of action of the interaction forces. We see that our assumptions (in contrast to the theory of Markov[1]) lead to the possibility of "dissociation" of nucleons for energies ≥ 450 mev, whereupon one ought to look for the *m*-particles among the heavy mesons with masses $\sim 1400\, m_e$. The dissociation level is higher than the mass of the hyperon Y_2 (mass $\sim 2570\, m_e$, excitation energy ~ 360 mev[6], the highest of the known exicted states of a nucleon.

Calculation of the excited states of the nucleon for our model is possible only in numerical fashion (for large distances, one needs also to take into account the meson interaction of the *m*-particles). We note that the only possible bound states with $l = 0$ (l is the orbital quantum number)[5] and equal values of n (principal quantum number) are possible. The number of such states will evidently be small (of the order of 3-4). The selection rules connected with the interaction with the meson field may allow the explanation of the comparatively large lifetimes of the hyperons.

A similar but somewhat more complicated discussion can be carried out for consideration of the spin of *m*-particles [an equation of the Dirac type in place of Eq. (3)] and for interaction with a pseudoscalar meson field.

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* Evidently it must be assumed that the χ-charges (similar to the electric) have two signs, while the meson charges (like gravitational) have only one sign. It follows from this assumption that the nucleon as a whole does not have a χ-charge, but does have a meson charge g.

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The Problem of the Interaction of Ultrasonic Waves in Liquids

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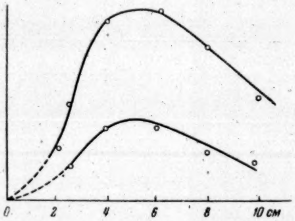
IF a quartz plate is excited as a resonator at two different frequencies, then combination frequencies can easily be observed in the liquid medium. These suggest the possibility of investigating the phenomenon of interaction between the fundamental waves. Below we give some preliminary data on the variation of the intensity of the combination waves with distance.

Let us give briefly the experimental conditions. In order to put two waves into the medium in one direction, we made use of a combination radiator, which was a quartz plate of 20 mm diameter. To one side of the plate was attached a foil membrane with air padding, and to the other side, foil electrodes in the form of two sectors, each covering half of the radiating face of the crystal. These sectors were separated by a distance of 1 mm. Variable voltages were applied from two independent generators. The foil membrane acted as the ground electrode. The generators developed variable voltages of 250-300 v. The thickness of the quartz plate was so chosen that the radiator was adjusted to resonance at one of the fundamental frequencies (either one). Frequencies of 1 mc and 1.5 mc were used as fundamental frequencies.



A barium titanate plate was used as the pressure receiver. The voltage developed across the receiver was amplified by a two-channel (for the sum and difference frequencies) tuned amplifier. An oscilloscope was used as the output meter. By displacing the receiver relative to the source (maintaining strict parallelism between the two surfaces), it was possible to observe (on the oscilloscope) the change in the intensity of the sum of difference frequency. As these observations showed, the intensity of these waves was spatially modulated by the base frequency . In the Figure the intensity of the modulated vibrations is plotted against distance; the distance from the source in cm is plotted along the abscissa, while the ordinate is a quantity proportional to the intensity of the modulated vibration. The results of the observation refer to the sum wave at 2.5 mc obtained in vaseline oil (at $t = 20^0$). Two curves are shown in the Figure, taken for different intensities of the source (the lower curve was taken for an intensity one-half the higher one). As the curves show, the intensity of the modulated combination waves increases with distance from the source, passes through a maximum and then decreases.

Thus we can draw the following conclusions:

1. The intensity of the modulated vibration has a maximum with respect to distance.
2. The location of the maximum does not depend on the intensity of the fundamental waves.
3. As the data show, the position of the maximum for a given frequency is determined principally by the vicosity of the medium.

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The Applicability of the Second Law of Thermodynamics for Large Volumes of a Gravitating Gas

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1. ACCORDING to Terletskii[1-3] the second law of thermodynamics is not applicable to large volumes of a gravitating gas. It is claimed[2] that processes begin to take place in large volumes of a gravitating gas which contradict the second law. Publication of a criticism by the present author[4] was followed by an assertion in discussion form[1] that the earlier conclusions[2,3] were still correct under conditions in which the gravitating gas is enclosed in a thermostatted vessel covered with a piston which works at a fixed force, the pressure being $P > NkT/4V$. We will show that even under these stated conditions, the conclusions[1-3] on the inapplicability of the second law of thermodynamics for large volumes of a gravitating gas cannot be considered correct.

Terletskii[1] assumes that all conditions of the gravitating gas for which $dP/dV < 0$, are entirely stable. Account is not taken of the fact, as we will show below, that the gravitating gas has a metastable region. It is for precisely these same conditions that a greater probability of fluctuations of a cosmic scale is assumed[1].

Let us call the mass of the gravitating ideal gas M, occupying a volume V, situated near the center of the cloud of diffused material, and in a quasistatistical equilibrium. Volume V is much smaller than the volume of the whole diffusion cloud, so that one can assume that the gas is distributed uniformly throughout the volume V. The identical case was considered by Jeans[5], and then in a more general way by Lebedinskii[6] with conclusions on the instability criterion for a gravitating gas. Let us say that the diffusion cloud outside the volume V can be considered as a weight applying a constant pressure P on the separated mass of gas. Then for the free enthalpy of the gas we can write

$$\Phi = -\alpha\varkappa M^2 V^{-1/3} - NkT \ln V + PV - C(T), \quad (1)$$

where α has a numerical value of unity and $\varkappa$ is the gravitational constant.

If the volume of the gas has the value $V_1 = (\alpha\varkappa M^2/3NkT)^3$ (condition 1), then $P = 0$, $dP/dV > 0$. The condition of the gas is unstable. The requirement $V < V_1$ is the equivalent of the known criterion for a gravitating unstable gas (see, for example, Ref. 6). If the volume of gas is $V_2 = (64/27) V_1$ then $P = NkT/4V_1$; $dP/dV = 0$. When $V > V_2$, $dP/dV < 0$ and it appears that the condition of the gas is thermodynamically stable. But this is not so. It follows from Eq. (1) that $\Phi_1 - \Phi_2 = NkT\,[\ln(64/27) - 1]$, i.e., $\Phi_1 < \Phi_2$. Consequently, the condition of the gas, immediately adjoining condition 2, appears to be metastable. Simple calculations show that the region of metastable equilibrium extends toward a minimum value of $V \approx 1.5\, V_2$. In this region the condition of the gravitating gas is in a certain sense, in a condition similar as to that of a supercooled vapor. The formation of small incidental condensation in the metastable region of the gravitating gas leads to the gravitational condensation of the whole gas. As we know, the fluctuation theory in its strict form is not applicable to the condition of metastable equilibrium. The relationship $\overline{(\Delta V)^2} = -kT\,\partial V/\partial P$,[1-3] is correct only for small deviations from equilibrium and under conditions such that the system is in a fixed thermodynamic equilibrium (see, for example, Ref. 7, p. 98). This consideration holds good for all the conditions of the gravitating gas models used in Ref. 1, both those obeying Newton's law and those not obeying it.

Thus the reasoning in Refs. 1-3 is appropriate only for a gravitating gas whose V is significantly greater than V_2. Under these conditions the factor $(1 - 4\alpha\varkappa M^2/9NkTV^{1/3})$ used in Eq. (2) of Ref. 1 significantly differs from zero and consequently cannot have any essential effect on the quantity $\overline{(\Delta V)^2}/V^2$. Furthermore, Eq. (2) or the analogous Eq. (11) in Ref. 1 cannot serve as a proof of the statement concerning the applicability of the second law of thermodynamics for a greater volume of a gravitating gas.

2. Statistical thermodynamics in principle cannot lead to results contrary to the second law of thermodynamics since it itself originated in the statistical interpretations of this law; otherwise, statistical thermodynamics would be internally inconsistent. In a similar way, classical mechanics cannot lead to results contrary to Newton's law on which it is based. Since only statistical thermodynamic methods are used in Refs. 1-3 one can consider to be mistaken, *a priori*, the statement concerning the inapplicability of the second law of thermodynamics for large volumes of a gravitating gas.

3. The hypothesis of J. P. Terletskii on the inapplicability of the second principle of thermodynamics to large volumes of a gravitating gas was introduced with the aim of defending the fluctuation hypothesis of Boltzmann. This hypothesis of Boltzmann is not true because of errors in the hypothesis of Clausius concerning the thermal death of the universe. In both these hypotheses the problem of the development of the universe is approached by the introduction of the universe as if it were an insulated system. This approach is physically unfounded. From the point of view of philosophy it appears to be idealistic. It is precisely the assumption of a finite or insulated universe and not the second law of thermodynamics which leads to the conclusion of the expectation of the thermal death of the universe[8]. To prove the error in the Clausius hypothesis it is not necessary to leave the framework of thermodynamics and go to statistical physics and to theories of clusters.

4. The existence of regions of metastability in the gravitating ideal gas has not been given much attention. We believe that the calculation of the metastable condition of a gravitating ideal gas will be useful for research into gravitational condensation of interstellar material.

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On the Nuclear Reaction Be^9 (pd) Be^8

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WE know that the nucleus of Be^9 can be regarded as consisting of Be^8 as a core, with an unpaired neutron moving in its field. This model proved to be quite satisfactory in connection with the photo- and electrodisintegration of this nucleus[1,2]. On the basis of this model, the Be^9 (pd) Be^8 reaction can be considered to be the result of the direct interaction of a proton and the unpaired neutron. The mechanism of this reaction then appears to be the following: an incident proton interacts with the unpaired neutron to form a deuteron, which escapes from the nucleus, leaving the Be^8 nucleus in its ground state. We here neglect particle spins, the Coulomb interaction and the recoil of the Be^8 nucleus.

The interaction between the proton and the nuclear neutron, whose radius vectors are designated by $\mathbf{r}_2$ and $\mathbf{r}_1$, respectively, is taken in the form of a Yukawa potential

$$V(r) = -g^2 r^{-1} e^{-\varkappa r}, \tag{1}$$

where $r = |\mathbf{r}_1 - \mathbf{r}_2|$. For the wave function $\psi(\mathbf{r}_1)$ of the unpaired neutron in the Be^9 nucleus we take an expression given in Ref. 1. The wave function $\psi(\mathbf{r}_2)$ of the incident proton is taken in the form of a plane wave with the wave vector $\mathbf{k}$ normalized to unit flux. The wave function of the deuteron with wave vector $\mathbf{k}'$ is

$$\begin{gathered}\psi_d(\mathbf{r}_1, \mathbf{r}_2) = (2\pi\hbar)^{-3/2} e^{-ik'(\mathbf{r}_1+\mathbf{r}_2)/2}\Phi(r),\\ \Phi(r) = \sqrt{\alpha_1/2\pi}\, r^{-1} e^{-\alpha_1 r},\end{gathered} \tag{2}$$

where $\Phi(r)$ is the wave function of the internal state of the deuteron.

The differential cross section of the process that we are considering, as calculated by the usual equation of perturbation theory, is given by

$$\begin{aligned}\frac{d\sigma}{d\omega} &= \sqrt{2\left(1+\frac{Q}{E_p}\right)}\, 48a^2 \frac{\mu^2 g^4 r_0^2 \alpha_1 r_0}{\beta r_0 \hbar^4}\\ &\times r_0^2 (|\mathbf{k}'/2 - \mathbf{k}|\, r_0)^{-2} \operatorname{arctg}^2 \frac{\left|\frac{\mathbf{k}'}{2} - \mathbf{k}\right| r_0}{\gamma r_0}\\ &\times [\varphi(E_p, \vartheta) + (\beta/\alpha)^2 \sin\beta r_0\, e^{\alpha r_0} J(E_p, \vartheta)]^2;\end{aligned} \tag{3}$$

$$\begin{aligned}\varphi(E_p, \vartheta) &= \frac{\beta r_0 \sin(\beta+q) r_0}{2(\beta+q) r_0} + \frac{\beta r_0 \sin(\beta-q) r_0}{2(\beta-q) r_0}\\ &+ \frac{\cos(\beta+q) r_0 - \cos(\beta-q) r_0}{2q r_0},\end{aligned}$$

$$\begin{aligned}J(E_p, \vartheta) &= \int_{r_0}^{\infty} \frac{(1+\alpha r)}{r} e^{-\alpha r} \cos qr\,dr\\ &- \frac{1}{q}\int_{r_0}^{\infty} \frac{(1+\alpha r)}{r^2} e^{-\alpha r} \sin qr\,dr,\end{aligned}$$

where $\mathbf{q} = \mathbf{k}' - \mathbf{k}$. The integrals in the expression for $J(E_p, \vartheta)$ are not in explicit form. They must be calculated for given values of the energy of the incident proton E_p and for the scattering angle ϑ. In Eq. (3) we have used the notation

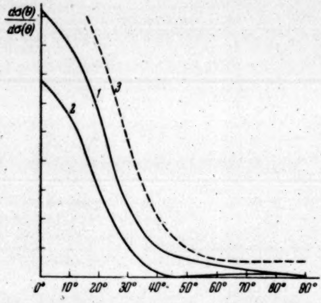
$$\alpha = \hbar^{-1}\sqrt{2\mu\varepsilon}, \quad \beta = \hbar^{-1}\sqrt{2\mu(V_0 - \varepsilon)}, \quad \gamma = \varkappa + \alpha_1,$$

where μ is the effective neutron mass with relation to the Be^8 core in the Be^9 nucleus, $\epsilon = 1.66$ mev is the binding energy of the neutron in the Be^9 nucleus, V_0 is the depth of the square potential well in the interaction between the unpaired neutron and the nuclear core, and r_0 is the width of this well. As has been shown in Refs. 1 and 2, $r_0 = 5 \times 10^{-13}$ cm and $V_0 = 12$ mev. Q is the energy liberated in the reaction and a is obtained from

$$\begin{aligned}\frac{a^2}{2}\Big\{\beta r_0 + \Big[(2+\alpha r_0)\frac{\beta^4}{\alpha^4}\\ + (1+\alpha r_0)\frac{\beta^2}{\alpha^2} - 1\Big]\frac{\sin^2\beta r_0}{\beta r_0}\Big\} = 1.\end{aligned} \tag{4}$$

Experimental curves are given in Ref. 3 for the angular distribution of deuterons from the Be^9 (pd) Be^8 reaction at proton energies of 5, 6, 7, 8 and 22 mev.

We have calculated the angular distribution from Eq. (3) at 5-8 mev. In the Figure, Curve *1* is the angular distribution at $E_p = 8$ mev. The angular



distributions at 5, 6 and 7 mev do not differ appreciably from the distribution at 8 mev, and the distribution at 5 and 6 mev for angles from 70^0 to 90^0 is in better agreement with the experimental curve than curve *1*. At 5 to 8 mev, the experimental distributions are identical within the limits of experimental error and are represented by curve 3. The reaction could also be calculated by Butler's theory[4], from the reverse reaction $\mathrm{Be}^8(dp)\mathrm{Be}^9$. The corresponding distribution at $E_p = 8$ mev is given by curve *2*. From a comparison of curves *1* and *2* with each other on the one hand, and with the experimental curve *3* on the other hand, we can see that curve *1* is in somewhat better agreement with experiment than curve *2* (especially for angles from 35° to 70°), although it does not differ appreciably as a whole from Butler's curve *2*.

It is well known that Butler's approximation is equivalent to a Born approximation in which the interior of the nucleus is neglected. Our result shows that an approximate calculation, which includes the interior of the nucleus, does not essentially change the angular distribution of deuterons in the $\mathrm{Be}^9(pd)\mathrm{Be}^8$ reaction that is obtained on the basis of Butler's theory.

We mention in conclusion that we have also calculated the angular distribution for a square well (instead of a Yukawa potential) interaction between the proton and the neutron of the nucleus. The result is practically the same as the distribution obtained from Eq. (3).

I consider it an obligation to express my gratitude to Professor V. I. Mamasakhlisov for his interest and for a number of suggestions.

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Polarization of Elastically Scattered Deuterons

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IT is known that the scattering of polarized particles provides important information regarding the spin dependence of their interactions. High energy polarized particles can be obtained by scattering on nuclei. The polarization is due to the spin-orbit interaction. A number of papers have presented calculations of the polarization of particles with spin 1/2. Lepore[1] has given a calculation of the scattering of polarized nucleons by nuclei with zero spin. The present note is concerned with the elastic scattering of polarized deuterons by nuclei with zero spin. This problem is similar to the problem of triplet nucleon-nucleon scattering[2].

The polarization of a deuteron beam is characterized by the three components of the spin vector and the five components of the symmetric second rank spin tensor with zero trace. Before collision the deuteron is described by the distorted plane wave

$$\Psi_0 = \exp\left[i\left(\mathbf{K}_0\mathbf{r} - \alpha \ln 2Kr\right)\right]\chi_0, \tag{1}$$

where χ_0 is the initial deuteron spin function. Polarization of the ingoing deuteron beam is characterized by the following quantities:

$$\mathbf{P}_{\text{in}} = (\chi_0 \mathbf{S} \chi_0^+), \qquad \langle T_{ik}\rangle_{\text{in}} = (\chi_0 T_{ik} \chi_0^+), \tag{2}$$

where T_{ik} is a symmetric spin tensor with zero trace:

$$T_{ik} = {}^1/_2\,(S_i S_k + S_k S_i) - {}^2/_3\,\delta_{ik}.$$

the constant $\alpha = Ze^2\mu / K\hbar$.

For the purpose of obtaining the scattered amplitude the wave function of the system must be expanded in a series of eigenfunctions of the operators

J^2, J_z, L, S, where J = L + S; L is the orbital moment of relative motion and S is the spin of the deuteron . Such an expansion of the wave function is accomplished through the use of projection operators[2]:

$$\Pi_l^{+} = [l + 1 + (l+2)\,(\mathbf{SL}) + (\mathbf{SL})^2] \,/\, (l+1)\,(2l+1); \tag{3}$$

$$\Pi_l^{0} = [\,l\,(l+1) - (\mathbf{SL}) - (\mathbf{SL})^2] \,/\, l\,(l+1);$$

$$\Pi_l^{-} = [-l - (l-1)\,(\mathbf{SL}) + (\mathbf{SL})^2] \,/\, l\,(2l+1).$$

These operators enable us to write the wave function of the system in the form

$$\Psi = \sum_{l=0}^{\infty} (2l+1)^{1/2}\, i^l \left[A_l^{+}\, \Pi_l^{+}\, \frac{U_l^{+}(Kr)}{Kr} + A_l^{0}\, \Pi_l^{0} \frac{U_l^{0}(Kr)}{Kr} + A_l^{-}\, \Pi_l^{-}\, \frac{U_l^{-}(Kr)}{Kr} \right] Y_l^0 \chi_0. \tag{4}$$

The usual method of calculating the scattered amplitude[1,3] gives

$$F(\theta) = A(\theta) + B(\theta)\,(\mathbf{Sn}) + C(\theta)\,(\mathbf{Sn})^2 + {}^1\!/_2 D(\theta)\,\{(\mathbf{Sk}_0)(\mathbf{Sk}) + (\mathbf{Sk})(\mathbf{Sk}_0)\}, \tag{5}$$

where $\mathbf{k}_0$ is a unit vector parallel to the momentum of the ingoing deuteron; **k** is a unit vector parallel to the momentum of the scattered deuteron; **n** $= [\mathbf{k}_0, \mathbf{k}] / \sin\theta$;

$$A(\theta) = \frac{1}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \left\{ \exp(i\delta_l^{+}) \sin\delta_l^{+} \left(Y_l^0 + \frac{2\cos\theta}{l+1} \frac{\partial V_l^0}{\partial(\cos\theta)} \right) \right.$$
$$+ (2l+1) \exp(i\delta_l^{0}) \sin\delta_l^{0} \left(Y_l^0 - \frac{2\cos\theta}{l\,(l+1)} \frac{\partial Y_l^0}{\partial(\cos\theta)} \right)$$
$$\left. - \exp(i\delta_l^{-}) \sin\delta_l^{-} \left(Y_l^0 - \frac{2\cos\theta}{l} \frac{\partial Y_l^0}{\partial(\cos\theta)} \right) \right\};$$

$$B(\theta) = \frac{i\sin\theta}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \left\{ \frac{2l+3}{2\,(l+1)} \exp(i\delta_l^{+}) \sin\delta_l^{+} \right.$$
$$\left. - \frac{2l+1}{2l\,(l+1)} \exp(i\delta_l^{0}) \sin\delta_l^{0} - \frac{2l-1}{2l} \exp(i\delta_l^{-}) \sin\delta_l^{-} \right\} \frac{\partial Y_l^0}{\partial(\cos\theta)};$$

$$C(\theta) = \frac{1}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \left\{ \frac{1}{l+1} \exp(i\delta_l^{+}) \sin\delta_l^{+} - \frac{2l+1}{l(l+1)} \exp(i\delta_l^{0}) \sin\delta_l^{0} \right.$$
$$\left. + \frac{1}{l} \exp(i\delta_l^{-}) \sin\delta_l^{-} \right\} \left\{ l\,(l+1)\, Y_l^0 - 2\cos\theta \frac{\partial Y_l^0}{\partial(\cos\theta)} \right\};$$

$$D(\theta) = -\frac{1}{K} \sum_{l=0}^{\infty} \frac{1}{(2l+1)^{1/2}} \left\{ \frac{1}{l+1} \exp(i\delta_l^{+}) \sin\delta_l^{+} \right.$$
$$\left. - \frac{2l+1}{l\,(l+1)} \exp(i\delta_l^{0}) \sin\delta_l^{0} + \frac{1}{l} \exp(i\delta_l^{-}) \sin\delta_l^{-} \right\} \frac{\partial Y_l^0}{\partial(\cos\theta)},$$

where δ_l^+, δ_l^0, δ_l^- are scattering phases corresponding to the total quantum number $j = l + 1,\ l,\ l - 1$.

We can now calculate the differential cross section and the polarization. It is convenient to use a density matrix[4,5] for the calculation of these quantities. For a deuteron beam this matrix is[5]

$$\rho = {}^1\!/_3\, \{1 + {}^3\!/_2\, (\mathbf{P}_{\text{in}}\, \mathbf{S}) + {}^3\!/_2\, \langle T_{ik} \rangle_{\text{in}}\, (\mathbf{S}_i \mathbf{S}_k + \mathbf{S}_k \mathbf{S}_i)\}, \tag{6}$$

so that $\mathbf{P}_{\text{in}} = \mathrm{Sp}\,(\mathbf{S}\,\rho)$,

$$\langle T_{ik} \rangle_{\text{in}} = \mathrm{Sp}\, \{[{}^1\!/_2\, (S_i S_k + S_k S_i) - {}^2\!/_3\, \delta_{ik}]\, \rho\}.$$

Two special cases are of interest: a) the polarization of an unpolarized incident deuteron beam, and b) the differential cross section for a polarized beam. In the first case the polarization vector and the polarization tensor will be expressed as follows:

$$\mathbf{p} = \frac{4}{3} \frac{\mathrm{Re}\left[\left(A + C + D\dfrac{\cos\theta}{2}\right) B^{*}\right]}{(d\sigma / d\Omega)_0}\, \mathbf{n} = P(\theta)\, \mathbf{n}, \tag{7}$$

$$\langle T_{ik} \rangle = P_1(\theta)\, (n_i n_k - {}^1\!/_3\, \delta_{ik})$$

$$+ P_2(\theta)(k_{0i}k_k + k_i k_{0k} - {}^2/{}_3\delta_{ik}\cos\theta)$$

$$+ P_3(\theta)(k_{0i}k_{0k} - {}^1/{}_3\,\delta_{ik}) + P_4(\theta)(k_i k_k - {}^1/{}_3\,\delta_{ik}),$$

$$P_1(\theta) = \frac{|B|^2 + |C|^2 + 2\mathrm{Re}\,[(A + D\cos\theta)C^*]}{3\,(d\sigma/d\Omega)_0};$$

$$P_2(\theta) = \frac{{}^3/{}_4\,|D|^2\cos\theta + \mathrm{Re}\,[(A+C)D^*]}{3\,(d\sigma/d\Omega)_0};$$

$$P_3(\theta) = \frac{-{}^1/{}_4\,|D|^2 - \dfrac{1}{\sin\theta}\,\mathrm{Im}\,(B^*D)}{3\,(d\sigma/d\Omega)_0};$$

$$P_4(\theta) = \frac{-{}^1/{}_4\,|D|^2 + \dfrac{1}{\sin\theta}\,\mathrm{Im}\,(B^*D)}{3(d\sigma/d\Omega)_0}.$$

In the second case,

$$d\sigma/d\Omega = (d\sigma/d\Omega)_0 + (d\sigma/d\Omega)_1 + (d\sigma/d\Omega)_2; \tag{8}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = |A|^2 + \frac{2}{3}|B|^2 + \frac{2}{3}|C|^2 + \left(\frac{\cos^2\theta}{2} + \frac{1}{6}\right)|D|^2 + \frac{4}{3}\mathrm{Re}\,[A^*(C + D\cos\theta)] + \frac{2}{3}\mathrm{Re}\,(C^*D)\cos\theta; \tag{9}$$

$$(d\sigma/d\Omega)_1 = 2\,\mathrm{Re}\,[(A + C + {}^1/{}_2\,D\cos\theta)B^*](\mathbf{P}_{\mathrm{in}}\mathbf{n}); \tag{10}$$

$$\begin{aligned}(d\sigma/d\Omega)_2 = {} & \{|B|^2 + |C|^2 + 2\,\mathrm{Re}\,[(A + D\cos\theta)C^*]\}\, n_i n_k \langle T_{ik}\rangle_{\mathrm{in}} \\ & + \{{}^3/{}_4\,|D|^2\cos\theta + \mathrm{Re}\,[(A+C)D^*]\}(k_{0i}k_k + k_i k_{0k})\langle T_{ik}\rangle_{\mathrm{in}} \\ & + \{-{}^1/{}_4\,|D|^2 + (\sin\theta)^{-1}\,\mathrm{Im}\,(B^*D)\}\,k_{0i}k_{0k}\langle T_{ik}\rangle_{\mathrm{in}} \\ & - \{{}^1/{}_4\,|D|^2 + (\sin\theta)^{-1}\,\mathrm{Im}\,(B^*D)\}\,k_i k_k \langle T_{ik}\rangle_{\mathrm{in}}.\end{aligned} \tag{11}$$

The cross section for an unpolarized deuteron beam is given by (9), whereas (10) and (11) result from initial polarization of the deuteron beam, with (10) corresponding to the polarization vector and (11) corresponding to the polarization tensor.

I take this opportunity to express my thanks to G. R. Khutsishvili for his interest and for valuable discussions.

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Charge Distribution of Mesons in Nucleon-Antinucleon Annihilation

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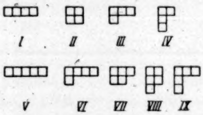
BELEN'KII and Rozental'[1] have studied the production of stars in antinucleon annihilation. On the basis of a statistical theory of multiple particle production they calculated the probabilities for processes of different multiplicites. We



present here the charge distribution which is calculated on the basis of isotopic spin conservation (see Refs. 2 and 3). As usual, p and n denote a proton and a neutron, while $\overline{p}$ and $\overline{n}$ denote an antiproton and an antineutron; annihilation products (π-mesons) are denoted by the signs of their charges. The charge distribution for $p\overline{n}$ is obtained from the distribution for $\overline{p}n$ by reversing the signs of meson charges. Table I shows the subdivision of processes of given multiplicity according to the charge states. If, for example, the annihilation cross section for $p\overline{p}$ into two mesons is σ_2, it can be seen from Table I that 0.167 of this cross section is due to the process $p + \overline{p} \to \pi^0 + \pi^0$ and 0.833 is due to $p + \overline{p} \to \pi^+ + \pi^-$.

If statistical theory is not used but only conservation of total isotopic spin, the charge distribution for a given multiplicity can be obtained only for processes that are characterized by a definite isotopic spin T, its projection T_3 and a definite Young scheme[4]. Such distributions are given in Refs. 5-7 for two and three mesons. We have done the same for four and five mesons. The results are given in Tables II and III. The Roman numerals at the top of the Tables indicate the Young schemes which correspond to the numerals in the Figure.

TABLE I

Number of Mesons	Charge distribution in $\bar{p}p$ and $n\bar{n}$ annihilation: Charge States	Relative statistical weight	Charge distribution in $\bar{p}n$ annihilation: Charge States	Relative statistical weight
2	0 0	0.167	− 0	1
	+ −	0.833		
3	0 0 0	0.150	+ − −	0.700
	+ − 0	0.850	− 0 0	0.300
4	+ + − −	0.400	+ − − 0	0.800
	+ − 0 0	0.578	− 0 0 0	0.200
	0 0 0 0	0.022		
5	+ + − − 0	0.640	+ + − − −	0.286
	+ − 0 0 0	0.340	+ − − 0 0	0.629
	0 0 0 0 0	0.020	− 0 0 0 0	0.085

TABLE II

$T=1$ $T_3=1$	III	IV	$T=1$ $T_3=1$	III	IV	$T=0$	I	II
+ + − 0	3/5	1	+ + − −	4/5	0	+ + − −	8/15	1/3
+ 0 0 0	2/5	0	+ − 0 0	1/5	1	+ − 0 0	2/15	2/3
						0 0 0 0	3/15	0

TABLE III

$T=1$ $T_3=1$	V	VI	VII	VIII	$T=1$ $T_3=0$	V	VI	VII	VIII	$T=0$	IX
+ + + − −	24/35	4/10	4/10	0	+ + − − 0	8/35	8/10	2/10	1	+ + − − 0	2/3
+ + − 0 0	8/35	3/10	6/10	1	+ − 0 0 0	12/35	2/10	8/10	0	+ − 0 0 0	1/3
+ 0 0 0 0	3/35	3/10	0	0	0 0 0 0 0	15/35	0	0	0	—	—

In conclusion I wish to thank Professor S. Z. Belen'kii who suggested this problem.

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Account of Retardation in the Interaction of Neutral Atoms

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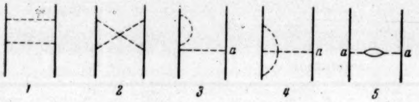
CASIMIR and Polder[1] considered retardation in the interaction of two neutral atoms. They showed that the energy of interaction for distances



large compared to the wavelengths in the spectrum of the atom decreases with the distance as R^{-7} and not as R^{-6} as follows from the theory of London for the Van der Waals forces. Subsequently, these deductions were verified in Ref. 2 for the independent macroscopic approach to the question. However, the calculations of Casimir and Polder have a number of shortcomings. In their original method of asymmetric perturbation theory there appeared divergent expressions which could not be completely and correctly removed until the appearance of the present technique of renormalization. Moreover, when integrating resonance denominators, basic to their thoery, the authors used rules of circumvention, which did not naturally follow from their method. In connection with the above, there appeared recently a paper (Ref. 3) in which the author doubts the results obtained by Casimir and Polder. Using a cumbersome nonrelativistic form of the perturbation theory this author obtained the physically scarcely detectable R^{-3} dependence for the interaction energy. We did not check the calculations in Ref. 3, and in order to solve this question we made calculations for the case of two hydrogen atoms in the ground state using the contemporary invariant technique of Feynman.

It is known that the interaction between neutral atoms is an effect of the fourth order in the charge e in perturbation theory. The S-matrix, averaged in the photon vacuum, is in the fourth approximation:

$$S^{(4)} = \frac{\pi^2}{2}\iiiint P[j_\mu(1)\, j_\mu(2)\, j_\nu(3)\, j_\nu(4)] \times D^F(12)\, D^F(34)\, dx_1\, dx_2\, dx_3\, dx_4.$$

The matrix elements, corresponding to the diagrams shown in Fig. 1 contribute to the effect which interests us. From these diagrams the divergent diagrams 3, 4 and 5 are zero after renormalization, since they will be proportional (neglecting the relativistic effects and multipole interactions), to the average value of the dipole moment of the atoms (point a). The contribution from diagrams 1 and 2 is finite and is equal to

$$S^{(4)} = 2\pi^2 \iiiint P[j_\mu^{(1)}(1)\, j_\nu^{(1)}(3)]\, P[j_\mu^{(2)}(2)\, j_\nu^{(2)}(4)] \times D^F(12)\, D^F(34)\, dx_1\, dx_2\, dx_3\, dx_4.$$

The current density $j_\mu(x)$ in our case has operators with the following matrix elements:

$$\langle \mathbf{j}\rangle_{nm} = e\psi_n^*\alpha\psi_m, \quad \langle j_4\rangle_{nm} = ie(\psi_n^*\psi_m - \delta(\mathbf{r})\,\delta_{nm}),$$

where ψ_n is the wave function of an atom in the state n.

Let us use the known expansion of D^F in the Fourier integral and then integrate for all times. Then for the effective energy of interaction U, corresponding to the given element of the S-matrix (see for instance, Ref. 4), we get

$$U = \frac{i}{16\pi^5}\iiiint d\mathbf{r}_1\, d\mathbf{r}_2\, d\mathbf{r}_3\, d\mathbf{r}_4 \times \iint d\mathbf{p}'\, d\mathbf{p}'' \exp\{i\mathbf{p}'(\mathbf{r}_1 - \mathbf{r}_2) + i\mathbf{p}''(\mathbf{r}_3 - \mathbf{r}_4)\} \times \int_{-\infty}^{\infty} \frac{d\omega}{(p'^2 - \omega^2)(p''^2 - \omega^2)}$$

$$\times \sum_{n,m}' \left\{ \frac{j^{(1)}_{\mu on}\, j^{(1)}_{\nu no}}{\omega_{no} - \omega} + \frac{j^{(1)}_{\nu on}\, j^{(1)}_{\mu no}}{\omega_{no} + \omega} \right\} \left\{ \frac{j^{(2)}_{\mu 0m}\, j^{(2)}_{\nu m0}}{\omega_{m0} + \omega} + \frac{j^{(2)}_{\nu 0m}\, j^{(2)}_{\mu m0}}{\omega_{m0} - \omega} \right\}.$$

The known rule of Feynman is used for the circumvention of the poles when integrating with respect to ω.

Neglecting relativistic effects and the effects of higher multipoles, and also taking into consideration that the ground state of hydrogen is an s-state, we obtain finally

$$U = \frac{i}{16\pi^5}\iint d\mathbf{p}'\, d\mathbf{p}''\, e^{-i(\mathbf{p}' + \mathbf{p}'')\mathbf{R}} \int_{-\infty}^{\infty} \frac{d\omega}{(p'^2 - \omega^2)(p''^2 - \omega^2)} \times \alpha_1(\omega)\, \alpha_2(\omega)\, [3\omega^4 - \omega^2(p'^2 + p''^2) + (\mathbf{p}'\mathbf{p}'')^2].$$

Here $\alpha(\omega)$ is the real part of the polarization of atom

$$\alpha(\omega) = \sum_n 2\omega_{n0}\, |d_{0n}|^2 / (\omega_{n0}^2 - \omega^2),$$

where d_{0n} is the matrix element of the dipole moment.

Divergent integrals with respect to p' and p'' must be calculated with a cutting-off multiplier of the type $e^{-\lambda p R}$, subsequently allowing λ to go to zero. Actually, in neglecting the effects of the higher multipoles, we assume that $p \ll 1/a$, where a is of the order of the size of the atom. In this sense $\lambda \to 0$ means neglecting values of the order of magnitude of a/R.

For $R \ll \lambda_0$ (λ_0 is the order of magnitude of the wavelengths in the spectrum of the atom) U is the form of London's formula. In the other limiting case for $R \gg \lambda_0$ in the integrals with respect to p and ω, the regions $p \sim \omega \sim \lambda_0/R$ are important, and the main member in U will be

$$U = \frac{i}{16\pi^5}\alpha_1(0)\ \alpha_2\ (0)$$

$$\times \lim_{\lambda\to 0}\iint d\,\mathbf{p}'\,d\,\mathbf{p}'' e^{-i(\mathbf{p}'+\mathbf{p}'')\mathbf{R}-\lambda R(p'+p'')}$$

$$\times \int_{-\infty}^{\infty}\frac{d\omega}{(p'^2-\omega^2)\,(p''^2-\omega^2)}\,[3\omega^4-\omega^2(p'^2+p''^2)$$

$$+(\mathbf{p}'\mathbf{p}'')^2\} = -\frac{23\alpha_1(0)\,\alpha_2(0)}{4\pi R^7};$$

which exactly coincides with the results obtained by Casimir and Polder.

I express my gratitude to L. P. Gorkov for taking part in the discussion of the above problem.

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The Energy Spectrum of γ-Quanta from Decaying π^0-Mesons

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THE relationship which determines the energy spectrum of the γ-radiation $W(\epsilon_\gamma, \theta_N)$ observed in any coordinate system at some angle θ_N, as a function of the energy and angular distributions of neutral mesons $F(\epsilon_\pi, \theta)$ in the same coordinate system is

$$W(\varepsilon_\gamma, \theta_N) = \int_{\varepsilon_{\min}}^{\infty}\frac{d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}}\cdot\frac{1}{2\pi}\int_0^{2\pi}F(\varepsilon_\pi, \theta)\,d\varphi'. \tag{1}$$

Here ϵ_π and ϵ_γ are the total energy of the π^0-meson and the energy of the γ-quantum in units of the π^0-meson rest energy: $\epsilon_{\min} = \epsilon_\gamma + 1/(4\epsilon_\gamma)$

$$\theta = \arccos(\sin\vartheta\cos\varphi'\sin\theta_N + \cos\vartheta\cos\theta_N);$$

$$\vartheta = \arccos\frac{\varepsilon_\pi - 1/2\varepsilon_\gamma}{\sqrt{\varepsilon_\pi^2-1}}.$$

For π^0-mesons whose angular distribution is proportional to $\cos^2\theta$ we obtain

$$W(\varepsilon_\gamma, \theta_N) = \frac{1}{2}(3\cos^2\theta_N - 1)\int_{\varepsilon_{\min}}^{\infty}\frac{\cos^2\vartheta\,F(\varepsilon_\pi)\,d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}} \tag{2}$$

$$+\frac{1}{2}\sin^2\theta_N\int_{\varepsilon_{\min}}^{\infty}\frac{F(\varepsilon_\pi)\,d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}}.$$

From this it follows that the γ-spectrum recorded at angle $\theta^*_N = \arccos(1/\sqrt{3})$ is logarithmically symmetrical with respect to the energy $\epsilon_\gamma = 1/2$ just as in the case of isotropic distribution of π^0-mesons. This means that from the γ-spectrum at a given angle and for an angular distribution of mesons of the form $a + b\cos^2\theta$, it is possible to obtain directly the energy distribution of the mesons and their mass by the method described in Ref. 1. Angles which are characterized as noted above will hereinafter be called "isotropic".

Another characteristic of an "isotropic" angle is the dependence of the total gamma flux at a given angle on the ratio of the constants a and b in the angular distribution of the π^0-mesons.

By integrating (2) with respect to the energy we obtain the angular distribution of the γ-rays produced through the decay of π^0-mesons whose angular distribution is proportional to $\cos^2\theta$, in the form

$$W_\gamma(\theta_N) = \frac{1}{2}(3\cos^2\theta_N - 1) \tag{3}$$

$$\times\int_0^{\infty}d\varepsilon_\gamma\int_{\varepsilon_{\min}}^{\infty}\frac{\cos^2\vartheta\,F(\varepsilon_\pi)\,d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2-1}} + \frac{1}{2}\sin^2\theta_N.$$

It follows that the gamma flux at angle $\theta^*_N = \arccos(1/\sqrt{3})$ remains unchanged in the transition from the $\cos^2\theta$ law of meson angular distribution to an isotropic distribution if only there is no change in the total number of mesons produced per unit time.

It also follows from (3) that when the angular distribution of π^0-mesons is $a + b\cos^2\theta$ the angular distribution of γ-rays must have the form $A + B \cos^2\theta$, where

$$A = a + \frac{b}{2}\left[1 - \int_0^\infty d\varepsilon_\gamma \int_\alpha^\infty \frac{\cos^2\vartheta\, F(\varepsilon_\pi)\, d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2 - 1}}\right];$$

$$B = \frac{b}{2}\left[3 \int_0^\infty d\varepsilon_\gamma \int_{\varepsilon_{\min}}^\infty \frac{\cos^2\vartheta\, F(\varepsilon_\pi)\, d\varepsilon_\pi}{\sqrt{\varepsilon_\pi^2 - 1}} - 1\right].$$

The same conclusion was reached earlier in Ref. 2. Using this result and Rosenfeld's[3] suggestion regarding the existence of an "isotropic" angle θ_N^* for charged mesons we can also conclude that the gamma flux at angle θ_N^* is independent of the ratio of a and b in the angular distribution of π^0-mesons.

The above-mentioned properties of the gamma spectrum and flux for the "isotropic" angle are retained in the more general case when the angular and energy distributions of the π^0-mesons are of the form $a(\epsilon_\pi) + b(\epsilon_\pi) \cos^2\theta$. It should be noted that in this instance when the derivative of the measured gamma spectrum is multiplied by the gamma-ray energy we obtain the function $a(\epsilon_\pi) + 1/3\, b(\epsilon_\pi)$.

I take this opportunity to acknowledge my indebtedness to B. M. Pontecorvo for a discussion of the above results.

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On the Stability of the Phase Boundaries Between Normal and Superconducting States

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NOT long ago, Beck's paper[1] appeared, in which the author, using London's equations for the magnetic field in a superconductor, found an instability in the boundary between the n- and s-phases as regards a periodic (along the surface) perturbation of the shape of the boundary. It is immediately clear from Eqs. (25) and (34) of Ref. 1 that the instability found by the author, at least as regards a perturbation with a period much greater than the penetration depth of the magnetic field in the superconductor, is explained by the well-known fact that London's equations lead to a negative surface energy on the boundary[2]. Since a negative surface tension contradicts experimental results for thin films, the analysis of the problem of stability should be based, not on London's equations, but on the theoretical calculations of Landau and Ginzburg[3], which give a positive value to the surface energy. In Landau and Ginzburg's theory, the problem of stability becomes the problem of a unique solution at infinity under corresponding boundary conditions. A strict analysis such as this can scarcely be performed by means of the non-linear equations of the theory. The only complete solution is for a perturbation with a period much greater than the penetration depth. In this case it can be made equal to zero, so that B = 0 in the superconducting phase. Also, in agreement with the theory of Landau and Ginzburg, we attribute a positive energy to the boundary between the n- and s-phases, which we write in the usual form, $(H_k^2/8\pi)\Delta$, where H_k is the critical field and Δ is a constant with the dimensions of length. The free energy change taking place with a variation in the shape of the boundary is written (for this case)

$$\delta F = \frac{H_k^2}{8\pi}\,\delta V_n + \Delta\frac{H_k^2}{8\pi}\,\delta S - \frac{1}{8\pi}\,\delta \int_{V_n} H^2 dV. \qquad (1)$$

The equilibrium of a plane boundary is studied in relation to an arbitrary (but not specifically oriented, as in Ref. 1) periodic perturbation. The stability of the boundary of arbitrary form is analyzed in the same way because any small part of the boundary can be thought of as a plane. The integral on the right side of Eq. (1) is easily transposed so that to calculate δF, correct to a second degree term over a small variation in the boundary δz, it is sufficient to know the magnetic field variation $\delta \mathbf{H}$ with an accuracy to a term of the first order of δz. By means of a simple transformation we can show that

$$-\frac{1}{8\pi}\,\delta \int_{V_n} H^2 dV = -\frac{H_0^2}{8\pi}\,\delta V_n + \frac{1}{8\pi}\int_S [\mathbf{A}_0 \delta \mathbf{H}]\, ds, \qquad (2)$$

where $\mathbf{A}_0$ is the vector potential of the unperturbed constant field $\mathbf{H}_0$; the integral on the right-hand side of Eq. (2) is taken over the surface of the perturbed boundary. In the derivation of Eq. (2) the vector potentials $\mathbf{A}$ and $\mathbf{A}_0$ are so normalized that

$A_0 = 0$ when there is no perturbation and $A = 0$ where there is a perturbation of the boundary.

If a small periodic perturbation is put on the boundary

$$\delta z = a(\mathbf{f})\, e^{i\mathbf{r}\mathbf{f}},$$

then for the component δH, parallel to f, correct to a first order quantity for $a(f)$, we have

$$\delta H_{\parallel} = \mathrm{H}_0 \mathbf{f} a(\mathbf{f}) e^{i(\mathbf{f}\mathbf{r}) - fz}, \tag{3}$$

$\delta H_{\perp}$ being reduced to zero. Using (2) and (3) we determine the free energy change (at the same time $H_0 = H_k$, which appears to be a necessary condition of equilibrium).

$$\delta F = \frac{SH_k^2}{8\pi} \sum_{\mathbf{f}} |a(\mathbf{f})|^2 \left(\frac{\Delta f^2}{2} + f\cos^2\varphi\right), \tag{4}$$

where φ is the angle between f and H_0.

In this way $\delta F \geq 0$, which shows the stability of the boundary in relation to a smoothly changing form (it should be $|\operatorname{grad} z| \ll z/\delta$, where δ is the penetration depth).

Equation (4) allows us to calculate the mean square of the fluctuation of the displacement of the boundary. By using the general theory of thermodynamic fluctuation[4] we find

$$\overline{|a(\mathbf{f})|^2} = \frac{8\pi kT}{SH_k^2\,(\Delta f^2 + 2f\cos^2\varphi)},$$

from which

$$\overline{(\delta z^2)} = \frac{2kT}{\pi H_k^2} \int \frac{d\mathbf{f}}{\Delta f^2 + 2f\cos^2\varphi}. \tag{5}$$

The integral diverges logarithmically for large f, but since our analysis is correct only insofar as $f \ll 1/\delta$ (δ is the penetration depth), we should stop the integration at $f_0 = 1/\lambda$, $\lambda \sim \delta$. The calculation gives

$$\overline{(\delta z)}^2 = (4kT/\Delta H_k^2)\ln(\Delta/\lambda). \tag{6}$$

For mercury when $T \sim 1°$ K, with the exception of the region near the lambda point, $\delta z \sim 10^{-7}$ cm.

We note that a difference from the usual result for the fluctuation of the displacement of the boundary in the absence of magnetic field, where $\overline{(\delta z)^2} \sim \int f^{-2} d\mathbf{f}$, is that the integral (5) corresponds to lower limit.

I gratefully acknowledge the help of E. M. Lifshitz in preparing this paper and in obtaining the results.

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Mechanical Phase Analyzers for Treatment of Experimental Data on the Scattering of Particles without Spin on Particles with Spin 0 or 1/2

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As is known, the scattering amplitude for elastic scattering of particles without spin on particles with spin 1/2 in a state with a definite isotopic spin is of the form

$$f(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} [(l+1)(\rho_l^+ - 1) \tag{1}$$

$$+ l(\rho_l^- - 1)] P_l(\cos\theta) + \frac{\sigma \mathbf{n}}{2k} \sum_{l=1}^{\infty} (\rho_l^- - \rho_l^+) P_l^1(\cos\theta),$$

where $P_l(\cos\theta)$ and $P_l^1(\cos\theta)$ are the Legendre polynomials and the associated Legendre functions, k and θ are wave number and scattering angle in the center-of-mass system, and **n** is a unit vector perpendicular to the plane of scattering. Here we introduce the notation: $\rho_l^{\pm} = \exp 2i\delta_l^{\pm}$, where $\delta_l^{\pm}$ are the scattering phases. With the plus sign we denote the magnitudes for the states in which the total momentum j is equal to $l + 1/2$, and with the minus sign, for the states in which $j = l - 1/2$. The amplitude in Eq. (1) satisfies the relationship

$$\tfrac{1}{2}\,\mathrm{Sp}\, Im f(0) = (k/4\pi)\,\sigma, \tag{2}$$

where σ is the total scattering cross section. For the scattering on the nonpolarized particles we have for the differential cross section and polarization

$$\sigma'(\theta) = \frac{1}{2}\,\mathrm{Sp}\, f^+(\theta) f(\theta) \tag{3}$$

$$= \frac{1}{4k^2} \left\{ \left| \sum_{l=0}^{\infty} [(l+1)(\rho_l^+ - 1) + l(\rho_l^- - 1) P_l(\cos\theta) \right|^2 \right.$$

$$+\left|\sum_{l=1}^{\infty}(\rho_l^- - \rho_l^+ P_l^1(\cos\theta)\right|^2\Big\} = \frac{1}{k^2}\sum_{l=0}^{\infty} A_l P_l(\cos\theta),$$

$$\sigma'(\theta)P(\theta) = \frac{1}{2}\,\mathrm{Sp}\, f^+(\theta)\,\sigma \mathbf{n} f(\theta) \tag{4}$$

$$= \frac{1}{4k^2}\,2\left\{\sum_{l=0}^{\infty}[(l+1)(\rho_l^+ - 1) + l(\rho_l^- - 1)]P_l(\cos\theta)\right\}$$

$$\times\left\{i\sum_{l=1}^{\infty}(\rho_l^- - \rho_l^+)P_l^1(\cos\theta)\right\},$$

where the product of the complex numbers is taken in the sense of the scalar product $2Z_1Z_2 \equiv Z_1^*Z_2 + Z_2^*Z_1$. The coefficients A_l are determined experimentally. From Eqs. (3) and (4) it is obvious that $\sigma'(\theta)[1 \pm P(\theta)]$ and particularly $\sigma'(0)$, $\sigma(\pi)$ and $\sigma'(\pi/2)[1 \pm P(\pi/2)]$ are expressed as the squares of the moduli of some linear combinations of the quantities $\rho_l^{\pm}$. If we add an arbitrary real number λ under the square sign of the modulus in the expression for $\sigma'(0)$, then, because of Eq. (2), we obtain

$$4k^2\left\{\sigma'(0) - \lambda\frac{\sigma}{4\pi}\right\} + \lambda^2 \tag{5}$$

$$= \left|\sum_{l=0}^{\infty}[(l+1)(\rho_l^+ - 1) + l(\rho_l^- - 1)] + \lambda\right|^2.$$

Furthermore, if the number of phases different from zero is finite, and there are present both phases of the highest orbital momentum p taking part in the interaction, then for the last coefficient of the expansion of the differential cross section we get the following, as can be easily verified:

$$|(p+1)\rho_p^+ + p\rho_p^- - p - 1|^2 \tag{6}$$

$$= p^2 + \frac{2(4p-1)!\,[(p-1)!]^2\,p!\,(p+1)!}{[(2p-1)!]^3\,(2p+1)!}A_{2p}.$$

If the last phases are both phases of the maximum total momentum $j = p - 1/2$ then

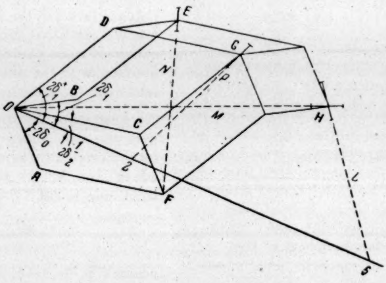
$$|\rho_{p-1}^+ + \rho_p^- - 1|^2 \tag{7}$$

$$= 1 + \frac{4(4p-3)!\,[(p-1)!]^4}{(2p-2)!\,[(2p-1)!]^3}A_{2p-1},$$

and the coefficient for $P_{2p}(\cos\theta)$ in σ' is identically zero. In cases, when the polarization is not experimentally known, one can use the relationships which follow directly from Eq. (3); for instance, letting $\theta = \pi/2$.

If in Eqs. (1)-(3), (5) and (6) we let $\rho_l^+ = \rho_l^-$ then we get the relationship for the scattering of particles without spin.

For the construction of mechanical apparatus which would allow the determination of phases $\delta_l^{\pm}$ from the experimental data for angular distribution and polarization of the particles, we will consider the magnitudes $\rho_l^{\pm}$ as vectors in a plane. Then it is obvious that the mentioned relationships determine the absolute magnitudes of some linear combinations of these vectors.



Scheme of the apparatus for 4 phases for spins (0, 1/2):

$$A = \rho_0,\ B = \rho_1^-,\ C = 2\rho_2^-,\ D = 2\rho_1^+,\ E = 2\rho_1^+ + \rho_1^-,$$

$$F = 2\rho_2^- + \rho_0,\ G = 2\rho_1^+ + 2\rho_2^-,\ H = \rho_0 + \rho_1^- + 2\rho_1^+ + 2\rho_2^-,$$

$$L = 2k\sqrt{\sigma'(0)},\ M = \sqrt{4k^2\{\sigma'(0) - 6\sigma/4\pi\} + 36},$$

$$N = 2k\sqrt{\sigma'(\pi)},\ P = 2\sqrt{1 + {}^{10}/_{9}A_2}.$$

Let us consider in more detail the case of finding four phases for the scattering of particles with spins (0, 1/2). From vectors ρ_0, ρ_1^-, $2\rho_1^+$ and $2\rho_2^-$, let us construct a lever system, shown on the picture, and superimpose on it four connections which are determined by relationships (5) (for $\lambda = 6$), (7), and by relationships following from the expressions for $\sigma'(0)$ and $\sigma'(\pi)$. On the picture the connections are shown by the dotted lines. Moreover we fix all 4 degrees of freedom of the system and can read off the angles 2δ which the vectors ρ in point 0 make with the abscissa. Because of the arbitrariness of λ in Eq. (5), the point H can be connected with two arbitrary points of the real axis, choosing them in such a way that the angle between these two connections is closest to a right angle--which increases the accuracy of the apparatus. For this purpose, because of Eq. (2), it is convenient to choose λ_1 and λ_2 in such a way that $\lambda_1 < 2k^2\sigma/4\pi < \lambda_2$, wherein, in the joining of the connection to the point x of the abscissa, $\lambda = (1+p)^2 - x$ or $\lambda = p^2 + p - x$ [depending on which of the Eqs. (6) or (7), is applicable]. Because of Eq. (2), the left side of (5) is positive, and the experimental data, which do not satisfy this condition, are not compatible, since for them the relationship $|f(0)|^2 \geq [\mathrm{lm}\, f(0)]^2$ is not satisfiel. As is known (Ref. 1) the cross section (3) does not change if one substitutes for all phases the phase of the states of the same total momentum but of the opposite parity. This property of the cross section can be utilized to double the region of the angles which can be found by this apparatus without moving any of its parts through the point 0; for this purpose it is sufficient to rename the vectors according to the scheme $\rho_0 \leftarrow \rho_1^-$, $2\rho_1^+ \leftarrow 2\rho_2^-$. Note that only 4 relations [(5), (6) or (7) and relations for $\sigma'(0)$ and $\sigma'(\pi)$] have a simple appearance and at the same time do not include polarization data. Therefore, for finding a larger number of phases it is necessary either to consider polarization data, or to realize more complicated relations of the type (3), or, finally, to extrapolate some phases based on the measurements for other energies.

Mechanical analyzers, with an adequate amount of experimental data, permit one quickly to find phases and thus to solve the system of trigonometric equations which arise during a comparison of the expansion coefficients for the experimental angular distributions and polarization with the expressions of these coefficients by means of phases. In this type of apparatus, the connection between the phases and magnitudes, introduced from the experiment, is two-sided. Therefore, if the limits are known in which each of n independent experimental magnitudes can be found with the probability q, the apparatus allows us to find the limits in which the phases lie with the probability Q, where $q^n \leq Q \leq q$ (the right equality is realized for the complete correlation of errors of the experimental data, the left for the complete absence of the correlation). For this purpose the lengths of the connections must allow a free change within the errors of the corresponding magnitudes (in the picture these limits are shown by the segments of the couplings). Varying the position of the apparatus within the limits allowed by the connections, one can find the limits for the phases and the sensitivity of the solution to a change of experimental data.

Up to the present time the solution of the corresponding system of equations (e.g., for the scattering of π-mesons by nucleons, or neutrons by α-particles) was either carried out numerically (Refs. 2-5), or with the help of electrical analogues (Ref. 6), or graphically (Refs. 7-9). The first two methods have the disadvantage that the connection between the experimental data and phases is one-sided: only analytical expressions for the coefficients in the expansion in phases of the cross sections and polarization are known, and not vice versa. Therefore, the solution is obtained by a large number of successive approximations and does not indicate the shortcomings of the phases. True, the numerical calculation allows one to obtain the "best" phases using the method of least square; however, the speed of numerical processing will doubtless be considerably greater if the phases are defined more accurately within the limits determined by the apparatus. Graphical solution is possible only for the case when the number of obtained phases does not exceed three; it also does not allow one to find the errors in the phases.

The lever apparatus has already been described in the literature (Refs. 10 and 11) for determining three phases for the scattering of π-mesons by nucleons, but in this apparatus only the relation (2) is utilized and the relation for $\sigma'(0)$; also, instead of utilizing the relations for $\sigma'(\pi)$ or (6), there is used a very inconvenient condition for the equality of two angles whose apexes are at different points for all situations of the apparatus.

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Electrical Properties of Germanium at Very Low Temperatures

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HUNG[1] found that there is a change in the activation energy of the carrier current in germanium in the region of helium temperatures and later this was verified by experiments[2,3]. But until the present time there has not appeared a satisfactory theoretical clarification. It seems to be proper at this time then to study the properties of germanium at very much lower temperatures.

We obtained very low temperatures by the adiabatic demagnetization of iron-ammonium alum. The apparatus allowed us to cool the germaium sample to 0.15° K. The temperature of the sample was controlled by a calibrated carbon thermometer, with an accuracy of 5×10^{-3}° K. The electrical conductor for measuring the resistance was passed into the dewar to the sample through an evacuated steel tube, and covered with polystyrene washers. The conductor is cooled to helium temperatures by means of a quartz rod and to very low temperatures by means of a block of alum. This made possible the attainment of very low temperatures in a matter of a few hours, so that we could carry on the experiments at a temperature that remained relatively stable.

The thermal and electrical contacts to the sample of germanium were made using springs tightly enveloping the sample, the ends of which were electrolytically covered with copper. The electrical resistance of the sample was measured with an electrometer with a reversable connection and with a current sensitivity equal to 1×10^{-14} A.

For measurements in the region of hydrogen and helium temperatures, the apparatus was filled with atmospheric helium. For measurements at very low temperatures, several centimeters of gaseous helium were put in at room temperature by which the isothermal magnetization was hastened. The adiabatic demagnetization depends on the adsorption of the gaseous helium over the surface of the cooled salt.

Several samples of germanium were studied having a specific resistance of the orderof 1 ohm-cm at room temperature. The samples were prepared in the Institute of the Metallurgical Academy of Sciences, USSR, and in the semiconductor section of Moscow State University.

The temperature dependence of the specific resistance of the sample is shown in Fig. 1. The resistance of the sample was measured at a gradient of 50-100 mv/cm. At this voltage the resistance for all practical purposes still does not depend on the field. The effect of geometry on the resistance was studied. To clarify the role of the contacts, the space between them was variedfor one of the samples from 15.8 to 8 mm at a constant cross section area of about 0.25 cm^2. Within the limits of the accuracy of the experiment (around 20%) the calculated specific resistances were equal. There is the same degree of accuracy in the results for samples subjected to these different surface treatments: 1. polishing, 2. dipping in a boiling mixture of hydrochloric and nitric acids. The results did not change when the ends of the sample were coated with copper by means of electrolysis.

We have shown two sets of curves on Fig. 1. One of these curves corresponds to the resistance of the samples in the region of hydrogen and helium temperatures, the other, in the region of helium and very low temperatures. The results of the measurements allow us to make some conclusions on the existence in the region of temperature from 0.15 to 1°K of an energy of activation smaller than in the region of temperature between 1.6 and 4.2° K. The small amount of activation energy of the current carrier in the region of helium and very low

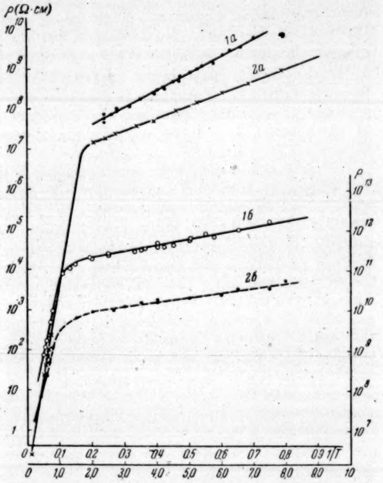

FIG. 1. ρ (ohm-cm) *vs.* 1/*T* (left scale), ρ *vs* 1/*T* (right scale). The dependence of resistance on the temperature: *1a, 2a,* in the region of hydrogen and helium temperatures (left scale), *1b, 2b,* in the region of helium and very low temperatures (right scale).

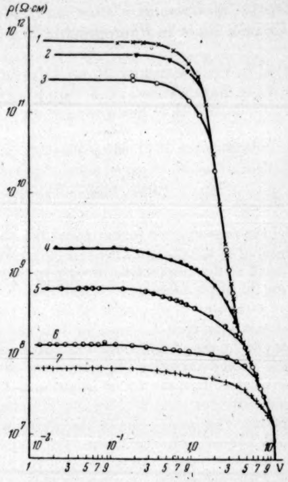

FIG. 2. ρ(Ω · cm) *vs.* V. The dependence of the resistance on the external electrical field: (v/cm): *1*--0.15; *2*--0.2; *3*--0.4; *4*--1.77; *5*--2.23; *6*--3.12; *7*--4.2° K; (V_{cr} = 11 v/cm).

temperatures makes germanium sensitive to a very small surface effect. Thus, for example, room temperature radiation lessens the resistance of the sample by a factor of ten. This circumstance led to the necessity of shielding the sample with a baffle at the temperature of liquid helium.

We know[4] that the presence of comparatively small electrical fields also leads to an increase of conductivity. Figure 2 shows curves taken at various temperatures between 0.15 and 4.2° K and the dependence of the sample resistance on the electrical field. The results for the samples exposed to various surface treatments are self-consistent. Changing the linear dimensions of the sample displaces all the curves so that the ratio of V_{cr} to the dimensions of the sample remains constant.

With an external voltage differing from the "critical" by only 10-15%, a thermometer fastened to a lower part of the sample, i.e., on the side opposite from the coolant, did not show more than a 0.1° K rise. The application of a "critical" field led to an instantaneous heating of the sample to the temperature of the outer bath, i.e., to 1.7-1.8° K. On the curves of Fig. 2 in the region of voltage lower than the critical we can see two different slopes, one of which is inherent in the curve in the region of very low temperatures; the other, in all curves in all temperature intervals from 0.15-4.2° K.

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Energy Spectrum of π^--Mesons Produced by Cosmic Rays in Photographic Emulsions

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1. IN the research we investigated the energy spectrum of π^--mesons (with energies < 70 mev) produced in the interaction of cosmic rays with the nuclei of photographic emulsions. We made use of two piles of emulsion layers of diameter 10 cm of emulsion type P, irradiated in the stratosphere (56 layers of 330μ each and 60 layers of 450μ). In the exposure the piles were assumed to be vertical so that the mean direction of the primary cosmic particles was directed into the plane of the emulsion. The processing of the plates was carried out according to the method described in Ref. 1. In the systematic examination of the plates under a microscope, all the tracks of stopped π-mesons were observed and then traced to the point of meson creation or of its entry into the plate. In the determination of the energy of the π-mesons from their total range, use was made of the tables[2] of the dependence of the range-energy for the emulsion Ilford G-5, inasmuch as the stopping action of the emulsion actually used differed insignificantly from the Ilford emulsion.

2. In all, about 1000 tracks of stopped π-mesons were traced, of which 195 π^+- and 328 π^--mesons were formed in the emulsions. Their energy spectra are shown in Figs. 1 and 2 (dashed line histograms). In view of the fact that the π-mesons of different energies formed in the emulsion have different probabilities of stopping in the plate, appropriate geometrical corrections were introduced to take account of the finite size of the emulsion volume.

The correction coefficients were computed in the following way. First, for each track of a π-meson which makes an angle θ with respect to the vertical the total interval $\Delta\varphi$ was determined of those angles φ (φ is the azimuth angle) for which the track of a π-meson with range R will end within the emulsion chamber. The first correction coefficient $K_\varphi = 2\pi/\Delta\varphi$ determines the ratio of the number of all π-mesons which leave the given point with angle θ and with energy $E(R)$ to the number of those of them which stop in the emulsion. Second, that part of the layer ΔS on which the π-mesons created in the plate could be stopped if they emerge from different parts of the chamber with energy E at the given angle θ and at different φ. It was determined (Fig. 3) as the area of the intersection of the circle O_1 of diameter 10 cm with one of the similar circles O_2 or O_3, whose centers, together with the center of O_1, form an isosceles triangle with legs R and apex angle 2θ (shaded part of Fig. 3). The second correction coefficient K_s was computed as the ratio of the area of the entire layer to the size of ΔS.

As a result of the introduction of the correction coefficients $K = K_\varphi K_s$ for each case, we obtained energy spectra for the π^-- and π^+-mesons, formed in the emulsion (which are shown in Figs.

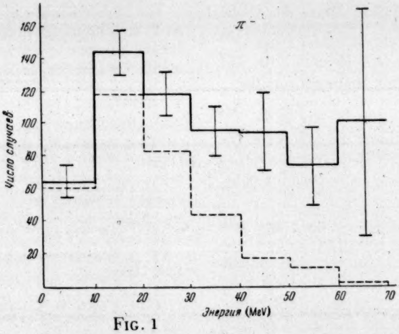

FIG. 1

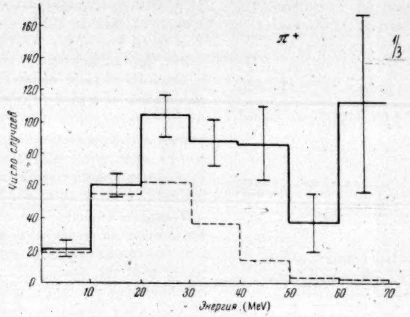

FIG. 2

1 and 2, respectively by the solid line histograms). It appears that the statistical sampling is too small to make a definitive judgement on the spectra; however, it seems possible to us to note a certain observed difference in the spectra of π^- - and π^+ - mesons.

First, the initial part of the π^--meson spectrum is displaced to the left, relative to the corresponding part of the π^+-meson spectrum, by ~ 10 mev. This shift can be explained by the Coulomb interaction of π-mesons with the nuclei from which they emerge, in the case of the magnitude of the mean Coulomb barrier of the the nuclei being of the order of several mev.

Second, the spectrum of the π^--mesons, in contrast to that of the π^+-mesons, is evidently not monotonic: in the region 10-30 mev a small maximum is observed. If this maximum actually exists, then it is possible to explain it by the decay of Λ^0-particles, absorbed by the same nuclei in which they are formed. The position of the maximum does not contradict this hypothesis. Actually, for π-meson decay of a Λ^0-particle connected with the original nucleus, an energy

$$Q = 37 \text{ mev} - (B_{\Lambda^0} - B_n) - (m_n - m_p) - (M_2 - M_1),$$

will be produced, where M_1, m_n and m_p are the masses (in mev) corresponding to the original stable Λ^0-nucleus, neutron and proton, M_2 is the total mass of all the products of the decay of the Λ^0-nucleus, other than the π-meson, B_{Λ^0} is the binding energy of the Λ^0-particle in the Λ^0nucleus, and B_n is the binding energy of the neutron in the corresponding stable nucleus.

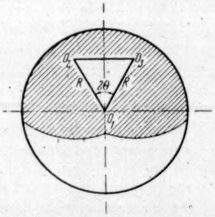

FIG. 3

As the distribution of primary stars according to the number of "black" or "gray" rays (the average number of "black" and "gray" rays is equal to 10) shows, the π-mesons are formed principally on the heavy nuclei of the photographic emulsion--Ag and Br. In such nuclei, the Λ^0-particle can appear to be at an energy level that is much lower than the lowest bound neutron, since its levels are all free. As a consequence, the quantity $(B_{\Lambda^0} - B_n)$ is greater than zero, the latter leading to a reduction of Q. Moreover, the π^--meson, in leaving the nucleus, will be retarded by the Coulomb field and lose several million electron volts. All this could lead to a shift of the maximum in the interval from 30 to 10 mev. The size of the maximum does not contradict (with accuracy of one order) data on the quantity of excited fragments which decay with the emission of π^--mesons.

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Disintegration of Beryllium and Carbon Nuclei as a Result of π^-–Meson Capture

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TO obtain information on the disintegration of light nuclei in stopping π^-– mesons, an investigation was carried out on σ-stars in beryllium and carbon. Powdered beryllium and carbon (crystal size $5-10\,\mu$) were introduced into the emulsion to the amount $4\,\mu$ gm per cm.^{3} The method developed for introducing the powders guaranteed a uniform distribution of the crystals over the emulsion layer. A plate of total thickness 2 – 2.5 mm was prepared from emulsion layers of diameter 42 mm,

The powder of the material under investigation was contained in only 3 – 4 layers out of 10. The plates were exposed to a beam of slow π^-–mesons in the synchrocyclotron of the Institute for Nuclear Problems of the Academy of Sciences, USSR. The method of processing the layers[1] differed little from the procedures used previously for the processing of emulsions in layers.[2] (The work was completed in 1953.[3]) A layer with beryllium was fixed at low temperature $\beta \sim 5°$) to preserve the blackening.

Each fixed case which resembled the nuclear absorption of a pion in the crystal was subjected to processing under a microscope. After all the necessary measurements had been completed, the tracks were again subjected to examination. For improvement of the visibility of the layer, the emulsion below the crystal was removed with chamois and wetted in a solution of methyl alcohol in ethyl. We succeeded in reducing the layer to several microns thickness. The emulsion was soaked in water at the place under investigation and was examined (in swollen condition) under the microscope. Only by means of such careful testing could it be reliably established whether the pion was absorbed in the crystal.

As a result of the investigation of 4 cm^3 of the emulsion, 7 cases of pion absorption in carbon and 12 in beryllium were observed. In the analysis of the tracks, we used a method of counting the grains on some plates and the δ -electrons on the others as functions of the remaining range of the particles. Singly-charged particles differed from doubly-charged ones in the remaining range by more than 50-100 μ. Information on the particles which arise in the nuclear disintegrations of carbon and beryllium are shown in Tables 1 and 2, respectively. The symbols H, p, d, T, α and f denote, respectively, singly-charged particles, protons, deuterons, tritons, doubly-charged particles and particles with short tracks, which could not be identified.

The average number of prongs in the stars in beryllium amounted to 1.15 ± 0.23. In nearly one fourth the cases (28 ± 12 %), the absorption of the pion did not lead to the release of charged particles. In each disintegration of a Be

TABLE 1

Number of prongs in the star	Nature of particles	Energy of particles in mev (range in μ)
0	—	—
1	p	14
1	f	(46)
2	p, α	10,10
2	α, f	12, (14)
3	p, f, f	19, (70), (12)
4	H. H. f. f	5.5. 4.7, (24), (37)

TABLE 2

Single-pronged stars		Double-pronged stars		Triple-pronged stars	
particle symbol	energy in mev (range in μ)	particle symbol	energy in mev (range in μ)	particle symbol	energy in mev (range in μ)
p	>21	α	11	p	19
p	11	p	9.5	p	16
H	6	α	11		0.9—1.2
α	13	H (d,T)	4.5—5.5	W	W
f	(17)	α	9	H	6
f	(8.5)	H (d,T)	8.5—9.5	H	1
f	(5.5)	f	(22)	H	3- -2
f	(3)	H	2—3	—	—
—	—	f	(15)	—	—
—	—	f	(4.5)	—	—

nucleus, there arose, on the average, not more than one single charged particle, the mean energy of the charged particles being 5-10 mev.

In such a light nucleus as Be, the particles which receive the energy in the initial act in the distribution of the rest pion between the nucleons, cannot undergo a large number of collisions with the rest of the nucleus. Consequently, in the energy spectrum of the particles emitted in the disintegration of the nucleus, one can make a direct judgement on the spectrum of primary particles.

Among the particles which are emitted from the star in Be and C, there are absent tritons with energy > 10 mev. Consequently, fast tritons are not observed in the primary acts in a significant number of cases. The data obtained do not agree with the model in which the pion is absorbed by a system similar to He^4, as a result of which a neutron is formed with energy ~ 95 mev and a triton with energy ~ mev.[4] This model also contradicts the fact that absorption of the pion by beryllium fairly frequently fails to result in the emission of charged particles.

A differeet model was proposed by Menon,[5] in which the pion was absorbed by a group of He^4 with a subsequent uniform distribution of energy among the four nucleons (three neutrons and a proton). From the point of view of this model, the absence in σ-stars in Be and C of a large number of tracks of protons with energy 20-40 mev remains unexplained (mean energy of the emitted protons does not exceed 10 mev).

The energy released in the emission of charged particles in the disintegraiion of a Be nucleus is equal on the average to 10-15 mev. Almost ten times more energy is released in the emission of neutral particles than in the emission of charged particles.

The resultant experimental information on σ-stars in Be and C testifies to the fact that 1 or 2 neutrons receive a large part of the energy of the rest pion. In such a light nucleus as Be, they rarely undergo collisions and thus retain an appreciable part of the energy without transmitting it to charged particles.

In conclusion, the authors thank I. I. Gurevich for his valued advice, A. P. Mishakov for his help in the microscopic examination, and D. M. Samoilovich for preparation of the emulsions.

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The Disintegration and Mass Difference of Heavy Neutral Mesons

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IN the researches of Pais, Gell-Mann and Piccioni[1,2] there were forecast very interesting characteristics of the behavior of heavy neutral mesons θ which are formed in a pair with Λ-particles, for example, by the reaction $\pi^- + p = \Lambda + \theta$. Along with θ, there ought to exist antiparticles $\bar\theta$; in this case, only the $\bar\theta$ and not the θ, are capable of bringing about the creation of Λ-particles in the interaction with nucleons through the reaction $\bar\theta + N = \Lambda$ (the momentum and energy can be given off to a pion or to the nucleus, into whose formation the nucleon enters). Strict laws of conservation of electric charge and of the number of heavy particles ("the nuclear charge")[3] do not forbid the interconversion $\theta \leftrightarrows \bar\theta$.

In the scheme of Gell-Mann[4], this conversion cannot be completed quickly under the action of a strong interaction. If the conversion $\theta \leftrightarrows \bar\theta$ could take place quickly, then the process

$$N + N = \Lambda + \theta + N = \Lambda + \bar\theta + N = \Lambda + \Lambda$$

would be possible according to a scheme with the virtual formation of θ, with a threshold much lower in comparison with the process of creation $N + N = N + \Lambda + \theta$. Experiment shows that the process $N + N \to \Lambda + \bar\Lambda$ is not realized.[5]

As a consequence of the invariance of the laws of nature relative to charge conjugation (the operator P) the eigenstates of p have a definite mass and a definite period of decay in a vacuum, symmetric

$$\theta_s = (\theta + \bar\theta)/\sqrt{2}, \qquad P\theta_s = +\theta_s$$

and antisymmetric

$$\theta_a = (\theta - \bar\theta)/\sqrt{2}, \qquad P\theta_a = -\theta_a.$$

The creation of θ must be regarded as the creation of a mixture of particles θ_s and θ_a, so related that a linear combination of the wave funciions of θ_s and θ_a describes θ at this particular moment. Later ,in flight, as a consequence of the difference in the masses of θ_s and θ_a, their phase relation changes; as a consequence of the different decay times of θ_s and θ_a, their amplitude ratio changes. As a result, at a certain distance from the point of creation of θ, the linear combination of θ_s and θ_a no longer contains only θ but also $\bar\theta$. The appearance of $\bar\theta$ in the beam could be discovered by the nuclear interaction $\bar\theta + N = \Lambda$. The quantity of $\bar\theta$ changes with distance according to a decaying sinusoid, whose period depends on the mass difference of θ_s and θ_a.

In the present note it is observed that a similar periodicity ought to be observed in the decay $\theta \to \mu + \pi + \nu$ and also considerations are made on the order of magnitude of the mass difference of θ_s and θ_a.

The decay of θ into μ, π, ν was noted by Thompson.[6] The constants of interaction, which govern the decay of θ into $\mu^+\pi^-\nu$ (g_1) and into $\mu^-\pi^+\nu$ (g_2) did not have to be identical. Carrying out charge conjugation, we find that the decay of $\bar\theta$ into μ^+ is characterized by the constant g_2 and into μ^- by the constant g. Here θ_s decays with constant $(g_1 + g_2)/\sqrt{2}$, giving μ^+ and μ^- with equal probability, and θ_a decays with constant $(g_1 - g_2)/\sqrt{2}$, also giving μ^+ and μ^- with equal probability. However, the ratio of the phase of μ^+ to the phase of μ^- in the superposition of states, which is formed upon the decay of θ_a, has a sign opposite to the ratio of the phases for the decay of θ_s into μ^+ and μ^-. Therefore, in the beam of θ-particles (which we ought to regard as a mixture of θ_s and θ_a), the ratio of the probability of decay with formation of μ^+ or μ^- oscillates in dependence on the ratio of the amplitudes and phases of θ_s and θ_a. With the passage of time, the quantity $\mu\pm$ changes in proportion to

$$|(g_1 + g_2)\exp(im_s - w_s)t \pm (g_1 - g_2)\exp(im_a - w_a)t|^2,$$

where m_s, m_a are the masses, w_s, w_q the probabilities of decay (total) of the particles θ_s and θ_a. Thus, even in this process one must expect damped oscillations of the ratio μ^+/μ^- with a period which depends on the mass difference, similarly to the oscillation of the nuclear interaction noted in Ref. 2.

The difference in masses of θ_s and θ_a depends on the possibility of the interconversion $\theta \leftrightarrows \bar\theta$. Such a conversion, accompanied by change in strangeness by two units, is a process of much higher order, and is much weaker in comparison with the decay $\theta \to \pi^+ + \pi^-$. At first glance, then, it follows that the mass difference ought to be significantly smaller than the probability of decay (in the system $\hbar = c = 1$). Actually, the probability of decay is proportional to the square of the matrix element for the process with change in strangeness $\Delta S = 1$, i.e., proportional to g^2, where g is the coupling constant. At the same time, the difference in masses is proportional to the first power of the matrix element for the transtion $\theta \rightleftarrows \bar{\theta}$, with change in strangeness $\Delta S = 2$. Actually, if we write symbolically

$$-i\partial\theta/\partial t = E_0\theta + f\bar{\theta},$$

$$-i\,\partial\bar{\theta}/\partial t = E_0\bar{\theta} + f\theta,$$

we get $E_s = E_0 + f$, $E_a = E_0 - f$; since we are dealing with the excitation of a created system, then the E_0 for θ and $\bar{\theta}$ are identically equal. According to considerations on the magnitude of ΔS for the conversion $\theta \to \bar{\theta}$, we can expect that $f \sim g^2$, so that $\Delta m \sim \hbar/\tau c^2$ (as was assumed by Pais and Piccioni), where τ is the period of decay $\sim 1.5 \times 10^{-10}$; numerically, we obtain $\Delta m = 10^{-11} m_e$, where m_e is the mass of the electron.

Another approach to the problem of the difference of the masses of θ_s and θ_a is based on the direct consideration of that coupling of the θ-particles with other fields, which determines their decay. If we assume that the spin of θ is zero, then the pair π^+, π^- which are generated in the decay, is found in a state which is even relaive to charge conjugation; only the decay $\theta_s = \pi^+ + \pi^-$ is possible, not the decay of θ_a. The decay of θ_s gives information on the coupling of the field of θ_s with the field of the pions*. According to the usual formulas of perturbation theory, such a coupling must produce a displacement of the level, i. e., a change of the energy of θ_s, along with the decay which produces a broadening of the level. We write down side by side the energy shift and the decay probability:

$$w = 2\pi M^2(E)\frac{dN}{dE}\bigg|_{E=E_0}, \quad \Delta E = \int_0^\infty \frac{M^2(E)}{E_0 - E}\frac{dN}{dE}\,dE,$$

$M(E)$ is the matrix element of the transition from the state θ_s into the state of continuous spectrum, i.e., into the pair π^+, π^- with energy E; dN/dE is the density of levels of the continuous spectrum. The integral in ΔE is taken in the sense of the principal value; therefore the immediate neighborhood of E_0 does not determine its values. In order that the integral converge, it is necessary that the falling off of $M(E)$ be sufficiently rapid for $E \to \infty$. To compute ΔE, not knowing the properties of $M(E)$ is impossible. From the expressions that have been given, it is evident only that ΔE is of the same order of magnitude as w; dimensional quantities – the coupling constants, etc. —enter into ΔE and w in the same degrees.

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* Decay into muons, which is less probable, is not considered here.

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Angular Correlation in Cascade Decay of Hyperons

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STUDY of the angular distribution of the decay products of hyperons can give evidence on the spin of the latter. The distribution of pions in the cascade decay $\Xi \to \Lambda \to p$ was considered in Ref. 1. Here we consider the cascade decay $\Sigma^0 \to \Lambda^0 + \gamma \to p + \pi^- + \gamma$. The wave function pertaining to the motion of a proton and a $\pi-$ particle has the following form:

$$\psi_{jm_j}(\mathbf{p}, \sigma) = C^{jm_j}_{lm\,1/2\,\sigma}\, Y_{lm}\left(\frac{\mathbf{p}}{p}\right)\chi_\sigma,$$

where $\mathbf{p} = \mathbf{p}_\pi - \mathbf{p}_p$ is the momentum of the relative motion, j is the spin of the Λ-particle, m_j = its projection, ψ_σ = the spin wave function of the proton, Y_{lm} is the spherical harmonic, $C^{\cdot\cdot}_{\cdots\cdot}$ are the coefficients of vector addition. Since the spin of the proton is 1/2 and the internal parity of the pion is odd, then l is uniquely determined by the spin and parity of the Λ-particle g : $l = j \pm 1/2$; $(-1)^l = (-1)^{g+1}$. $m = m_j - \sigma$ is determined in the same fashion.

The wave function of the entire system in the final state is

$$\Psi_{JM_J}(\mathbf{P}, \mathbf{p}, \sigma, m_j, \mu) = \sum_{L\lambda} \rho_{L\lambda} C^{JMJ}_{LMjm_j} \psi_{jm_j}(\mathbf{p}, \sigma)\, \Phi^{\lambda}_{LM}\left(\frac{\mathbf{P}}{P}, \mu\right),$$

where $\mathbf{P} = \mathbf{p}_\lambda - \mathbf{p}_\Lambda$ is the momentum of the relative motion of the photon and the Λ-particle (in the system in which the Σ-particle $\mathbf{P} = 2\,\mathbf{p}_\gamma$ is at rest), J = spin of the Σ-particle, M_J its projection, μ = the spin (polarization) variable of the photon, λ, L, M are the parity, moment, and projection of the momentum relative to the motion of the photon and the Λ-particle. For given spins and parity of the Σ- and Λ-particle, L and λ can be different in the general case; the values of L for different λ ought to differ in parity. Since m_j and M_J are given, then $M = M_J - m_j$. $\Phi^{(\lambda)}_{LM}$ is a vector spherical harmonic; its three values, corresponding to the three values of μ, are appropriately combined in the vector $Y^{(\lambda)\,2}_{LM}$; for given L and M, two vector spherical harmonics are possible ($\lambda = 0, 1$), which differ in parity. Similarly, we denote the wave function $\Psi_{JM}(\ldots\mu) \rightarrow \Psi_{JM}(\ldots)$. In such a fashion,

$$\Psi_{JM}(\mathbf{P}, \mathbf{p}; \sigma, m_j) = \sum_{L\lambda} \rho_{L\lambda} C^{JM_J}_{LMjm_j} C^{jm_j}_{lm\,{}^1/_2\,\sigma} \chi_\sigma \mathbf{Y}^{(\lambda)}_{LM}\left(\frac{\mathbf{P}}{P}\right) Y_{lm}\left(\frac{\mathbf{p}}{p}\right). \tag{1}$$

The angular distribution of the particles is determined by the function

$$I(\theta) = \sum_{\sigma m_j M_J} |\Psi_{JM}|^2.$$

Here $\theta = \mathbf{P}\cdot\mathbf{p}/Pp$; the Σ-particle is assumed to be unpolarized, in view of which summation is carried out over the M_J . It is appropriate to direct the Z axis along P . Then it is easy to get (from the properties of the spherical harmonics)

$$\mathbf{Y}^{(0)}_{LM}\mathbf{Y}^{(1)*}_{LM} = 0, \quad \mathbf{Y}^{(0)}_{LM}\mathbf{Y}^{(0)}_{L'M} = \mathbf{Y}^{(1)}_{LM}\mathbf{Y}^{(1)*}_{L'M} = \frac{1}{4\pi}\sqrt{(2L+1)(2L'+1)}\,\delta_{M,\pm 1}.$$

In this way the spherical vectors with different λ do not interfere, and in squaring (1), the terms which contain the index L will be multiplied by terms containing $L' + 2k$, where k is an integer. Making use of this, we find

$$I(\theta) = \sum_{L} \sum_{L'=L+2n} \sum_{M_J m_j \sigma} V_{L\lambda} V_{L'\lambda'} \times C^{JM_J}_{LMjm_j} C^{JM_J}_{L'Mjm_j} (C^{jm_j}_{lm\,{}^1/_2\,\sigma})^2 \,|\, Y_{lm}(\theta)\,|^2.$$

Making use of the well-known formulas for the decomposition of the square of the spherical harmonics in Legendre polynomials P_n and the properties of the sum of the coefficients of spherical harmonics[2,3] , we obtain

$$I(\theta) = \sum_{n=0}^{N} A_n P_{2n}(\cos\theta), \tag{2}$$

$$A_n = C^{2n0}_{l0l0}\, W(jjll; 2n\,{}^1/_2) \times \sum_{L} \sum_{L'=L+2k} V_{L\lambda} V_{L'\lambda} C^{2n0}_{L1L'-1}\, W(jjLL'; 2n\tilde{J}),$$

$$N = j - {}^1/_2; \qquad V_{L\lambda} = \sqrt{(2L+1)/4\pi}\,\rho_{L\lambda},$$

W = Racah coefficients.

Below are given the angular distribution $I(\theta)$ for $j = 3/2$ (for $j = 3/2$, the distribution will be spherically symmetric) for different values of J (the coefficients a , α , etc. are determined by the decay mechanism and are expressed by V_{LI} . If only the smallest L plays a role, then the two first terms remain in the formula) :

$$J = {}^1/_2 : I = 1 - 0{,}6 \cos^2\theta + a\,(1 + \cos^2\theta);$$

$$J = {}^3/_2 : I = 1 + 0{,}75 \cos^2\theta$$

$$+ \alpha\,(0{,}4 - 1{,}2\cos^2\theta) + \alpha^2\,(0{,}37 + 0{,}48\cos^2\theta) + b;$$

$$I = {}^5/_2 : I = 1 - 0{,}45 \cos^2\theta$$

$$+ \beta\,(0{,}4 - 1{,}2\cos^2\theta) + \beta^2\,(0{,}33 + 0{,}43\cos^2\theta)$$

$$+ c\,[(1 - 0{,}14\cos^2\theta) + \gamma\,(0{,}5 - 1{,}5\cos^2\theta)$$

$$+ \gamma^2\,(0{,}44 - 0{,}1\cos^2\theta)];$$

$$J = {}^7/_2 : I = 1 - 0{,}6 \cos^2\theta$$

$$+ \delta\,(0{,}5 - 1{,}36\cos^2\theta) + \delta^2\,(0{,}26 + 0{,}48\cos^2\theta)$$

$$+ d\,[1 + 0{,}23\cos^2\theta + \varepsilon\,(0{,}7 - 2{,}1\cos^2\theta)$$

$$+ \varepsilon^2\,(0{,}5 + 0{,}01\cos^2\theta)]$$

Note added in proof. The problem of the correlations in the decay of a Σ-particle have also been considered in the recent publication of Gatto.[4]

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Gamma Resonances in Reactions of Proton Capture by Silicon Isotopes

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THE reactions Si (p, γ) P were first studied[1] by means of the yield of γ-radiation from thick targets for the proton energy interval from 0.3 to 0.55 mev. Somewhat later,[2] Tangen reported the results of more detailed investigations of this reaction in the same proton energy interval. He found resonances for E_p = 326 and 414 kev, which he attributed to the reaction Si^{29} (p, γ) P^{30}, since he observed them by the activity of P^{30}, and also resoances for E_p = 367 and 499 kev, which he attributed to the reaction Si^{30} (p, γ) P^{31}. Recently, Milani, Cooper and Harris observed[3] γ-resonances in the reaction Si^{29} (p, γ) P^{30} at approximately E_p = 696, 727, 917, 956 kev. They carried out their investigations both on thin and thick targets of Si^{29}.

The integral excitation function of the Si (p, γ) P reactions was measured on the 4 mev electrostatic generator of the Physico-technical Institute of the Acadmey of Sciences, USSR, in the proton energy interval from 500 to 2600 kev. A thick target with the natural mixture of the istopes of silicon (Si^{28}, 92.28 %; Si^{29}, 4.67%; Si^{30}, 3.05%) was prepared from a pure (99.98 %) single crystal of silicon, which has been obtained by vacuum distillation. To test the various impurities in the silicon crystal, investigations were carried out on thick and thin targets, prepared from silicon which has served as the initial material in the preparation of the single crystal. The tests showed that Al, Fe and Pb, which appeared as impurities in the original material, had been removed. The energy of the accelerated protons was measured by an electrostatic analyzer with accuracy to within 0.05 %; the imhomogeneity in the energy of the proton beam amounted to 0.8 kev for the entire interval of proton energies, the γ-rays were detected by a copper counter. The current at the target was measured by a current integrator of the Watt type. During the measurements, the target temperature was maintained at the level 300-500° to avoid weakening of the carbon film; vapors of the oil of the diffusion pumps were carefully frozen out with a liquid nitrogen trap. In the measurments of the integral excitation function, the position and width of 26 new γ-resonances were determined. New γ-resonances were found for E_p = 619.5, 717, 753, 775, 800, 831, 895, 940, 980, 1520, 1618, 1635, 1647, 1663, 1680, 1699, 1774, 1810, 1849, 1879, 2520, 2543, 2553, 2557.5, 2570 and 2575 kev. The experimental widths were observed to lie within the limits 0.8 to 8 kev.

For identification of the reactions corresponding to the resonances, the differential excitation function was measured on thin targets by the yield of positron activity of P^{29} and P^{30}. At present, this work has been carried out only to proton energies of E_p = 1000 kev. Not a singleresonance was foundfor P^{29}. Also, no new resonances have been found for P^{30} up to 1000 kev. The positions of the two γ-resonances mentioned earlier[3] for the reaction Si^{29} (p, γ) P^{30} for E_p = 917 and 956 kev were determined more accurately by us; according to our measurements, E_p = 916.5 ± 0.5 and 956 ± 1 kev. The lower accuracy of the determination of the resonance for 956 kev was caused by its weak intensity. The first 9 of the new γ-resonances from 619.5 to 980 kev can be attributed to the reaction $Si^{30}(p, \gamma)P^{31}$, since they are not connected with appreciable positron activity. We note that the effectiveness of detection of charged particles is many times greater than the effectiveness of detection of γ-quanta.

The identification of the reactions for the remaining 17 γ-resonances that we have discovered will be carried out at a later time.

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Isotopic Invariance and "Strange" Particles

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THE basic peculiarity of "strange" particles— heavy mesons and hyperons— is that two different types of interaction connect them with the usual strongly interacting particles (π-mesons and nucleons) : strong and weak. The formation of strange particles from ordinary is a fast process (characteristic time $\sim 10^{-23}$ to 10^{-24} sec); decay of "strange" particles to ordinary is a slow process (characteristic time $\sim 10^{-8}$ to 10^{-11} sec). This, and a whole series of other peculiarities of "strange" particles can be explained if, following Gell-Mann,[1] we consider the concept of isotopic spin for these particles and assume that the fast processes with some of these particles is isotopically invariant, and in the slow processes, not only the isotopic spin T changes, but also its projection T_3.

The charge of the particles Q is shown to be connected with the projection of the isotopic spin T_3 by the relation

$$Q = T_3 + (n/2) + (s/2),$$

where n is the number of baryons minus the number of antibaryons, and s is the quantum number "strangeness," introduced by Gell-Mann. A consistent explanation of the properties of elementary particles demands that the following isotopic spin and "strangeness" s be assigned to them:

	π	K	$\tilde{K}$	N	$\tilde{N}$	Λ	$\tilde{\Lambda}$	Σ	$\tilde{\Sigma}$	Ξ	$\tilde{\Xi}$
s	0	−1	1	0	0	+1	−1	+1	−1	2	−2
T	1	1/2	1/2	1/2	1/2	0	0	1	1	1/2	1/2

(Along with particles, we have considered the corresponding antiparticles, which are designated by the tilde). One of the most direct ways of verifying the correctness of the hypothesis on the isotopic invariance of fast processes with some of the "strange" particles is the experimental test of the relation (arising from this hypothesis) between the cross sections of reactions which differ only in the charge states of the participating particles. We consider three types of such reactions.

a) *Reactions in which* 4 *particles with* $T = 1/2$ *take part.* For example, the scattering of K-mesons by nucleons $K + N \to K + N$ or, in more detail,

1) $K^+ + p \to K^+ + p,$
2) $K^+ + n \to K^+ + n,$
3) $K^+ + n \to K^0 + p,$
4) $K^0 + n \to K^0 + n.$
5) $K^0 + p \to K^0 + p$
6) $K^0 + p \to K^+ + n.$

From charge symmetry it follows that

$$\sigma_1 = \sigma_4, \ \sigma_2 = \sigma_5, \ \sigma_3 = \sigma_6.$$

Consideration of charge invariance in this case does not give additional equations; however, if we consider that there is a relation among the amplitudes a ($\sigma = |a|^2$: $a_1 = a_2 + a_3$, then the inequality $\sigma_1^{1/2} \leqq \sigma_2^{1/2} + \sigma_3^{1/2}$ and its cyclic permutations can easily be established.

b) *Reactions in which one particle with* $T = 1$ *and two particles with* $T = 1/2$ *take part.* (The presence in the reaction of any number of particles with $T = 0$ is not important for us.) For example, formation of a Λ-particle and a K-meson in the reaction $\pi + N \to \Lambda + k$, or, more generally:

1) $\pi^+ + n \rightarrow \Lambda^0 + K^+$,
2) $\pi^0 + n \rightarrow \Lambda^0 + K^0$,
3) $\pi^- + p \rightarrow \Lambda^0 + K^0$,
4) $\pi^0 + p \rightarrow \Lambda^0 + K^+$.

It follows from charge symmetry that $\sigma_1 = \sigma_3$, $\sigma_2 = \sigma_4$. A relation which is determined by the charge invariance can easily be obtained by the method assumed by Shmushkevich.[2,3] We imagine a nucleonic target which contains an equal number of protons and neutrons, on which falls a beam of pions which contains equal numbers of π^+, π^- and π° -mesons. Since the interaction in this reaction is isotopically invariant, but the target and the beam are not isotopically polarized, then the number of π^+ (π^-) -mesons which are knocked out of the beam must be equal to the number of π^+– mesons. The π^+ mesons are absorbed in reaction 1, π° in reactions 2 and 4. This gives, at once, $\sigma_1 = \sigma_2 + \sigma_4$ or $\sigma_1 = 2\sigma_2$.

c) *Reactions in which two particles with* $T=1$ *and two particles with* $T=2$ *take part.* For example, formation of a -Σ-particle and a K-meson in the reaction $\pi + N \rightarrow \Sigma + K$:

1) $\pi^+ + p \rightarrow \Sigma^+ + K^+$,
2) $\pi^0 + p \rightarrow \Sigma^0 + K^+$,
3) $\pi^0 + p \rightarrow \Sigma^+ + K^0$,
4) $\pi^- + p \rightarrow \Sigma^- + K^+$,
5) $\pi^- + p \rightarrow \Sigma^0 + K^0$,
6) $\pi^- + n \rightarrow \Sigma^- + K^0$,
7) $\pi^0 + n \rightarrow \Sigma^0 + K^+$,
8) $\pi^0 + n \rightarrow \Sigma^- + K^+$,
9) $\pi^+ + n \rightarrow \Sigma^+ + K^0$,
10) $\pi^+ + n \rightarrow \Sigma^0 + K^+$.

It follows from charge symmetry that

$$\sigma_1 = \sigma_6, \quad \sigma_2 = \sigma_7, \quad \sigma_3 = \sigma_8, \quad \sigma_4 = \sigma_9, \quad \sigma_5 = \sigma_{10}.$$

Again let us consider an isotopically nonpolarized K-meson beam and a nucleonic target. As in the previous case, the number knocked from the beam of $\pi^+(\pi^-)$ – mesons and π° –mesons ought to be equal:

$$\sigma_1 + \sigma_4 + \sigma_5 = 2(\sigma_2 + \sigma_3).$$

Moreover, the number of Σ^+ and Σ° -particles that are formed ought to be equal: $\sigma_1 + \sigma_3 + \sigma_4 = 2(\sigma_2 + \sigma_5)$. From the latter two equations, we obtain

$$\sigma_3 = \sigma_5, \quad \sigma_1 + \sigma_4 = 2\sigma_2 + \sigma_5.$$

Similar relations are obtained for the following reactions, each of which pertains to one of the types: a), b) or c):

REACTION OF FORMATION OF HEAVY MESONS AND HYPERONS:

1) $\pi + N \rightarrow \Lambda + \pi + K$ (c),
2) $\pi + d \rightarrow \Lambda + N + K$ (b),

3) $\pi + d \rightarrow \Sigma + N + K$ (c),
4) $\pi + d \rightarrow \Lambda + \pi + N + K$ (c).

REACTIONS OF CAPTURE OF K-MESONS:

5) $\tilde{K} + N \rightarrow \Lambda + \pi$ (b),
6) $\tilde{K} + N \rightarrow \Sigma + \pi$ (c),
7) $\tilde{K} + d \rightarrow \Sigma + N$ (b),

8) $\tilde{K} + d \rightarrow \Lambda + N + \pi$ (b),
9) $\tilde{K} + d \rightarrow \Sigma + N + \pi$ (c),

REACTIONS OF INTERACTION OF Σ-HYPERONS WITH NUCLEONS.

10) $\Sigma + N \rightarrow \Sigma + N$ (c),

11) $\Sigma + N \rightarrow \Lambda + N$ (b),

12) $\Sigma + N \rightarrow \Lambda + N + \pi$ (c).

REACTIONS OF CREATION OF ANTIHYPERONS AND K-MESONS:

13) $\pi + N \rightarrow N + \Lambda + \tilde{\Lambda}$ (b),
14) $\pi + N \rightarrow N + \Lambda + \overline{\Sigma}$ (c),
15) $N + N \rightarrow d + \Lambda + \tilde{\Sigma}$ (c),

16) $N + N \rightarrow d + \Sigma + \tilde{\Sigma}$ (b),
17) $N + N \rightarrow d + K + \tilde{K}$ (a).

REACTIONS OF ANNIHILATION OF ANTIHYPERONS:

18) $\tilde{\Lambda} + N \rightarrow K + \pi$ (b),

19) $\tilde{\Sigma} + N \rightarrow K + \pi$ (c),

20) $\tilde{\Xi} + N \rightarrow K + K$ (a),

Some of these relations were obtained earlier.[1,4] It should be emphasized that all the relations considered connect the differential cross sections of the corresponding reactions.

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